A study of electron scattering through noise spectroscopy

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NOISE : BASIC THEORETICAL CONCEPTS

The flow of electrons in a conductor is a stochastic random process due to the discrete nature and randomness in its transfer. The flow of electrons fluctuates around a mean value i.e. the average current. The fluctuation away from the mean current is known as current noise. It has two components: a) The equilibrium noise i.e. thermal noise also known as Johnson-Nyquist noise. It originates from the random motion of electrons in the conductor within the $k_B T$ width of the Fermi distribution function. b) The non-equilibrium noise i.e. shot noise. The origin of shot noise is due to the randomness in electron reflection at a scatterer. The first full expression for the shot noise in mesoscopic conductors was first derived by Lesovik and Levitov [1]. Since then much advancement has been made in theoretical understanding of the non equilibrium phenomena in mesoscopic devices. Scattering theory, Green function formalisms, Boltzmann-Langevin and counting field theory approaches have been developed to understand the noise in mesoscopic systems. The scattering approach, also known as Landauer-Büttiker approach is a simple quantum mechanical approach which relates the transport properties of the device to the scattering properties of quasiparticles. Here, I introduce scattering matrix formalism to relate the fluctuations in electron transport to the transport properties. Although this approach fails to take into account the effect of inelastic scattering and electron spin correlations in noise, which are central themes of this thesis, it explains beautifully the suppression of noise in atomic contacts. The extensions to inelastic scattering and spin correlations in noise will be developed in later chapter.

1.1 STATISTICS: AN INTRODUCTION TO RANDOM PROCESSES AND MOMENTS OF A DISTRIBUTION

LET us first look into some basic concepts of statistics[2]. The probability $P(x,t)$ of outcome x at time t is defined as the ratio of outcome x to all possible outcomes for a series of events $x(t)$. It defines the average behavior of events $x(t)$ for determination of $x's$ rate of occurrence. The distribution function $W(x_1,t_1)$ defining the probability can be written in terms of the probability itself *i.e.* $W(x_1,t_1) = P\{x(t_1) \leq x_1\}$. It states the probability for x at t_1 to have a value smaller or equal to x_1. If the distribution function of the random process is differentiable then its probability density function can be written in terms of its distribution function:

$$w(x_1,\ t_1) = \frac{\partial W(x_1,t_1)}{\partial x_1} \tag{1.1}$$

One can use the probability density function, to calculate the mean of the random processes.

$$\langle x(t)\rangle\ = \int_{-\infty}^{\infty} x(t)\, dW(x) = \int_{-\infty}^{\infty} dx\ x\ w(x,t) \tag{1.2}$$

Similarly, higher order moments can be computed from the probability density function using $\langle \delta x(t)\rangle = \langle x(t) - \langle x\rangle\rangle$ which is the deviation of a random quantity $x(t)$ from its mean value $\langle x\rangle$.

$$\langle [\delta x(t)]^r\rangle = \int_{-\infty}^{\infty} dx\ [\delta(x)]^r\ w(x,t) \tag{1.3}$$

The second order central moment is the most important moment of the random process. It is known as the variance. It is also known as fluctuations or noise. Hence, noise measurement is nothing but measurement of the variance of the random processes. Now let's try to understand why the study of fluctuations and higher order moments are important. Figure (1.1) depicts three different hypothetical experiments, counting the rate of occurrence of certain events. The mean rate of occurrence of the events is 10 in all three experiments. Hence by just looking at the mean we will not be able to see the difference between these experiments. If we look at the histograms of these experiments then we can easily see that while (a) and (b) have similar shapes of the probability distribution functions, (c) has a different one. Regarding the variance of (a) and (b), one observes that (a) has a higher variance than (b).

Hence, the higher moments of the fluctuations reveal more information than contained in just the mean values. Figure (1.2) shows the physical significance of the first few central moments. It shows that one should examine also the higher

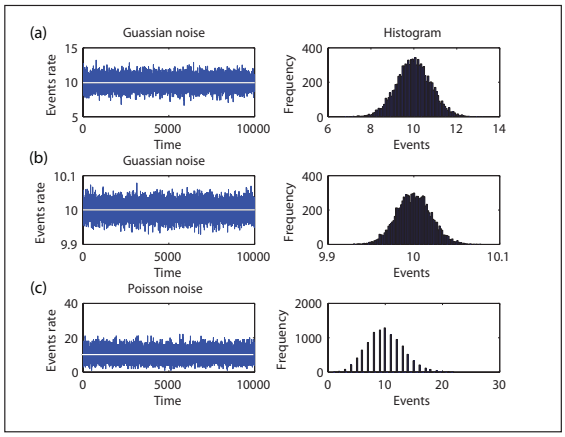

FIGURE 1.1: Fluctuations in measurement: Showing three different simulated measurements along with their frequency spectra. (a) Gaussian events with $\sigma = 0.8$. (b) Gaussian events with $\sigma = 0.02$. (c) Poissonian distribution with $\mu = 10$.

