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On topological Properties of Superconducting Nanowires

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On topological properties of superconducting nanowires

PROEFSCHRIFT

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VOLGENS BESLUIT VAN HET COLLEGE VOOR PROMOTIES
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Dmitry Igorevich Pikulin

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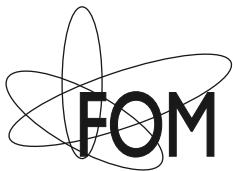
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Cover: *Dependence of the Andreev conductance in a narrow spin-orbit wire on the angle of applied magnetic field (horizontal axis) and the bias voltage. Compare with the right panel of Fig. 4.7.*

To Elena, to my parents, grandparents and all my family
Лене, моим родителям, бабушке с дедушкой и всей моей
большой семье

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