Superfluid helium-3 in cylindrical restricted geometries : a study with low-frequency NMR

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Chapter 1

Theory

1.1 Normal Fermi liquid

The ^{3}He nuclei exist of two protons and one neutron, and has a total spin $\mathbf{S} = 1/2$. Half quantum numbers makes the particle a fermion, and an ensemble of spin 1/2 particles should (for low enough temperatures) behave as formulated by the Fermi-Dirac statistics. At atmospheric pressure, ^{3}He becomes liquid at a temperature of 3.2 K, which is relatively close to the Fermi temperature T_F ($\sim$ 1 K). It is for this reason that at the moment ^{3}He is liquefied, it lets itself be described by the Fermi Liquid Theory FLT. This section will not derive any of the quantum properties of a Fermi system, for which there are good text books [1], [2], [3]. Here only the relevant FLT relations for this thesis are summarized.

In the ground state of a Fermi system the energy fills up till the Fermi energy ε_F. The energies can be expressed in k-space, where ε_F corresponds to the Fermi wave vector k_F. Concerning N particles in a cubic box one can solve the Schrödinger equation for the ground state, from here the relation k_F is obtained as:

$$k_F = \sqrt[3]{3\pi^2 n}, \tag{1.1}$$

where n is the particle density per unit volume.

The ^{3}He atoms/particles have, because of its extensiveness, hard core repulsion between each other. One can say: It creates a kind of 'screening cloud' for the other atoms. To include this interaction the mass m of the ^{3}He atoms should be replaced by an effective mass m^*. This changes the ^{3}He particles into 'quasi-particles', which is still described with the FLT, but with an effective mass m^*. The ratio between the effective mass m^* and the mass m is given by:

$$\frac{m^*}{m} = 1 + \frac{1}{3}F_1^s, \tag{1.2}$$

where F_1^s is a Landau parameter, which is pressure dependent and listed in the appendix A.1.

The Fermi energy for (quasi)-particles is expressed as,

$$\varepsilon_F = \frac{\hbar^2 k_F^2}{2m^*} = \frac{{p_F}^2}{2m^*} = \frac{m^* {v_F}^2}{2} = k_B T_F, \tag{1.3}$$

where $p_F = \hbar k_F$ is the Fermi momentum, $v_F = p_F/m^*$ the Fermi velocity and T_F the Fermi temperature.

The density of states for both spin components per unit volume per unit energy is given by:

$$N_F = 2N(0) = \frac{m^* k_F}{\pi^2 \hbar^2} = \frac{{p_F}^2}{\pi^2 \hbar^3 v_F} = \frac{3n}{p_F v_F} = \frac{3nm^*}{{p_F}^2}, \tag{1.4}$$

where $N(0)$ is the density of states for one spin component at the Fermi energy.

As shown empirically by experiments, see for example [4], the spin susceptibility χ of ^{3}He is hardly dependent on temperature when $T \ll T_F$. The Pauli paramagnetic spin susceptibility for normal liquid ^{3}He is then

$$\chi_N = \chi_N^0/(1 + F_0^a), \tag{1.5}$$

where F_0^a is a Landau parameter, see appendix A.1, and χ_N^0 the Pauli spin susceptibility for (quasi)-particles with an effective mass m^*.

$$\chi_N^0 = {\mu_0}^2 N_F = \frac{1}{4}\gamma^2 \hbar^2 N_F, \tag{1.6}$$

where μ_0 is the magnetic moment and γ the gyromagnetic ratio of the ^{3}He nucleus. The gyromagnetic ratio γ for ^{3}He is $-20.3801587 \cdot 10^7$ rad Hz T^{-1}.

1.2 Superfluidity in bulk helium-3

The first understanding of pair correlations in an interacting Fermi system was described by Bardeen, Cooper and Schrieffer [6]. They constructed a microscopic theory, which could explain the phenomena of superconductivity. As liquid ^{3}He is a Fermi system, one can expect the same phenomena. However, the ratio between the critical temperature T_c compared to the Fermi temperature T_F is typically $T_c/T_F \sim 10^{-3}$. While the T_F of electrons is rather high, about 10.000 K, for ^{3}He it is approximately 1 K. The low T_c ($\sim$ 1 mK) could only be reached after the invention of dilution fridges including a nuclear stage or Pomeranchuk cell. It is due to this technical difficulty that there is more than 60 years difference between the first observations of superconductivity in 1911 by Kamerling Onnes [7] and the superfluidity in liquid ^{3}He by Osheroff, Richardson and Lee [8].