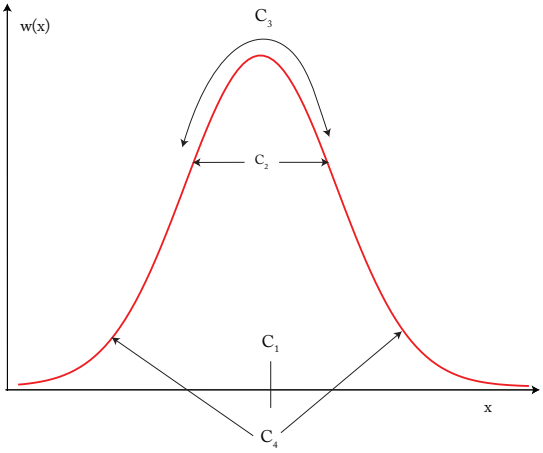

FIGURE 1.2: Probability density function showing the significance of the lowest order moments. C_1 is the mean, C_2 is the variance, C_3 is the skewness (asymmetry between left and right tails) and C_4 is the sharpness of the tails.

order of moments, not only the mean values to understand the underlying phenomena in random processes.

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1.2 CORRELATION TECHNIQUES

IN studies of random processes different correlation techniques are employed to look into any statistical regularity in the random processes. Coming back the noise, which is just the second order moment of the probability distribution function of the random process, it is usually measured in terms of the power in each frequency bin. The noise power can be calculated using the autocorrelation function. The autocorrelation function gives the correlation of a random signal $x(t)$ with the function itself but delayed by a time interval of τ. The autocorrelation function for a stationary stochastic random process is defined as:

$$\phi_x(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t+\tau)\,dt. \tag{1.4}$$

It measures the correlation of the random process *w.r.t.* time. The spectral density of noise can be calculated using the Fourier transformation of the autocorrelation

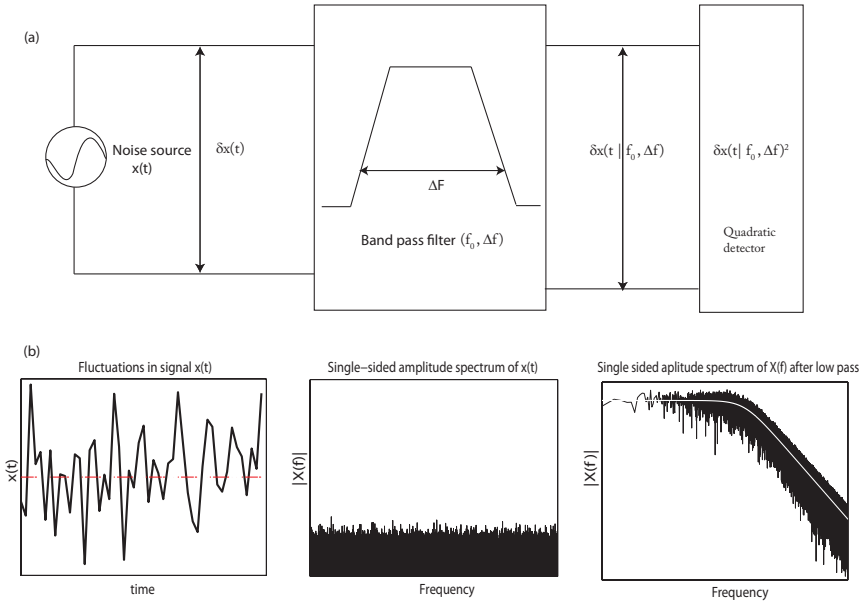

FIGURE 1.3: Fluctuations in the measurement and its bandwidth limited measurement using the quadratic detector: (a) A conceptual demonstration of the noise measurement (b) Noise and its spectral density. The roll off in the spectral density is due to effective low pass behavior of the measurement system.

function [Wiener - Khintchine formula][3, 4].

$$S_x(\omega) = \int_{-\infty}^{\infty} \phi_x(\tau) e^{-j\omega\tau}\, dt. \tag{1.5}$$

For a bandwidth limited measurement, which is more realistic to the experimental world, the power spectral density is measured per unit frequency. A simple schematic representation of noise measurement is shown in figure (1.3). Here the fluctuation power spectral density is measured using a quadratic detector. Power spectral density is plotted on a logarithmic scale. The signal is measured around a central frequency f_0 with effective bandwidth of δf.

