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Superfluid helium-3 in cylindrical restricted geometries : a study with low-frequency NMR

Benningshof, O.W.B.

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Appendix A

Appendix

A.1 Pressure dependent quantities and constants of helium-3

Most of the numbers presented in this appendix are taken from W.P. Halperin's ³He Calculator [142]. Their numbers based on the references [143], [144], [145] and [146].

Quantity	Symbol	Value	Units
modified Planck constant	\hbar	$1.0545727 \cdot 10^{-34}$	J s
gyromagnetic ratio ³ He	γ	$20.3801587 \cdot 10^7$	rad Hz T ⁻¹
Boltzmann constant	k_B	$1.380658 \cdot 10^{-23}$	J K ⁻¹
vacuum permeability	μ_0	$4\pi \cdot 10^{-7}$	N A ⁻²
³ He mass	m	$5.008234 \cdot 10^{-27}$	kg
cut off energy	ε_c	$\sim 0.7 k_B$	J
Riemann Zeta function	$\zeta(3)$	1.202	-

P bar	m^*/m	F_0^a	F_1^s	v_F m s^{-1}	T_c mK	P bar	m^*/m	F_0^a	F_1^s	v_F m s^{-1}	T_c mK
0.0	2.80	-0.701	5.40	59.03	0.93	3.5	3.22	-0.720	6.65	53.07	1.34
0.1	2.81	-0.701	5.44	58.83	0.94	3.6	3.23	-0.721	6.68	52.93	1.35
0.2	2.83	-0.702	5.48	58.64	0.96	3.7	3.24	-0.721	6.72	52.78	1.36
0.3	2.84	-0.703	5.52	58.45	0.97	3.8	3.25	-0.722	6.75	52.64	1.37
0.4	2.85	-0.703	5.55	58.26	0.98	3.9	3.26	-0.722	6.78	52.50	1.38
0.5	2.86	-0.704	5.59	58.07	1.00	4.0	3.27	-0.722	6.81	52.36	1.39
0.6	2.88	-0.704	5.63	57.88	1.01	4.1	3.28	-0.723	6.85	52.22	1.40
0.7	2.89	-0.705	5.67	57.69	1.02	4.2	3.29	-0.723	6.88	52.08	1.41
0.8	2.90	-0.706	5.70	57.51	1.04	4.3	3.30	-0.724	6.91	51.94	1.42
0.9	2.91	-0.706	5.74	57.33	1.05	4.4	3.31	-0.724	6.94	51.80	1.42
1.0	2.93	-0.707	5.78	57.14	1.06	4.5	3.32	-0.725	6.97	51.67	1.43
1.1	2.94	-0.707	5.82	56.96	1.07	4.6	3.34	-0.725	7.01	51.54	1.44
1.2	2.95	-0.708	5.85	56.79	1.09	4.7	3.35	-0.726	7.04	51.40	1.45
1.3	2.96	-0.708	5.89	56.61	1.10	4.8	3.36	-0.726	7.07	51.27	1.46
1.4	2.97	-0.709	5.92	56.43	1.11	4.9	3.37	-0.727	7.10	51.14	1.47
1.5	2.99	-0.710	5.96	56.26	1.12	5.0	3.38	-0.727	7.13	51.01	1.48
1.6	3.00	-0.710	6.00	56.09	1.14	5.1	3.39	-0.727	7.16	50.88	1.49
1.7	3.01	-0.711	6.03	55.92	1.15	5.2	3.40	-0.728	7.19	50.75	1.49
1.8	3.02	-0.711	6.07	55.75	1.16	5.3	3.41	-0.728	7.23	50.63	1.50
1.9	3.03	-0.712	6.10	55.58	1.17	5.4	3.42	-0.729	7.26	50.50	1.51
2.0	3.05	-0.712	6.14	55.41	1.18	5.5	3.43	-0.729	7.29	50.38	1.52
2.1	3.06	-0.713	6.17	55.25	1.19	5.6	3.44	-0.729	7.32	50.25	1.53
2.2	3.07	-0.713	6.21	55.08	1.20	5.7	3.45	-0.730	7.35	50.13	1.54
2.3	3.08	-0.714	6.24	54.92	1.22	5.8	3.46	-0.730	7.38	50.01	1.54
2.4	3.09	-0.714	6.28	54.76	1.23	5.9	3.47	-0.731	7.41	49.89	1.55
2.5	3.10	-0.715	6.31	54.60	1.24	6.0	3.48	-0.731	7.44	49.77	1.56
2.6	3.12	-0.716	6.35	54.44	1.25	6.1	3.49	-0.731	7.47	49.65	1.57
2.7	3.13	-0.716	6.38	54.28	1.26	6.2	3.50	-0.732	7.50	49.53	1.58
2.8	3.14	-0.717	6.42	54.13	1.27	6.3	3.51	-0.732	7.53	49.42	1.58
2.9	3.15	-0.717	6.45	53.97	1.28	6.4	3.52	-0.733	7.56	49.30	1.59
3.0	3.16	-0.718	6.48	53.82	1.29	6.5	3.53	-0.733	7.59	49.18	1.60
3.1	3.17	-0.718	6.52	53.67	1.30	6.6	3.54	-0.733	7.62	49.07	1.61
3.2	3.18	-0.719	6.55	53.52	1.31	6.7	3.55	-0.734	7.65	48.96	1.61
3.3	3.19	-0.719	6.58	53.37	1.32	6.8	3.56	-0.734	7.68	48.85	1.62
3.4	3.21	-0.720	6.62	53.22	1.33	6.9	3.57	-0.735	7.70	48.73	1.63