Superfluidity itself was already discovered in 1930 for the isotope ^{4}He [9], however here the phenomena is based on a different principle. For ^{4}He, which is a boson, the fluid condenses into a Bose-Einstein condensate. While a lot of the phenomena are the same, the superfluid phase diagram of ^{3}He is much richer. In both, superconductivity

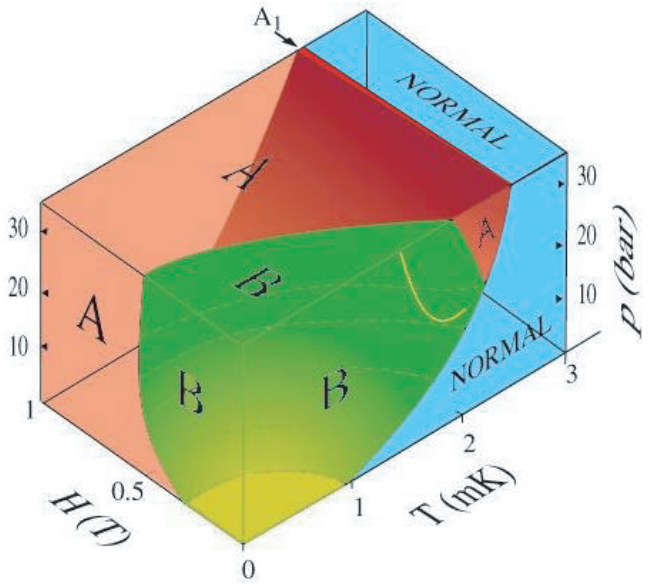

Figure 1.1: Phase diagram of bulk liquid ^{3}He, as function of temperature, pressure and magnetic field. The bulk shows the normal liquid helium-3 phase and 5 different superfluid phases. Figure is taken from [5].

in type I superconductors and superfluidity in ^{4}He, there exist 'one kind' of superconductivity/superfluidity, while in ^{3}He there exist 'more kinds' of superfluidity. The difference is due to higher symmetry in the liquid, which will be discussed in more detail in the next sessions.

The variety of superfluid phases can already been seen in the bulk properties of ^{3}He, see phase diagram in figure 1.1. In zero magnetic field two stable phases exist, named the A- and B-phase. They were observed for the first time in 1971 [1], when accurate measurements at the melting curve of ^{3}He were done. In magnetic field a third phase was discovered, the A_1-phase. Future studies pointed out that the A- and B-phases do break symmetry when they are put in a magnetic field and are in fact different phases. These phases are referred as the A_2- and B_2-phase. The richness of different phases performs themselves already in the bulk properties (including magnetic fields) of ^{3}He. Even more phases are predicted as one changes the geometric dimensionality of the system.

All phases let themselves describe by a different order parameter. The difference in isotropic (or better anisotropic) properties results in a preferable orientation of

[1] Initially it was thought that it was a transition in the solid [10].

the order parameter. A powerful technique to distinguish the different phases (order parameter) is with the help of nuclear magnetic resonance NMR. The line shape of the NMR spectrum gives good identification ('fingerprint') of each phase. The applied NMR techniques to obtain the data of this thesis and predicted NMR spectra of certain phases can be found in the chapters 3 and 4, respectively.

1.3 Ginzburg-Landau theory

In order to investigate the possibility of a possible new superfluid phase of liquid ^{3}He in restricted geometry (Dimensions), we discuss shortly the Ginzburg-Landau theory. Landau's theory of second order phase transitions was together with Ginzburg [11] extended to be able to describe the phenomena of superconductors and superfluids. It is a macroscopic theory, in which the order parameter of the superconductor is described with a wave function ψ. The order parameter has spatial properties and is complex. The free energy density difference between the normal state F_n (ground state of a Fermi system) and the superfluid state F_s is expanded into a power series of the order parameter. In the case we neglect the spatial variation (homogenous or bulk case) of the order parameter, we can write the free energy density as:

$$F_n - F_s = \alpha\psi^2 + \frac{\beta}{2}\psi^4. \tag{1.7}$$

The most simple temperature dependence for this model is when $\alpha(T) = -\alpha_0(1 - T/T_c)$ and β is constant, which is a good description for temperatures near the transition temperature T_c. Here the coefficients have the correct limits, and minimizing the free energy with respect to the order parameter gives:

$$\psi = \pm\sqrt{\frac{\alpha_0}{\beta}}\sqrt{1 - T/T_c}. \tag{1.8}$$

At zero temperature, the free energy density is maximally lowered. No more Cooper pairs can then be formed. The average gap energy [2] $\Delta(T)$ is then maximal and the free energy density, or total condensation energy, can be simplified as:

$$F_n - F_s = \frac{1}{2}N(0)\Delta(0)^2. \tag{1.9}$$

(Here $N(0)$ is the density of states for one spin component.) The product of $1/2$ $N(0)$ $\Delta(0)$ corresponds with the total amount of formed Cooper pairs and every Cooper pair lowers the free energy density with $\Delta(0)$.

