Summary

This thesis is devoted to the effects of disorder on two-dimensional systems of Dirac fermions. These quasiparticles appear in condensed matter in graphene (carbon monolayers), and also in superconductors with p-wave or d-wave symmetry of the order parameter, as well as in topological insulators.

The effect of disorder on which we focus our attention is the phenomenon of localization. It is known that ordinary electrons (described by the Schrödinger equation, rather than the Dirac equation) are localized by disorder, meaning that the wave function of an excitation decays exponentially, rather than being an extended plane wave. Localization thus transforms a metal into an insulator.

Dirac fermions respond qualitatively different to disorder. An early discovery was that electrostatic disorder cannot localize Dirac fermions in graphene, if it is smooth on the scale of the lattice constant. We concentrate on a different type of disorder, namely on a random mass term in the Dirac equation. It is realized in graphene by randomness in the substrate. We have discovered, somewhat unexpectedly in view of earlier work on this problem, that Dirac fermions in graphene are localized by a random mass, without any transition into a metallic state.

The situation is entirely different for Dirac fermions in a p-wave superconductor. There electrostatic disorder appears in the Dirac equation as a random mass, which localizes the excitation, but only if the disorder is relatively weak. For large mass fluctuations a transition into a metallic state appears, in contrast to what we found in graphene. We investigate the metal-insulator transition in p-wave superconductors using a lattice model of staggered fermions (originally proposed in the context of QCD). We calculate the critical exponents and identify a repulsive tricritical point at the phase diagram.

The qualitatively different response to disorder of Dirac fermions in graphene and in p-wave superconductors calls for an explanation, which we find in the appearance of Majorana bound states.

These midgap excitations in a p-wave superconductor allow for resonant tunneling and a metallic state. Graphene has no Majorana bound states, hence no metallic state in the presence of a random mass.

Electrostatic disorder in a d-wave superconductor manifests itself in an alltogether different form, as a random vector potential in the Dirac equation. A gauge transformation can eliminate this type of disorder at zero energy, if it is smooth on the scale of the lattice constant. The transmission of Dirac fermions through a d-wave superconductor is therefore only slightly affected by long-range electrostatic potential fluctuations. Short-range fluctuations do have a strong effect, exponentially suppressing the electrical current carried by the excitations, while leaving the thermal current unaffected.

We return to graphene in the final chapter of the thesis, to study two of its prominent properties: it forms a highly conducting two-dimensional electron gas and at the same time is a mechanically stable membrane. The interplay of electrical and mechanical properties is studied by calculating the correction to the conductivity of suspended graphene due to its deformation by a gate electrode.