1.3 QUANTUM TRANSPORT: A SCATTERING APPROACH

QUANTUM transport refers to the transport of electrons within their phase coherence length. A typical quantum conductor is shown in figure (1.4). The constriction in the narrow neck regime is provides the quantum confinement *i.e.* the width of the neck should be smaller than the scattering length. Due to the quantum confinement in the narrow neck regime, electrons in that regime can be treated as particles in a box. Here, the electrons are confined along the x and y axis but they are free to move along the z axis. The Schrödinger equation for the electrons in the confining potential can be written as:

$$\frac{\hbar^2}{2m^*}\nabla^2\psi(x,y,z) + V(x,y,z)\psi = E\psi(x,y,z) \tag{1.6}$$

where $V(x,y,z)$ is the confining potential and $\psi(x,y,z)$ is an eigen state of electrons confined in the box. Solving the above equation for the case of particles in a long box,

$$E = E_x + E_y + \frac{\hbar^2 k_z}{2m^*}, \tag{1.7}$$

where $E_x = \dfrac{\hbar^2}{2m^*}\left(\dfrac{n_x\pi}{w_x}\right)^2$ and $E_y = \dfrac{\hbar^2}{2m^*}\left(\dfrac{n_y\pi}{w_y}\right)^2$, and w_x and w_y are the width of the constriction in x and y directions, respectively. These eigen states are called transmission channels and the corresponding eigen vectors are called modes. A pictorial representation is shown in figure (1.4). Equation (1.7) states that for a given Fermi energy and constriction width, only certain transverse states are allowed. Such a phenomenon was first observed in a 2-dimensional electron gas by van Wees *et al.* and Wharam *et al.* [5, 6] in 1988, where they changed the constriction by means of a gate potential and observed a variation in the current in discrete steps. These discrete steps are integral multiples of the universal conductance quantity $2e^2/h$, that we will refer to as the quantum conductance G_0.

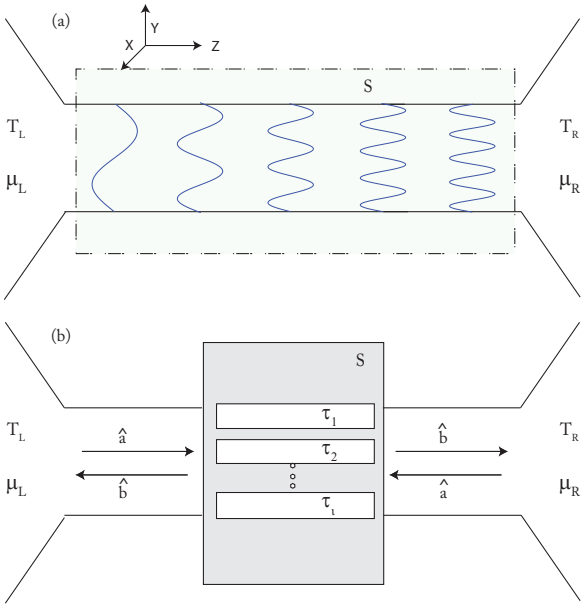

FIGURE 1.4: A schematic of a ballistic conductor attached to perfect leads: (a) The dotted-dashed line shows the quantum conductor. The electrons are emitted from the left and right leads into the quantum conductor, with an electron chemical energy μ_L and μ_R, respectively. (b) The grey shaded region shows the scattering region connected to bulk reservoirs through perfect leads having incoming and outgoing states.

The simple particle-in-a-box model helps to understand the quantized conductance of a narrow channel, but it fails to explain the fluctuations in the current. The simplest approach to a better understanding of quantum conductance is to use the scattering approach, also known as the Landauer-Büttiker formula. In this approach, a quantum conductor is modeled as a scattering regime connected to electron reservoirs through perfectly transmitting leads. There are four main assumptions in this formalism:

* The quantum conductor is modeled as a scattering regime connected to a reservoirs through perfectly transmitting leads.
* The reservoirs act as a perfect sources and sinks for the transmitted and reflected electrons irrespective of their initial energy states.
* The energy and phase of the electron states is preserved in the scasttering region and in the leads.

* The quantum conductor can be either in equilibrium or in a non-equilibrium state. This is included in the formalism through the Fermi Dirac distribution function of the reservoirs connected to the leads.

Now let us implement this approach for a two-terminal quantum conductor. Electrons propagate through the leads as plane waves with longitudinal momentum $k_\parallel$ and quantized transverse momentum $k_\perp$. If there are N independent scattering states in the quantum conductor then there will be 2N states in the leads (each scattering state will be accompanied by a partially reflected and a transmitted state) as shown in figure (1.4). Let us introduce four operators which can create and annihilates the electrons in the states defined by the quantum conductor: $\hat{b}_{\alpha i}$ and $\hat{b}^\dagger_{\alpha i}$ are the annihilation and creation operators in i^{th} outgoing eigen state of the lead $\alpha(= L, R)$. They annihilate and create the electrons in states moving away from the scattering center. $\hat{a}_{\alpha i}$ and $\hat{a}^\dagger_{\alpha i}$ are annihilation and creation operators for the i^{th} incoming eigen states in lead $\alpha(= L, R)$. The incoming and outgoing states can be related to each other using a scattering matrix s.

$$\begin{pmatrix} \hat{b}_{L1} \\ \hat{b}_{L2} \\ \vdots \\ \hat{b}_{Ln} \\ \hat{b}_{R1} \\ \hat{b}_{R2} \\ \vdots \\ \hat{b}_{Rn} \end{pmatrix} = s \begin{pmatrix} \hat{a}_{L1} \\ \hat{a}_{L2} \\ \vdots \\ \hat{a}_{Ln} \\ \hat{a}_{R1} \\ \hat{a}_{R2} \\ \vdots \\ \hat{a}_{Rn} \end{pmatrix} \tag{1.8}$$