A.2 Gap function

In the case of $^3\text{He-B}$ the gap function is isotropic, see figure 1.3. Since we deal with p-wave pairing, the gap function for this particular phase has the same symmetry as for the s-wave pairing (BCS theory). The behavior of the gap near T_c is given by

$$\Delta(T) \approx 3.06k_B T_c \sqrt{1 - \frac{T}{T_c}}, \quad (\text{A.1})$$

and for $T = 0$ the gap equals

$$\Delta(0) \approx 1.76k_B T_c. \quad (\text{A.2})$$

The gap function is not analytically solvable, however it is numerically solved by Mühlshlegel [147] from which we took the calculated gap values. The Δ^2 as function of temperature is plotted in figure A.1. The temperature dependence is fitted with a 4th order polynomial.

Best fit is obtained with least squares fitting techniques, and is given by

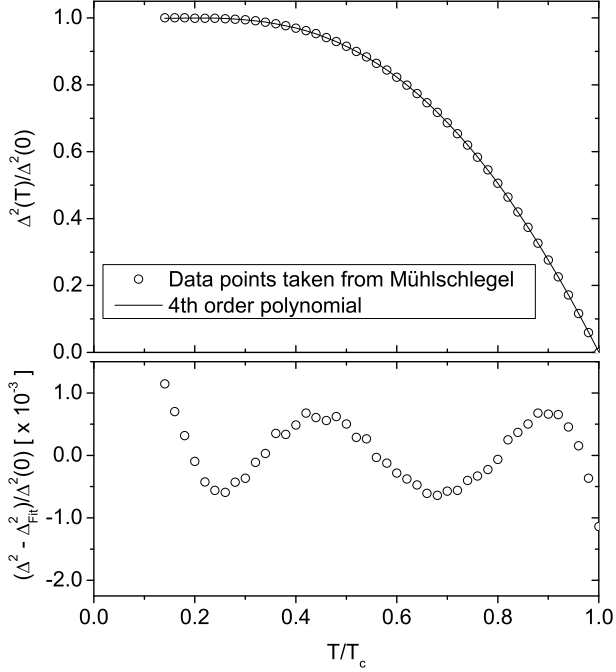


Figure A.1: Upper graph shows the squared gap Δ^2 as function of temperature and fitted by a 4th order polynomial. Lower graph shows the difference between the fit function and the numerical values.

$$\begin{aligned} \frac{\Delta(T)^2}{\Delta(0)^2} = & 1.00459 - 0.15842 (T/T_c) + 1.24905 (T/T_c)^2 \\ & - 3.06569 (T/T_c)^3 + 0.97161 (T/T_c)^4. \end{aligned} \quad (\text{A.3})$$

The differences between the numerical values and fit function is typically less than 0.1 %, this is sufficient for the purpose of this thesis.