If one wants to include the variation of the order parameter $\psi(\mathbf{r})$, a gradient term (or kinetic energy term) in the free energy density should be included.

[2] The average gap energy $\Delta(T)$ corresponds with ψ, but is the more general form for the order parameter. Strictly spoken ψ of equation (1.8), only valid near T_c, may not be extrapolated to zero temperature. To express the maximum condensation energy, equation (1.9), one should express it in terms of the average gap energy.

$$F_n - F_s = \alpha|\psi(\mathbf{r})|^2 + \frac{\beta}{2}|\psi(\mathbf{r})|^4 + k|\nabla\psi(\mathbf{r})|^2, \tag{1.10}$$

where k is determined by the normalization of $\psi(\mathbf{r})$. The gradient term prevents $\psi(\mathbf{r})$ to change 'quickly', which 'costs' too much energy. Instead it smoothest the order parameter over a typical size ξ, which we call the coherence length. Coherence lengths are temperature dependent. In the case of superconductors they have the form of $\xi(T) = (k/\alpha)^{1/2}$, which is proportional with $(1 - T/T_c)^{-1/2}$ near T_c.

1.4 Free energy density of the superfluid phases

The Landau-Ginzburg theory, as described in the previous section, explains phenomenologically the behavior of superconductors near the transition. A Cooper pair formed by 2 electrons, which is an s-wave pair, is a relative simple system. Here both, the angular momentum $\mathbf{L}$ and spin quantum number $\mathbf{S}$ of the Cooper pairs are 0. In such systems only one symmetry is and can be broken, namely the Gauge symmetry $U(1)_\phi$. Consequently there can only exist one kind of superconducting state. As shown in the previous section, it is described by a complex order parameter $\psi(\mathbf{r}) = \psi_0(\mathbf{r})e^{i\phi}$, where ψ_0 is the amplitude and ϕ the phase.

Like an electron, ^{3}He is a spin 1/2 particle. The most important difference is not the weight (the atom is ~ 5500 times heavier than the electron) but its spatial extensiveness. The hard core repulsion between the atoms prevents the wave functions to overlap. As a consequence the Cooper pairs prefer to form in p-wave pairs, instead of s-wave pairs. For p-waves pairing the angular moment and spin quantum number changes to $\mathbf{L} = \mathbf{S} = 1$. Such pairing systems have much more symmetry, next to the Gauge symmetry it has three dimensional rotation symmetry for both the orbital and spin space. The total symmetry G is given by a product of the three independent symmetries.

$$G = SO(3)_\mathbf{L} \times SO(3)_\mathbf{S} \times U(1)_\phi. \tag{1.11}$$

The total symmetry can at least be broken in 13 different continuous subgroups, and 4 discrete subgroups. All subgroups have a different order parameter structure [12], [13], [14], [15], [16]. An illustration of the broken $SO(3)_\mathbf{L} \times SO(3)_\mathbf{S} \times U(1)_\phi$ into continuous subgroups is shown in figure 1.2, which is taken from Bruder and Vollhardt [16].

All subgroups correspond to minima, local minima, stationary points or at worst saddle points in the free energy density given by a Landau type of expression. The order parameter is now a complex 3×3 matrix, and equation (1.7) should be up graded for such order parameters. Obeying the restrictions of invariance under spin and spatial rotations (and Gauge invariance) the free energy density for the homogenous case, including strong coupling corrections, till fourth order is given by Mermin and Stare [12], [17] as

$$F_n - F_s = \alpha\ tr(\mathbf{A}\mathbf{A}^\dagger) + \beta_1\ |tr\mathbf{A}\tilde{\mathbf{A}}|^2 + \beta_2\ [tr(\mathbf{A}\mathbf{A}^\dagger)]^2 + \beta_3\ tr[(\mathbf{A}^\dagger\mathbf{A})(\mathbf{A}^\dagger\mathbf{A})^*] + \beta_4\ tr[(\mathbf{A}\mathbf{A}^\dagger)^2] + \beta_5\ tr[(\mathbf{A}\mathbf{A}^\dagger)(\mathbf{A}\mathbf{A}^\dagger)^*], \tag{1.12}$$

where the 3×3 matrix $\mathbf{A}$ is related to the spin space matrix $\Delta_{\mu\nu}$. The coefficients in the limit of the BCS weak-coupling approach are given by:

$$\alpha = -\frac{N(0)}{3}\left(1 - \frac{T}{T_c}\right) \tag{1.13}$$

$$\beta_i = \tilde{\beta}_i \beta_{BCS} = \tilde{\beta}_i \frac{7\zeta(3)}{240\pi^2} N(0)\left(\frac{1}{k_B T_c}\right)^2 \tag{1.14}$$

where $\tilde{\beta}_i = (-1, 2, 2, 2, -2)$. Strong coupling effects are included as $\Delta\beta_i$ in β_i.