Here s is a scattering matrix of dimension $2N \times 2N$[1]. The block structure of s matrix is given as:

$$s = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \tag{1.9}$$

where $r_{N\times N}$ and $r'_{N\times N}$ are square sub matrices describing the reflected electron wave for the left and right leads respectively. The off-diagonal matrices $t_{N\times N}$ and $t'_{N\times N}$ describe the effective transfer of electrons through the scattering regime. Since the number of electrons is fixed, s is unitary. In absence of a magnetic field *i.e.* under time reversal symmetry, s is symmetric. The fact that s is unitary and

[1] Here the size of the scattering matrix depends on the number of modes in the left and the right leads. In general, the sizes of the left and right lead can differ, in which case the dimension would be $(M + N) \times (M + N)$

symmetric permits us to diagonalize the matrices into scattering matrices of independent channels. This allows us to determine the current in each eigen state or channel [2] and sum over all independent quantum channels to obtain the total current. The net current in a quantum channel is the difference between the incoming and the outgoing state occupation numbers of the electrons. The current in lead α and quantum channel n is given as:

$$\hat{I}_{\alpha n}(t) = \frac{2e}{2\pi\hbar}\int dE dE' e^{i(E-E')t/\hbar}\left(\hat{a}^{\dagger}_{\alpha n}(E)\hat{a}_{\alpha n}(E') - \hat{b}^{\dagger}_{\alpha n}(E)\hat{b}_{\alpha n}(E')\right). \tag{1.10}$$

Here α, β represent the L,R reservoirs connecting to the scattering regime. For simplicity we take the current in the left lead only, from here onwards. Using the scattering matrix relation 1.8 the current in lead $\alpha = L$ and for the single-channel case can be written in terms of $\hat{a_\alpha}$ and $\hat{a_\alpha}^{\dagger}$,

$$\hat{I}_L(t) = \frac{2e}{2\pi\hbar}\sum_{\alpha\beta}\int dE dE' e^{i(E-E')t/\hbar}(\hat{a}^{\dagger}_{\alpha} A_{\alpha\beta}(L;E,E')\hat{a}_{\beta}(E')) \tag{1.11}$$

and $A_{\alpha\beta}(L;E,E')$ represents the matrix defined from scattering matrix s *i.e.* $A_{\alpha\beta}(L;E,E') = \delta_{\alpha L}\delta_{\beta L} - s^{\dagger}_{L\alpha}(E)s_{L\beta}(E')$. The average current $\langle I_L(t)\rangle$ can be given as:

$$\langle\hat{I}_L(t)\rangle = \frac{2e}{2\pi\hbar}\int dE dE' e^{i(E-E')t/\hbar}\langle\hat{a}^{\dagger}_{\alpha} A_{\alpha\beta}(L;E,E')\hat{a}_{\beta}(E')\rangle \tag{1.12}$$

Using $\langle\hat{a}^{\dagger}_{\alpha}(E)\hat{a}_{\beta}(E')\rangle = \delta_{\alpha\beta}\delta(E-E')f_{\alpha}(E)$, where $f_{\alpha}(E)$ describes the Fermi Dirac distribution in reservoir α, and taking into account the unitary property of matrix s, the average current can be given as,

$$\langle\hat{I}(t)\rangle = \frac{2e}{2\pi\hbar}\int dE\left(f_L(E) - f_R(E)\right)t^*(E)\,t(E). \tag{1.13}$$

Neglecting the energy dependence of $t(E)$ on the scale of $k_{\mathrm{B}}T$ and eV, one obtains the conductance for the single channel case as,

$$G = \frac{2e^2}{h}\tau, \tag{1.14}$$

with $\tau = t^*(E)\,t(E)$. Generalizing the Landauer-Büttiker formalism for multiple channels gives simply a superposition of all transmission eigen channels. Since $t^{\dagger}t$ has been diagonalized the conductance G can be given as,

$$G = \frac{2e^2}{h}\sum_{i=1}^{N}\tau_i. \tag{1.15}$$

[2] The concepts of eigen channel and eigen states are interchangeable

Hence the Landauer-Büttiker formalism states that the effective conductance is the linear addition of the transmission probabilities of all eigen channels. It is quite intuitive to state that the effective numbers of channels is limited by the narrowest region in the quantum conductor[3].