A.3 Susceptibility of the B-phase of helium-3

The spin susceptibility in the normal phase, given in equation (1.5), is hardly temperature dependent when $T \ll T_F$. However, if one cools below T_c the spins will form a spin triplet configuration. At the lowest temperatures the populations will be equally distributed over these three states. Consequently, the susceptibility should reduce to $2/3 \chi_N$, as the $m_s = 0$ state (spin up-down state) does not respond to magnetic fields. Simultaneously, the 'screening cloud' effect of the atom in the normal phase decreases

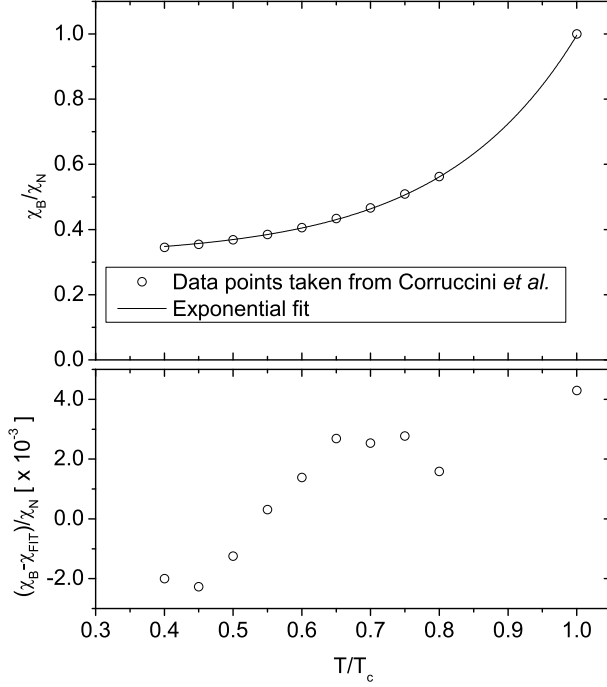


Figure A.2: Upper graph is the normalized susceptibility of the $^3\text{He-B}$ as function of temperature and fitted by an exponential function. Lower graph is the difference between the fit function and the susceptibility χ_B/χ_N .

in the superfluid phase as well [148]. At zero temperature it results in a susceptibility of the B-phase χ_B of approximately $1/3 \chi_N$.

As is shown in literature the temperature dependence of χ_B , for all pressures, show approximately the same universal law as function of T/T_c . In figure A.2 the susceptibility as function of temperature is plotted, data is taken from [149] [38].

The fit, obtained with least squares fitting techniques, is given by

$$\frac{\chi_B(T)}{\chi_N} = 0.00417 \exp\left(5.09372 \frac{T}{T_c}\right) + 0.31601. \quad (\text{A.4})$$

While the data taken from literature are obtained at pressures higher than 18 bar, we assume that the behavior of equation (A.4) will not change significantly for lower pressures. Well it is most likely, that it may differ in detail. However, it should not take off much of the quantitatively description of the susceptibility's temperature dependence.

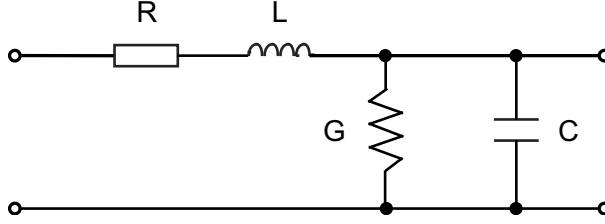


Figure A.3: Schematic representation of the elementary component of a transmission line.

A.4 Transmission Line

The coaxial cable or transmission line can be seen as a repetition of elementary components. Such an elementary component is shown in figure A.3. Here the conductor's resistance is represented by a series resistor R . The self-inductance is represented by a series inductor L . The parasitic capacitance between the shield and core is represented by a shunt capacitor C , and its losses by a shunt resistor G . As the transmission line in principle exists of an infinite series of elements the components are specified per unit length.

Our experiments are performed at relatively low-frequencies (500 kHz), for which we assume that the self-inductance of the transmission line does not play an important role, and will be neglected for low-frequencies. Also the losses due to the conductor's resistance R are considerably larger than in G , so we neglect G , leaving only the capacitor and resistor in the elementary component of the transmission line.

In our set-up the transmission lines are coax cables, of which the outer shield is connected to the ground. The transmission lines are 2 meters long, due to the length of the cryostat, resulting in a relatively high (total) parasitic capacitance, which coincides with low impedance $Z_{C_{Total}}$ between shield and core. In our set-up the transmission lines are connected between a high input impedance (high Q tank circuit) and high output impedances (pre-amplifier), meaning that the current through the transmission line is 'short circuited' by Z_C to the ground.