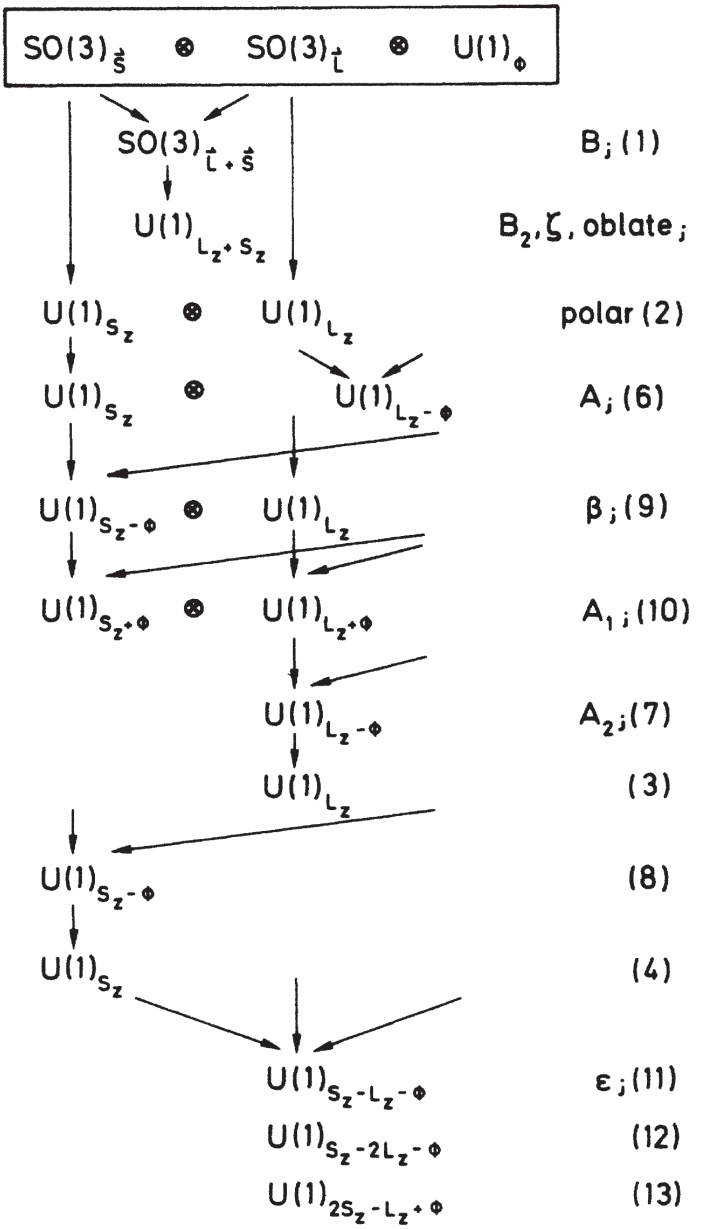

Figure 1.2: Scheme of $SO(3)_{\mathbf{L}} \times SO(3)_{\mathbf{S}} \times U(1)_{\phi}$ broken into continuous subgroups. Broken relative symmetries arise in factorizations involving more than one group, indicated by diagonal arrows. On the right name of phase (if any) is mentioned.

1.5 B-phase

Most experiments on the superfluid ^{3}He in this thesis are done in the B-phase of ^{3}He. This phase was for the first time described by Balian and Werthamer [18], and was called the BW-state. It is the lowest energy state of the free energy density (1.12), which corresponds with Mermin and Stare [12] when they investigate different gap structures in the weak coupling BCS theory. The gap structure, like in the case of type I superconductors, is isotropic (see figure 1.3). Here no preferred directions are demanded in spin-space.

In the B-phase the symmetry of the rotation in spin- and orbital space are broken, but not independently. The relative orientation is still ordered, and has still rotational invariance ($SO(3)_{\mathbf{L}+\mathbf{S}}$). This relative rotation lets itself be described with a rotation matrix $\mathbf{R}$ characterized by $\hat{\mathbf{n}}$ and θ. The rotation axis $\hat{\mathbf{n}}$ is perpendicular to the plane formed by the $\mathbf{L}$ and $\mathbf{S}$ vectors of the Cooper pairs and θ is the angle between those two vectors.