Equation (1.15), does not yet explain the fluctuations in the current. The fluctuations in the current due to the random flow of discrete electrons has two components: equilibrium noise *i.e.* thermal noise and non-equilibrium noise *i.e.* shot noise. Noise is characterized by its spectral density as stated in equation (1.4). The power spectral density is the Fourier transform of the current - current correlation function.

$$S(\omega) = 2\int_0^\infty dt e^{i\omega t}\langle \delta I(t+t_0)\delta I(t_0)\rangle \tag{1.16}$$

For the simplest case of a single-channel two-terminal device the noise spectrum can be written as

$$S(\omega) = \frac{1}{2}\int_0^\infty dt e^{i\omega t}\langle \delta\hat{I}_\alpha(t)\delta\hat{I}_\beta(0) + \delta\hat{I}_\beta(0)\delta\hat{I}_\alpha(t)\rangle, \tag{1.17}$$

where $\delta\hat{I}(t) = \hat{I} - \langle\hat{I}\rangle$. From equation(1.10) and (1.11) $\delta\hat{I}(t)$ can be written as:

$$\delta\hat{I}(t) = \frac{2e}{2\pi\hbar}\int dE dE' \sum_{\alpha\beta} A_{\alpha\beta}[\hat{a}^\dagger_\alpha(E)\hat{a}_\beta(E') - \langle\hat{a}^\dagger_\alpha(E)\hat{a}_\beta(E')\rangle]e^{i(E-E')t/\hbar} \tag{1.18}$$

Using the above equations, the noise power can be further simplified to

$$S_I(\omega) = \frac{2e^2}{2\pi\hbar}\sum_{\alpha\beta}\int dE A^2_{\alpha\beta}\left[f_\alpha(E)(1-f_\beta(E+\hbar\omega)) + f_\alpha(E+\hbar\omega)(1-f_\beta(E))\right] \tag{1.19}$$

In the zero frequency limit, $\omega \to 0$ the noise power density can be written as:

$$S_I = \frac{2e^2}{2\pi\hbar}\sum_{\alpha\beta}\int dE A^2_{\alpha\beta}\left[f_\alpha(E)(1-f_\beta(E) + f_\alpha(E)(1-f_\beta(E)\right] \tag{1.20}$$

For the case of a single quantum channel $A_{\alpha\beta}(L) = \delta_{\alpha L}\delta_{\beta L} - s^\dagger_{L\alpha}s_{L\beta}$, hence the noise power spectral density is:

$$S_I = \frac{2e^2}{h}\int dE\,\tau(E)\left[(f_\alpha(1-f_\alpha) + f_\beta(1-f_\beta)) + \tau(E)(1-\tau(E))(f_\alpha - f_\beta)^2\right]. \tag{1.21}$$

[3] In 2 DEG quantum point contacts the steps of quantum conductance are seen in the conductance due to the constriction that is controlled by the applied gate potential. Similar kinds of quantized steps are seen in atomic conductors, but here the conductance channels are attributed to the hybridized valence orbitals of atoms or molecules bridging the bulk leads. The interpretation is based on calculations, for example, using a tight a binding approach by Cuevas *et al.*[7], and was experimentally verified by Scheer *et al.*[8] and Brom *et al.*[9].

Since $\tau(E)$ changes insignificantly on the experimental energy scale we can neglect its energy dependence, so that we obtain

$$S_I = \frac{2e^2}{h}\left[eV\coth(\frac{eV}{2k_BT})\tau(1-\tau)+2k_BT\tau^2\right]. \tag{1.22}$$

The above equation (1.22) can be generalized to the multi-channel case, known as the Lesovik-Levitov expression,

$$S_I = \frac{2e^2}{h}\left[eV\coth(\frac{eV}{2k_BT})\sum_{n=1}^{N}\tau_n(1-\tau_n)+2k_BT\sum_{i=1}^{N}\tau_n^2\right]. \tag{1.23}$$

In the limit of $V \to 0$ the noise power spectral density reduces to

$$S_I(0) = 4k_BTG, \tag{1.24}$$

which is the equilibrium noise as the both leads are in equilibrium within the k_BT width of their Fermi Dirac distribution. This is also known as thermal noise or Johnson-Nyquist noise. The first term in equation(1.22) is voltage dependent and hence contributes to the non-equilibrium noise which is also known as shot noise. The cross over from the shot noise to thermal noise is quite smooth. For $eV \ll k_BT$, thermal noise dominates over shot noise and the non-linear dependence on V is gradually smoothed to a linear dependence of the noise on V at lower temperatures. This cross over phenomenon was first observed by H. Birk *et al.*[10]. Thermal noise does not give much information about the conductor apart from the impedance or electron temperature of the sample. Thermal noise is often used as a calibration measurement for the shot noise measurement. I will discuss this procedure in the section on Shot noise analysis in more detail below. For $T \to 0$ shot noise in the conductor is given as:

$$S_I(0) = \frac{e^3V}{\pi\hbar}\sum_{n=1}^{N}\tau_n(1-\tau_n). \tag{1.25}$$

Hence, shot noise is not determined just by the conductance, like thermal noise, but it is dependent on the effective transmission and reflection coefficients of the quantum channels. In the case of completely closed and open channels, the shot noise vanishes completely. The maximum shot noise level is seen for a half-open quantum channel. Taking $\tau_n \to 0$, for all n, the shot noise takes the Poisson value as stated by Schotkky in 1926.