Figure A.4 illustrates the situation of N elementary components connected between a high in- and output impedance. The total current I_{Total} coming from the input impedance will be divided over the parasitic capacitance of each element. For our transmission lines we claim that $Z_C \gg R$ for each element, and we even assume that the total resistance of the transmission line, $R_{Total} = R_1 + R_2 + R_3 + \dots + R_{N-1} + R_N$, is much smaller than the total impedance $Z_{C_{Total}}$.

In that case the current will be (approximately) equally divided over all the parasitic elements, hence $I_1 = I_2 = I_3 = \dots = I_{N-1} = I_N = I$, and the total parasitic capacitance $C_{P_{Total}}$ equals the (parallel) adding of each parasitic capacitance per element, hence $C_{P_{Total}} = C_1 + C_2 + C_3 + \dots + C_{N-1} + C_N = N \cdot C$. However, one correction with respect to the resistance should be made, the current through R_1 is much more than in R_N . It is important to know the proper effective resistance of the

transmission line for our simulation, because it determines the additional dissipation 'seen' in our tank circuit. We want to approximate the transmission line of figure A.4 with the total impedance of the coax capacitance in series with an effective resistor, hence $Z_{C_{total}} + R_{eff}$.

If we look how much elementary resistors are 'seen' by the current through each elementary component we get:

$$\begin{aligned}
 I_1 &\rightarrow R_1 \\
 I_2 &\rightarrow R_1 + R_2 \\
 I_3 &\rightarrow R_1 + R_2 + R_3 \\
 &\vdots \\
 I_{N-1} &\rightarrow R_1 + R_2 + R_3 + \dots + R_{N-1} \\
 I_N &\rightarrow R_1 + R_2 + R_3 + \dots + R_{N-1} + R_N
 \end{aligned}$$

Using that all elementary components R are equal, $R_1 = R_2 = R_3 = \dots = R_{N-1} = R_N = R$, than in average the resistors 'seen' by the current is:

$$\begin{aligned}
 I_1 + I_2 + I_3 + \dots + I_{N-1} + I_N &\rightarrow \frac{NR + (N-1)R + \dots + 3R + 2R + R}{N} \\
 I_1 + I_2 + I_3 + \dots + I_{N-1} + I_N &\rightarrow \frac{\sum_{i=1}^N iR}{N}
 \end{aligned}$$

Here $\sum_{i=1}^N i = \frac{(N-1+1)(N+1)}{2} = \frac{(N)(N+1)}{2}$, for large N the sum equals $N^2/2$. We assumed that all currents are equal; $I_1 = I_2 = I_3 = \dots = I_{N-1} = I_N = I$, this gives:

$$NI = I_{Total} \rightarrow \frac{NR}{2} = \frac{R_{Total}}{2} \equiv R_{eff} \equiv R_C$$

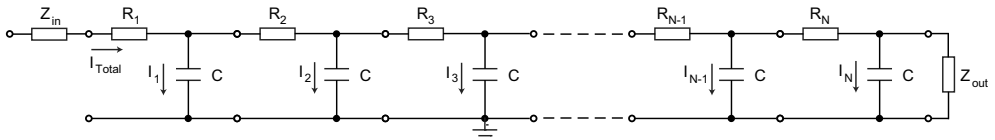


Figure A.4: Transmission line existing of N elementary components. In this representation L and G are neglected, due to relatively low operating frequency (500 kHz). The transmission is connected between a high in- and output impedance, consequently the current uses the parasitic capacitance of the transmission line to reach the ground.

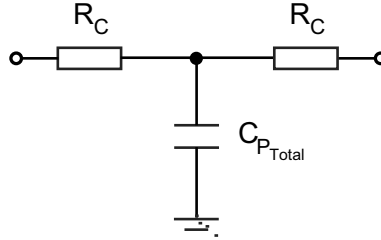


Figure A.5: The transmission line approximate for our conditions as a simple circuit. $C_{P_{Total}}$ is equal to the total parasitic capacitance and R_C is half the value of the total resistance of the transmission line.

For enough elements one can approximate the effective resistance by $R_{Total}/2 \equiv R_C$. So, with our imposed conditions we can approximate the transmission line, as illustrated in figure A.4, by a resistance R_C (half the value of the total resistance in the transmission line) and the total parasitic capacitance $C_{P_{Total}}$. However, one should take into account that the transmission line is symmetric when the current comes from the other direction, so to complete this symmetry the resistance R_C should be placed on both sides of $C_{P_{Total}}$, as illustrated in figure A.5 which is the circuit representing our transmission lines between room temperature and the tank circuit.

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