The order parameter matrix of equation (1.12), for the B-phase, can be written as:

$$A_{\mu j} = 3^{-1/2} e^{i\phi} R_{\mu j}(\hat{\mathbf{n}}, \theta) \tag{1.15}$$

$$R_{\mu j} = (1 - \cos\theta)\hat{n}_\mu \hat{n}_j + \cos\theta\ \delta_{\mu j} - \sin\theta \sum_k \epsilon_{\mu j k} \hat{n}_k \tag{1.16}$$

where ϕ is the overall phase variable.

It should also be emphasized that the B-phase, contains all the triplet components for both the spin and the orbital angular momentum pairing, hence $\mathbf{S} = \mathbf{L} = 1$. Concerning spin-space, it is convenient to combine the three spin components into the order parameter vector of spin-space $\mathbf{d}(\mathbf{k})$. The energy gap matrix of spin space $\Delta_{\mathbf{k}\alpha\beta}$ expanded in Pauli matrices is then

$$\Delta_{\mathbf{k}\alpha\beta} = \sum_\mu d_\mu(\mathbf{k})(\boldsymbol{\sigma}_\mu i\boldsymbol{\sigma}_2)_{\alpha\beta} = \begin{bmatrix} -d_1 + id_2 & d_3 \\ d_3 & d_1 + id_2 \end{bmatrix} = \begin{bmatrix} \Delta_{\uparrow\uparrow} & \Delta_{\uparrow\downarrow} \\ \Delta_{\downarrow\uparrow} & \Delta_{\downarrow\downarrow} \end{bmatrix}. \tag{1.17}$$

The energy gap in k-space is shown in figure 1.3. At the equator the gap is formed by combination of up-up and down-down spin pairs, while at the poles it is formed by combination of up-down and down-up spin pairs. The anisotropy of the spin pairs in the k-space mentions the B-phase also as 'pseudo'-isotropic.

1.6 B_2-phase

In the ^{3}He-B phase the gap parameter is isotropic, for which it can be referred to as the spherical state. If an external magnetic field is applied to the B-phase, the gap parameter becomes anisotropic. The gap in the z-direction (direction of magnetic field) is flattened, and one may speak of the squashed spherical state. This is a

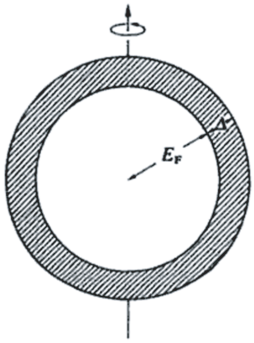

Figure 1.3: Scheme of the energy gap, indicated by the shaded area, of the B-phase in k-space. Ratio of Fermi energy E_F and the gap Δ is not to scale. Figure is taken from [19].

consequence of the fact that the Cooper pairs formed in a magnetic field prefer $|\uparrow\uparrow>$ and $|\downarrow\downarrow>$ spins pairs instead of $|\uparrow\downarrow>$ spin pairs. Here the $SO(3)_{\mathbf{L}+\mathbf{S}}$ symmetry gets broken. The symmetry, which is conserved, is the rotation around the z-axis, hence $U(1)_{L_z+S_z}$. The phase corresponding with this symmetry is the B_2-phase, which was first described by Barton and Moore [13]. The order parameter matrix is given by:

$$A_{\mu j} = p_2 e^{i\phi} \begin{bmatrix} A & B & 0 \\ \pm B & \mp A & 0 \\ 0 & 0 & C \end{bmatrix} \tag{1.18}$$

where,

$$\begin{aligned} p_2 &= [2(|A|^2 + |B|^2) + |C|^2]^{-1/2} \\ A &= \frac{1}{2}(\Delta_{\uparrow\uparrow} + \Delta_{\downarrow\downarrow}) \\ B &= -\frac{1}{2} i(\Delta_{\uparrow\uparrow} - \Delta_{\downarrow\downarrow}) \\ C &= \Delta_{\uparrow\downarrow}. \end{aligned}$$

1.7 Dipole energy

The dipole interaction, coupling of the nuclear spins of the ^{3}He atoms, has influence on the relative orientation of $\mathbf{L}$ and $\mathbf{S}$. However, this energy is so small that the influence on the forming of the order parameter itself can be neglected. Nevertheless, it is the most important orientation force in the superfluid, which cannot be 'turned off' [3].

[3] in contrast with magnetic fields and surfaces.