$$S_I(0) = \frac{e^3V}{\pi\hbar}\sum_{n=1}^{N}\tau_n \tag{1.26}$$

This is known as full shot noise. Comparing equation (1.25) with (1.26) shot noise in a quantum conductor always deviates from the full shot noise as $S_I(0) = 2eIF$. The factor measuring the deviation is known as the Fano factor F, which can be written as,

$$F = \frac{\sum_{n=1}^{N} \tau_n (1-\tau_n)}{\sum_{n=1}^{N} \tau_n}, \tag{1.27}$$

with $0 \leq \tau_n \leq 1$. The Lesovik-Levitov expression, equation(1.23) is a function of the Fano factor F, temperature T and the bias voltage V. Expression (1.23) can be written as the difference between noise at non-equilibrium $S_I(0)$ and the thermal noise $S_{th}(0)$,

$$S_I(0) - S_{th}(0) = \frac{2e^2}{h}\left[2k_BT\sum_{n=1}^{N}\tau_n^2 + eV\coth\left(\frac{eV}{k_BT}\right)\sum_{n=1}^{N}\tau_n(1-\tau_n)\right] - 4k_BT\frac{2e^2}{h}\sum_{n=1}^{N}\tau_n. \tag{1.28}$$

Let us define the excess noise as $S_{ex} = S_I(0) - S_{th}(0)$, then the expression for the normalized excess noise can be simplified to,

$$\frac{S_{ex}}{S_{th}(0)} = F\left[\frac{eV}{2k_BT}\coth\left(\frac{eV}{2k_BT}\right) - 1\right] \tag{1.29}$$

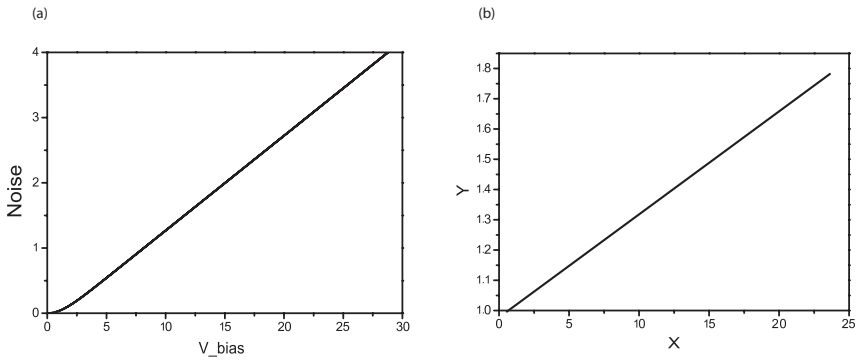

FIGURE 1.5: Shot noise *w.r.t.* bias voltage: (a) Noise is a non linear function of V_{bias} for $eV \leq k_BT$ and approaches a linear function for $eV > k_BT$ (b)The normalized excess noise, *i.e.* $\frac{S_{exc}}{S_{th}(0)}$ plot *w.r.t.* the reduced bias $X = \frac{eV}{2k_BT}\coth\left(\frac{eV}{2k_BT}\right)$ is a simple linear function for which the slope gives the Fano factor.

Hence, expression (1.29) shows that slope of the normalized excess noise *w.r.t.* reduced bias $X = \frac{eV}{2k_BT}\coth\left(\frac{eV}{2k_BT}\right)$ gives the Fano factor, as can be seen from figure (1.5).

A note on shot noise

From the Lesovik-Levitov expression (1.23), it follows that for $\tau_n = 1$ or $\tau_n = 0$, for all channels n noise is absent in the quantum conductor. But for a channel with $0 < \tau_n < 1$, the electron wave packet splits into a reflected and a transmitted wave packet and this occurs randomly. The electron is detected as transmitted or reflected and this occurs randomly. This randomness gives the shot noise in the quantum conductor. Hence shot noise is due to both the wave and the particle nature of the electron. This is shown in figure (1.6) as an example. The different kinds of interaction of the traversing electron in the conductor leads to deviations of noise from the Lesovik-Levitov value. These deviations can be lumped into the factor called the Fano factor. Hence, the study of the Fano factor gives information about the kinetics of the electron transport in the quantum conductor. In mesoscopic physics, shot noise is currently extensively used to determine the different kinds of underlying kinetics of the electron during its transport.

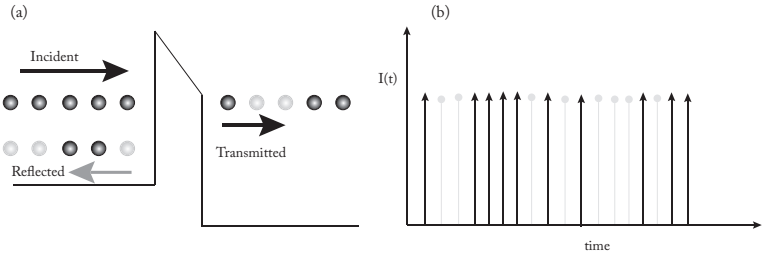

FIGURE 1.6: Shot noise in a tunnel junction due to random motion of the electron: Electrons approach the contact at regular intervals but get reflected randomly and hence the pulses of transmitted electrons are random, giving noise in measurement. Each transmitted electron pulse gives a tick in the current meter. These series of ticks over the large measurement time ensemble gives a measure of the average noise.

