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## Effects of spin-orbit coupling on quantum transport

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# Effects of Spin-Orbit Coupling on Quantum Transport

Jens Hjørleifur Bárðarson

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# Effects of Spin-Orbit Coupling on Quantum Transport

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*Fyrir mömmu og pabba,  
Guðmund, Kristjönu, Helga og Hlyn.  
Takk fyrir allt.*



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