The standard expression for magnetic-dipole interaction is

$$H_D = \frac{1}{2}(\gamma\hbar)^2 \int d^3\mathbf{r} \int d^3\mathbf{r}' \left\{ \frac{\boldsymbol{\sigma}(\mathbf{r}) \cdot \boldsymbol{\sigma}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} - 3\frac{[(\mathbf{r}-\mathbf{r}') \cdot \boldsymbol{\sigma}(\mathbf{r})][(\mathbf{r}-\mathbf{r}') \cdot \boldsymbol{\sigma}(\mathbf{r}')]}{|\mathbf{r}-\mathbf{r}'|^5} \right\}. \tag{1.19}$$

Where $\sigma(\mathbf{r})$ is the spin density operator. This Hamiltonian is solved for a p-wave condensate by tremendous work of Leggett [20], [21], [22]. By expressing the dipole energy density F_D in terms of the order parameter matrix, one obtains

$$F_D = \frac{3}{5} g_D(T) \sum_{ij} [A^*_{ii}A_{jj} + A^*_{ij}A_{ji} - \frac{2}{3}A^*_{ij}A_{ij}], \tag{1.20}$$

where $g_D(T)$ is the dipole coupling constant

$$g_D(T) \approx \frac{1}{2}\pi\gamma^2\hbar^2 < R^2 >_{av} \left[\frac{1}{2} N_F \Delta(T) \ln\left(\frac{2\varepsilon_c}{\Delta(0)}\right) \right]^2. \tag{1.21}$$

The renormalization factor $< R^2 >_{av}$ is there because it is not entirely obvious that the expression of equation (1.19) is the same for quasi-particles, as discussed by Leggett and Takagi [22] [23] . However, experimentally it is shown that it should be very close to 1 (in fact it might be 1), which is the reason we put it to 1 for the rest of this thesis. The logarithm includes the cut off energy ε_c, which is proportional with the pair interaction potential:

$$\ln\left(\frac{2\varepsilon_c}{\Delta(0)}\right) \sim \frac{1}{N(0)V_1}. \tag{1.22}$$

The product $N(0)V_1$ can be obtained by fitting the experimentally found T_c, and is proportional with $\ln(T_c/T_F)$ [24]. So, the cut off energy ε_c is hardly pressure dependent and is roughly $\sim 0.7k_B$ [25]. Due to the logarithm, small deviations in ε_c have little or no consequence and, for this reason, it is kept fixed for all pressures.

If the B-phase order parameter matrix (1.15) is substituted in de dipole free energy density (1.20) we obtain:

$$F_D^B = \frac{8}{5} g_D(T) \left(\cos\theta + \frac{1}{4}\right)^2 + const. \tag{1.23}$$

The angle $\theta = \theta_L = \cos^{-1}(-1/4) \approx 104°$ minimizes the dipole energy (and of course $\theta = 2\pi - \theta_L$). The L refers to Leggett and the angle θ_L is in the literature known as the Leggett angle. Consequently the angle between $\mathbf{L}$ and $\mathbf{S}$ in the B-phase will be θ_L. The $\hat{\mathbf{n}}$-vector does not have a preferred orientation in the bulk B-phase, so it does not contribute to lower the dipole energy.

In the case of the B_2-phase the $\hat{\mathbf{n}}$-vector does have a preferred orientation, which influences the total dipole energy. Putting the B_2-phase order parameter matrix (1.18) in equation (1.20) we obtain:

$$F_D^{B_2} = \frac{1}{5}\lambda_D N_F \left\{ f_0(\theta) + f_1(\theta)(\hat{\mathbf{n}}\cdot\hat{\mathbf{H}})^2 + f_2(\theta)(\hat{\mathbf{n}}\cdot\hat{\mathbf{H}})^4 \right\} + const, \tag{1.24}$$

where

$$\lambda_D N_F \equiv g_D(T)/\Delta(T)^2, \tag{1.25}$$

and

$$\begin{aligned} f_0(\theta) &= 8\Delta_\perp^2(\cos\theta + 1/4)^2 + 8(\Delta_\parallel - \Delta_\perp)\Delta_\perp \cos\theta(\cos\ \theta + 1/4) \\ &\quad + 2(\Delta_\parallel - \Delta_\perp)^2\cos^2\theta, \\ f_1(\theta) &= 2(\Delta_\parallel - \Delta_\perp)[\Delta_\perp(3 + \cos\theta - 4\cos^2\theta) \\ &\quad + 2(\Delta_\parallel - \Delta_\perp)\cos\theta\ (1 - \cos\theta)], \\ f_2(\theta) &= 2(\Delta_\parallel - \Delta_\perp)^2(1 - \cos\theta)^2. \end{aligned} \tag{1.26}$$

Here the relations $\Delta_{\uparrow\uparrow} = \Delta_{\downarrow\downarrow} = \Delta_\perp$ and $\Delta_{\uparrow\downarrow} = \Delta_{\downarrow\uparrow} = \Delta_\parallel$ are used.