I give a short overview of experimental studies of shot noise on different systems: The first observations of suppression of shot noise by high-transmission conductance channels in point contacts were made in two dimensional electron gas systems by Renikov *et al.* [11] and Kumar *et al.* [12]. They measured the noise for a point contact. The shot noise evolves from the full shot noise with $F \sim 1$ at the nearly pinched-off regime to $F \sim 0$ on the first plateau of the quantum conductance, as expected from the theory. In another series of experiments, by de-

Picciotto *et al.* [13] and Saminadayar *et al.* [14], the fractional charge in the fractional quantum hall regime was measured. This experiment was a direct manifestation of Laughlin's concept of the quasiparticles having fractional charge and taking part in the charge transport in the fractional quantum Hall regime.

In a completely different system, a superconductor-normal metal diffusive contact, the shot noise measurement revealed the $2e$ charge of elementary processes taking part in the conduction [15]. This shot noise measurement revealed the effective charge of the quasiparticles taking part in the conduction. In similar 2DEG point contacts as mentioned above, non-integer conductance steps are seen at $0.7G_0$, widely known as the 0.7 anomaly. This 0.7 anomaly has been reported to be related to the intrinsic electron spin. This spin interaction effect was first revealed in shot noise measurements by Roche *et al.* [16]. The shot noise measurement shows the spin polarized channel taking part in the conductance. Parallel to above developments, shot noise measurements in atomic contacts have been used to identify the number of eigen channels participating in the conductance. First such measurement were reported by van den Brom *et al.* [9] on Au and Al atomic contacts, revealing 1 and 3 channels taking part in the conduction, respectively. In a similar system, Dj/"ukic *et al.* [17] have shown the presence of a single dominant channel in the single-molecule junction Pt-D_2-Pt. Shot noise measurements were used as a tool by Tal *et al.* [18] to show the cross over between point contact spectroscopy and inelastic tunneling spectroscopy at $\tau = 0.5$. In this thesis we report on the use of noise to show the effect of electron-electron and electron-vibron interaction in quantum electron transport in atomic and molecular junctions.

1.4 RANDOM TELEGRAPH NOISE

RANDOM telegragh noise is associated to Markovian processes where a random process x(t) is a continuous time function randomly jumping between two well defined states. A process is called Markovian if the probability of the transition between the states depends upon the present state and does not depend upon its previous history. Hence Markovian processes are memoryless.

Such processes are common in systems like atomic contacts, molecular junctions, single electron transistors, tunnel junctions *etc.*, where the resistance of the system fluctuates between a higher resistance R_u and a lower resistance R_l due to the movement of atoms between two metastable states or due to charge fluctuations in a double-well potential. A schematic of such process is shown in figure (1.7). The probability of making a single transition from state R_i to state R_j in a small interval time δt is inversely proportional to the mean life time τ_i of state R_i. Hence the decay rate for occupational probability of states R_l and R_u can be defined as $\delta t/\tau_l$ and $\delta t/\tau_u$. The probability to find the system in state R_i can be

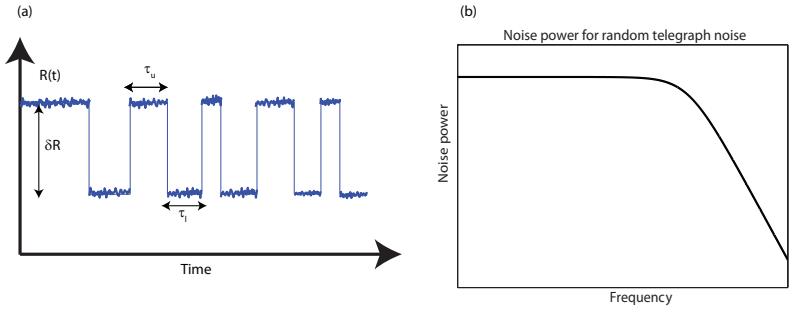

FIGURE 1.7: Illustration of two-level fluctuations. (a) A quantum system is jumping between two metastable states with finite mean life time and the electrons participating in the quantum transport sense the two states. Hence the effective transmission probability of the electron fluctuates between two values. Here, such phenomenon is translated in a change in the resistance of the system δR with finite life time τ_u for the higher resistive state and τ_l for the lower resistive state. (b) The power spectral density of the corresponding two level fluctuations with a Lorentzian line shape with cut-off frequency given by the mean life time *i.e.* $f_0 = \frac{1}{\tau_u + \tau_l}$ of the two states.

given as,

$$P_i = \frac{\tau_i}{\sum_{i=u,l} \tau_i}. \tag{1.30}$$

Since a random telegraph signal is memoryless, the switching between two states with known average rates can be described by a Poisson distribution and its characteristic function. The autocorrelation function for this Poisson process is given as:

$$\begin{aligned} \phi(\tau) &= P_1 P_2 (R_l - R_u)^2 e^{-(\nu_1+\nu_2)\tau} \\ &= P_1 P_2 (\delta R)^2 e^{-\nu\tau} \end{aligned} \tag{1.31}$$

where $\nu = (\nu_1 + \nu_2)$ and $\nu_i = 1/\tau_i$. The Fourier transform of $\phi(\tau)$, using the Wiener-Khintchine theorem, gives its power spectral density.

$$S_R(\omega) = 4\delta R^2 P_1 P_2 \left[\frac{\tau}{1+\omega^2\tau^2} \right] \tag{1.32}$$

The power spectrum of a random telegraph signal (RTS) is Lorentzian in shape with a cutoff frequency given by the mean life time of the two states, and one such spectrum is shown in figure (1.7).

