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## On localization of Dirac fermions by disorder

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# On localization of Dirac fermions by disorder

PROEFSCHRIFT

TER VERKRIJGING VAN  
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Cover: Artistic impression of the phase diagram of Dirac fermions in symmetry class D and of Majorana-Shockley bound states on the lattice, as it is calculated in chapter 2 of this thesis (see Fig. 2.1) and in chapter 4 (see Fig. 4.4) respectively.

*to my Father, Mother,  
Grandfather, Grandmothers,  
Sister, Uncle,  
and to all other people, whom I call my Family*

моей Семье

Optimism is the faith that leads to achievement.  
Nothing can be done without hope and confidence.  
*Albert Einstein*

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