Equation (1.24) is minimal in energy, if the $\hat{\mathbf{n}}$-vector and the magnetic field $\mathbf{H}$ are parallel. This is true for all configurations of $\Delta_\perp$ and $\Delta_\parallel$ and at the minimum equation (1.24) becomes:

$$F_D^{B_2} = \frac{8}{5}\lambda_D N_F \left(\cos\theta + \frac{1}{4}\frac{\Delta_\parallel}{\Delta_\perp}\right)^2 + const.\ , \quad \hat{\mathbf{n}} = \pm\hat{\mathbf{H}}. \tag{1.27}$$

Thus the total dipole energy is now minimized if the angle between $\mathbf{L}$ and $\mathbf{S}$ is [26], [27], [28]

$$\theta_0(H) = \cos^{-1}\left(-\frac{1}{4}\frac{\Delta_\parallel}{\Delta_\perp}\right). \tag{1.28}$$

The angle, now depending on the ratio of $\Delta_\parallel/\Delta_\perp$, varies between θ_L (if populations are the same, as in the case of the B-phase) and 90° ($\Delta_\parallel$ is unoccupied).

Magnetic fields change the population ratio, but also influence the orientation of the $\hat{\mathbf{n}}$-vector. From equation (1.24) we can determine the orientation energy density due to magnetic field. The magnitude given to second order:

$$\Delta f_H^{B_2} = \lambda_D N_F \frac{(\Delta_\parallel - \Delta_\perp)\Delta_\perp}{H^2}(\hat{\mathbf{n}}\cdot\mathbf{H})^2, \tag{1.29}$$

where [19]

$$(\Delta_\perp - \Delta_\parallel)\Delta_\perp = \frac{5}{12}\left(\frac{\gamma\hbar H}{1 + F_0^a}\right)^2 \frac{\beta_{345}^{WC}}{\beta_{345}}\frac{T}{T_c}. \tag{1.30}$$

Two points should be emphasized which are relevant for the experiments presented in this thesis. First the orientation energy density $\Delta f_H^{B_2}$ is a fraction of the dipole energy F_D (which is already small by itself) and makes it only temperature dependent to the lowest order. Secondly, the magnetic fields used in our experiments are kept small for purpose to stay close to the B-phase. Consequently, the difference in population is negligible, and also the angle θ_0 is nearly equal to θ_L. Nevertheless, the effect of the magnetic field on the direction of $\hat{\mathbf{n}}$ is important for the forming of the textures, which will be discussed later.

1.8 Coherence lengths

If we deal with spatial inhomogeneity in the superfluid, hence gradients in the order parameter, we should include the gradient free energy density part in equation (1.12). It is the equivalent of the gradient part in the Ginzburg Landau theory, see equation (1.10), and is given by:

$$F_{grad} = \frac{1}{2}\sum_{jl\mu}[K_1(\nabla_j A_{\mu l})(\nabla_j A^*_{\mu l}) + K_2(\nabla_j A_{\mu l})(\nabla_j A^*_{\mu j}) + K_3(\nabla_j A_{\mu j})(\nabla_j A^*_{\mu l})]. \tag{1.31}$$

In the Ginzburg-Landau regime the coefficients do not depend on the order parameter, and are isotropic. However in a more general case the coefficients can be anisotropic in orbital and spin spaces. In the weak-coupling regime the coefficients [4] have been calculated [29]

$$K_1 = K_2 = K_3 \equiv K = \frac{1}{5}N_F\xi_0^2. \tag{1.32}$$

where N_F is again the density of quasi-particle states of both spin components in the normal phase at the Fermi energy and ξ_0 the coherence length at zero temperature. This length is corresponding to the size of the Cooper pair. The coherence length expressed in natural constants, Fermi velocity v_F and critical temperature T_c is given by:

$$\xi_0 = \left[\frac{7\zeta(3)}{48\pi^2}\right]^{1/2}\frac{\hbar v_F}{k_B T_c}. \tag{1.33}$$

At higher temperatures the coherence length grows and its temperature dependence is given by:

$$\xi(T) = \xi_0(1 - T/T_c)^{-1/2}, \tag{1.34}$$

showing how it diverges at the transition temperature.