1.5 1/f NOISE

$\frac{1}{f}$ NOISE or flicker noise as it was termed by Schottky, often hampers low frequency noise measurements. It has universal presence in all kinds of conductors and semiconductors. Its power spectral density increases with the decrease of frequency as suggested by its onomatopoetic name $1/f$. There is a famous paradox related to $1/f$ noise, "If $1/f$ noise behavior holds down to zero frequency then its noise power would be infinite"[19]. Until today no experimental evidence has shown this behavior. All the experiments have finite frequency bandwidths. The lower bound of frequency(f_l) is limited by the measurement time(t) *i.e.* $f_l \propto \frac{1}{t}$ and hence to observe such an infinite power, one needs to measure for infinite duration of time. However, until now none of the experiments has shown any plateau appearing for f approaching zero. At higher frequency, the power spectral density of $1/f$ noise diminishes until it reaches the noise floor of amplifier and hence it becomes immeasurable at higher frequency. The corner frequency of $1/f$ noise is defined as the frequency point at which its noise spectral density sinks into the noise floor of the amplifier. Depending upon the nature of the system, its corner frequency ranges from 10^2 to 10^6Hz. Figure (1.8b) shows the typical $1/f$ noise in an atomic or molecular junction.

Although its presence is universal in all kinds of conductors, there is no central cause for its origin. Its origin is often more complex and widely unknown in condensed matter physics. The most accepted view for its origin in metals is that

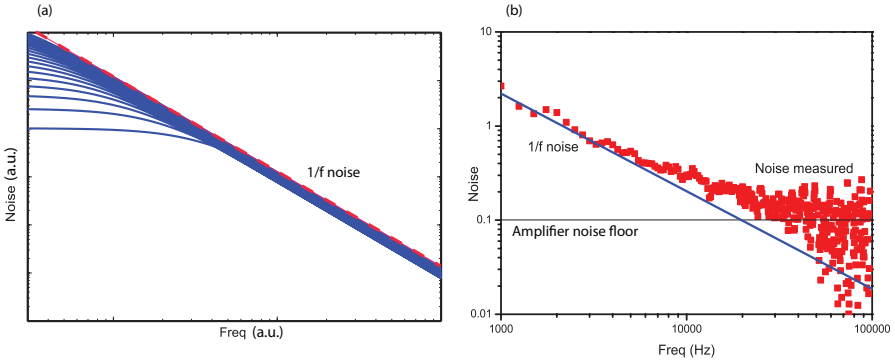

FIGURE 1.8: (a) Conceptual model for the origin of $1/f$ noise as a superposition of different two-level fluctuations with different effective mean life times, adding up to $1/f$ noise behavior. The envelope of the cumulative Lorentzian shapes is shown as the dashed, resembling $1/f$ behavior. (b) Power spectral density of $\frac{1}{f}$ noise in a $Au - O_2 - Au$ junction. The typical corner frequency for such systems is around 30KHz.

it can be interpreted as a superposition of many two-level fluctuations with different amplitudes δR_i and a range of decay rates ν_i. Its power spectrum can be calculated as the sum over all individual RTS. If the time constant of the ensemble is within the limit $\tau \in \{\tau_1, \tau_2\}$ and the probability distribution of the system to stay in state R_i at time t is given as P_i with its power spectral density$S_{RTS}(\omega, \tau_i)$ given by equation(1.32), then total power spectrum would be given by,

$$S(\omega) = \int_{\tau_1}^{\tau_2} S_{RTS}(\omega, \tau_i) P_i d\tau_i \propto \frac{1}{f} \tag{1.33}$$

for the frequency range $\frac{1}{\tau_2} < f < \frac{1}{\tau_1}$. A conceptual demonstration is shown in figure (1.8a), where different Lorentzians with different time constants are integrated to obtain a $1/f$-like spectral density behavior.

Detailed studies on the properties of $1/f$ noise have been done on metallic and semiconductor systems by Hooge [20]. He has proposed a phenomenological expression for metallic systems,

$$S_v(f) = \gamma \frac{V^{2+\beta}}{N_c f^{\alpha}} \tag{1.34}$$

This formula is known as Hooge's formula. Here α, β and γ are constants. N_c is the number of charge carriers in the sample. The typical values for metallic systems are: $0.9 \leq \alpha \leq 1.4$, $\gamma \simeq 2 \times 10^{-3}$, $\beta \simeq 0$.

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