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## Effects of spin-orbit coupling on quantum transport

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### Citation

Bardarson, J. H. (2008, June 4). *Effects of spin-orbit coupling on quantum transport*. *Casimir PhD Series*. Retrieved from <https://hdl.handle.net/1887/12930>

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# Effects of Spin-Orbit Coupling on Quantum Transport

Jens Hjørleifur Bárðarson

Cover designed by Guneeta.

# Effects of Spin-Orbit Coupling on Quantum Transport

PROEFSCHRIFT

TER VERKRIJGING VAN DE GRAAD  
VAN DOCTOR AAN DE UNIVERSITEIT LEIDEN, OP GEZAG VAN  
DE RECTOR MAGNIFICUS PROF. MR. P.F. VAN DER HEIJDEN,  
VOLGENS BESLUIT VAN HET COLLEGE VOOR PROMOTIES  
TE VERDEDIGEN OP WOENSDAG 4 JUNI 2008  
TE KLOKKE 15.00 UUR

DOOR

Jens Hjörleifur Bárðarson

GEBOREN TE REYKJAVÍK IN 1979

**Promotiecommissie:**

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Casimir PhD Series, Delft-Leiden, 2008-01  
ISBN: 978-90-8593-040-2

The research described in this thesis was supported by the European Community's Marie Curie Research Training Network under contract MRTN-CT-2003-504574, Fundamentals of Nanoelectronics for the first three years, and by the Leiden Institute of Physics for the fourth year.

*Fyrir mömmu og pabba,  
Guðmund, Kristjönu, Helga og Hlyn.  
Takk fyrir allt.*



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