1.8.1 Coherence lengths of different interactions

The gradient energy density is minimized when the transition of the orientation of the order parameter is as smooth as possible. Typical length scales of this transition are indicated with a coherence length ξ. To give an estimation for the order of magnitude of such coherence length it is convenient to use the *London Limit*. Here the order parameter attains its equilibrium structure everywhere in bulk, where weak perturbation does not change its structure but merely influence the preferred direction in orbital and spin space (and phase) [19]. The gradient free energy density can be

[4] The weak-coupling limit is valid when the strongest interaction parameter V_L, forming the Cooper pairs, times the density of states $N(0)$, equals $N(0) \mid V_L \mid \ll 1$.

rewritten in terms of gradients of the symmetry variables only, and will be referred as the bending free energy density F_{bend}.

This bending free energy density for the B-phase, where ϕ is kept constant and $\theta = \theta_L$ is then given by:

$$F_{bend} = 2\frac{1}{2}\left(\frac{1}{2}K\Delta(T)^2\right)\left\{4(\hat{\mathbf{n}}\times(\nabla\times\hat{\mathbf{n}}))^2 + \frac{13}{4}(\nabla\cdot\hat{\mathbf{n}})^2 + \frac{11}{4}(\hat{\mathbf{n}}\cdot\nabla\times\hat{\mathbf{n}})^2\right. \quad (1.35)$$
$$\left. - \frac{\sqrt{15}}{2}(\nabla\cdot\hat{\mathbf{n}})(\hat{\mathbf{n}}\cdot\nabla\times\hat{\mathbf{n}}) + 4\nabla\cdot[(\hat{\mathbf{n}}\cdot\nabla)\hat{\mathbf{n}} - \hat{\mathbf{n}}(\nabla\cdot\hat{\mathbf{n}})]\right\} \approx \frac{1}{2}K\Delta(T)^2\frac{1}{\xi_L^2},$$

where $\Delta(T)$ is the energy gap of the B-phase and ξ_L is the healing length, which corresponds to the spatial variation of the texture. Comparing other energy densities with the bending energy density, one can make an estimation of the coherence length corresponding to that energy.

Dipole healing length

By comparing the dipole free energy (1.20) with the bending free energy (1.35) we find

$$\frac{1}{2}K\Delta(T)^2\left(\frac{1}{\xi_D^B}\right)^2 \approx \frac{3}{5}g_D(T). \quad (1.36)$$

Using the definition of the dimensionless dipole coupling parameter λ_D (1.25) and K (1.32), the dipole healing length [5] can be expressed as:

$$\xi_D^B \approx \sqrt{\frac{1}{6\lambda_D}}\xi_0. \quad (1.37)$$

While the expression is most accurate in the Ginzburg-Landau regime, the most important identity is that the healing length ξ_D^B is temperature independent. However, it does depend on pressure, since it is proportional with ξ_0. Staring from melting pressure to zero bar, it varies between approximately 7 to 32 μm.

Magnetic healing length

The order of magnitude of the magnetic healing length ξ_H^B [6] can be derived by comparing the magnetic field orientation energy (1.29) with the bending free energy (1.35).

$$\frac{1}{2}K\Delta(T)^2\left(\frac{1}{\xi_H^B}\right)^2 \approx \lambda_D N_F\frac{5}{12}\left(\frac{\gamma\hbar H}{1+F_0^a}\right)^2\frac{\beta_{345}^{WC}}{\beta_{345}}\frac{T}{T_c}. \quad (1.38)$$

[5] Also called: dipole coherence length.

[6] Technically one should put $\xi_H^{B_2}$, however there should be no difference between the two, since the isotropic B-phase does not have a preferred direction, so it does not have a magnetic healing length.

Using the definition of K (1.32), the magnetic healing length can be expressed as:

$$\xi_H^B = \sqrt{\frac{12}{50}} \frac{1}{\sqrt{\lambda_D}} \frac{(1+F_0^a)}{\gamma\hbar} \sqrt{\frac{\beta_{345}}{\beta_{345}^{WC}}} \frac{\xi_0}{H} \sqrt{\frac{T_c}{T}} \Delta(T). \tag{1.39}$$

Due to the approximations [7] the model is not accurate in detail for the whole temperature range. The limits, especially temperature near zero temperature, do not agree with the empirically found relations published in literature (see section 3.6 of this thesis.) However, it does describe the physical behavior. The healing length is zero at T_c and grows as temperature decreases. With the exception close to T_c the magnetic healing length ξ_H^B is longer than the dipole coherence length ξ_D^B. The inverse relation of the strength of the magnetic field H seems to be natural to first order, as one expects that the healing length will become smaller with higher field.

[7] Till second order of the magnetic field orientation, approximations in the relation $(\Delta_\perp - \Delta_\parallel)\Delta_\perp$ and derived in the Ginzburg-Landau regime.