Comments on "A generalization of Fisher's exact test in PxQ contingency tables using more concordant relations"

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COMMUN. STATIST.-SIMULA., 16(1), 301-306 (1987)

COMMENTS ON "A GENERALIZATION OF FISHER'S EXACT TEST IN PXQ CONTINGENCY TABLES USING MORE CONCORDANT RELATIONS" Communications In Statistics, Volume B 14, 633-645.

The purpose of this note is to criticize Nguyen (1985) for his account of the literature on the generalization of Fisher's exact test and to point out parallels with existing algorithms of the algorithm proposed by Nguyen. Subsequently we will briefly raise some questions on the methodology proposed by Nguyen.

Nguyen (1985) suggests that all literature on exact testing prior to Nguyen & Sampson (1985) is based on the "more probable" relation or Exact Probability Test (EPT) as a test statistic. This is not correct. Yates (1934 - Pearson's X^2), Lewontin & Felsenstein (1965 - X^2), Agresti & Wackerly (1977 - X^2, Kendall's tau, Kruskal & Goodman's gamma), Klotz (1966 - Wilcoxon), Klotz & Teng (1977 - Kruskall & Wallis' H), Larntz (1978 - X^2, loglikelihood-ratio statistic G^2, Freeman & Tukey statistic), and several others have investigated exact tests with other statistics than the EPT. In fact, Bennett & Nakamura (1963) are incorrectly cited as they investigated both X^2 and G^2, rather than EPT. Also, Freeman & Halton (1951) are incorrectly cited for they generalized Fisher's exact test to $\underline{p}$xq tables and not $\underline{2}$xq tables as stated. And they are even predated by Yates (1934) who extended the test to 2x3 tables.

As is evident from Verbeek & Kroonenberg's (1985) survey of algorithms for just this problem, Nguyen's algorithm is basically similar to that of Agresti & Wackerly (1977) and Verbeek, Kroonenberg, & Kroonenberg (1983). A survey of the methodological literature on exact testing in contingency tables with fixed margins can be found in Verbeek & Kroonenberg (1979).

As an aside we should mention that the usefulness of the newly proposed statistic by Nguyen is not readily apparent. It is difficult to compute, and difficult to interpret. Moreover, no comparisons are made with the many existing statistics and models for ordinal associations (cf. Agresti, 1983, 1984), and no examples or circumstances are given where the new statistic would be applicable or superior. Furthermore, the generalizability of the simulation results with respect to the power are unclear, and, again comparisons with the power of existing methods are lacking.

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RESPONSE from Author

First of all, I would like to thank Kroonenberg and Verbeek for showing some mistakes in refering I made in my paper "A Generalization of Fisher's Exact Test in pxq Contingency Tables using More Concordant Relations". Else, I do not think that I can agree with their opinion about this paper. Based on their Comment, it seems that their generating method is well enough for any kind of statistics we use for generalizing Fisher's exact test. In general, every generating method gives out the set of all non-negative integer matrices having the same row sum and column sum vectors as the observed matrix. The only difference among these methods is in the procedure to find the significance level of the observed matrix. A method of generation is appropriate for a test statistic if in this procedure we need to generate a least number of matrices. In this point, I do not think that it exists one method appropriate for all statistics. And this is the point Kroonenberg and Verbeek do not want to mention in their Comment. This is also a way to improve the effectiveness for the exact test. For good examples see Mehta and Patel (1980, 1983).

The main purpose of my paper is to generalize the Fisher's exact test under restricted alternative, the positive quadrant dependent set of distributions. This is an application of exact test to order restrictions (See Barlow, Bartholomew, Bremner and Brunk (1972). To see how complicated the likelihood ratio statistic in 2xc is, I refer to the paper of Grove (1980). In this paper, we also know how is this statistic in rxc case and a comparison of some test statistics using the Monte Carlo method. The simulation results of Grove give the similar conclusions as in my paper, i.e. no test statistic is superior in all cases of alternative. But the main point here is Kroonenberg and Verbeek do not want to mention about the purpose of their paper

(1983) about generalization the Fisher's exact test with no restriction in the alternative and mine in the restriction to to the PQD set.

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RESPONSE *from Kroonenberg and Verbeek*

In Nguyen's reaction to our Comments he emphasizes that we do not acknowledge that different alternative hypotheses lead to different enumerations to achieve optimal efficiency. We concede that in theory his point of view is correct, but like to point out that in practice one would generally prefer a fast algorithm and implementation that work efficiently for many alternative hypotheses, and can be easily adapted to accomodate other test statistics.

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Received by Editor January, 1986; Final response and retyping October, 1986.

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