Programmable mechanical metamaterials

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Citation

Florijn, H. C. B. (2016, November 29). *Programmable mechanical metamaterials*. *Casimir PhD Series*. Retrieved from https://hdl.handle.net/1887/44475

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Title: Programmable mechanical metamaterials
Issue Date: 2016-11-29

CHAPTER 1

INTRODUCTION

Metamaterials are artificial materials with properties that may not be found in nature. They derive their unusual properties from their architecture, rather than from their composition [1]. Metamaterials were first introduced in optics to create materials with a negative refractive index [2–4].

1.1 Optical Metamaterials

The refractive index (n) of a material defines how light is refracted when entering a medium. By definition, $n = 1$ for vacuum. For all naturally occurring transparent materials the refractive index is real and larger than zero. This implies, according to Snells law, that when a light beam is traveling from a medium with low refractive index (say $n = 1$) and entering a medium with a higher refractive index ($n > 1$), with an angle smaller than the critical angle, the beam bends toward the line normal to the surface [5] (Fig. 1.1(a))

In a paper published in 1968 [2], soviet physicist Victor Veselago predicted that materials with simultaneously negative electric permittivity (ϵ) and *negative magnetic permeability* (μ) have a negative refractive index. Note, these materials are also often referred to as Left Handed Materials, LHM, as electromagnetic waves travel opposite to the direction of energy flow. When a light beam is traveling from a medium with low refractive index ($n = 1$) and entering a medium with a *negative* refractive index ($n < 0$), the beam bends away from the line normal to the surface (Fig. 1.1(b)). Artis-

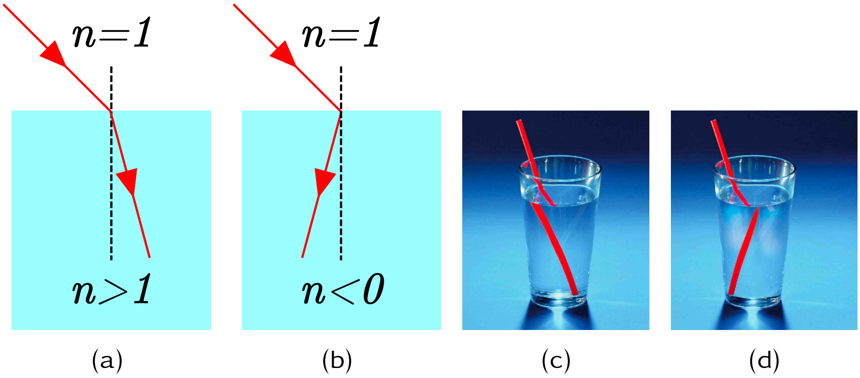

FIGURE 1.1: (a) Beam of light (red line, triangles indicate direction of the group velocity vector) traveling from a medium of low refractive index ($n = 1$) and entering a medium of higher refractive index ($n > 1$), bends towards towards the line normal to the surface (black dashed line). (b) Beam of light (red line, triangles indicate direction of the group velocity vector) traveling from a medium of low refractive index ($n = 1$) and entering a medium with *negative* refractive index ($n < 0$), bends away from the normal. (c) Artistic impression of a straw in a fluid with $n = 1.33$ (water). (d) Artistic impression of a straw in a fluid with $n = -1.33$. Images (c) and (d) are adopted from [6].

tic impressions of a fluid with $n = 1.33$ (water) and with $n = -1.33$ are shown in Fig. 1.1(c)-(d) [6]. Materials with negative refractive index are extremely desired as they can be used to make super (flat) lenses [4, 7] with sub wavelength resolution, beating Abbes diffraction limit [3, 8, 9]. However, bulk materials with negative magnetic permeability can not be found in nature, and the study of materials with negative refractive index was seen as an exclusive theoretical concept.

In the last decade, optical metamaterials have been introduced [10–13], consisting of a periodic array of split ring resonators (Fig. 1.2). These materials have the required magnetic coupling to an electromagnetic field (light) to have an effective negative magnetic permeability, and thus can have a negative refractive index. Ongoing research has dramatically reduced the size of these unit cells and nowadays there exist negative index materials at resonance wavelengths close to the wavelength of visible light [14–16]. Moreover, bulk ultraviolet negative refractive index materials have been made by stacking plasmonic waveguides [17]. The design of optical metamaterials is also a boost for the field of transformation op-

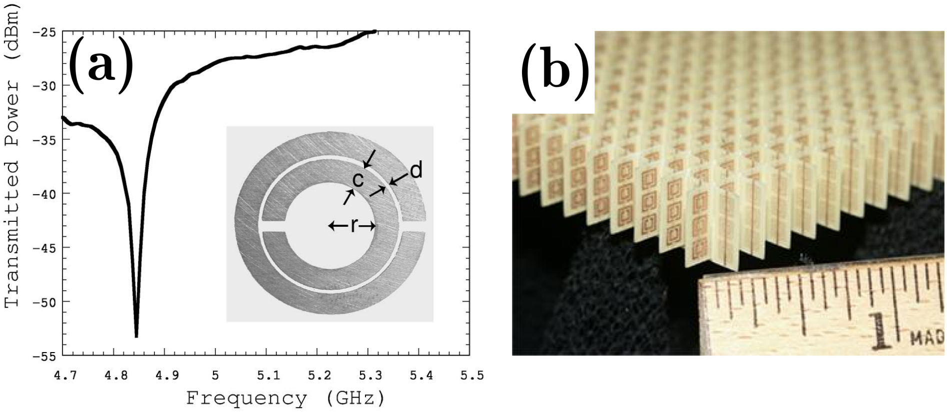

FIGURE 1.2: (a) A (copper) split ring resonator ($c = 0.8$ mm, $d = 0.8$ mm, $r = 1.5$ mm) has a strong coupling with an incident (time varying) magnetic field, visible in the dip in the transmitted power at the resonance frequency (4.845 GHz). Image adopted from [10]. (b) An array of split ring resonators acting as a negative refractive index material around the resonance frequency of the split ring resonators. The resonance wavelength of a single split ring resonator can be made much larger than the lattice spacing between individual split ring resonators in a metamaterial, hence the array of split ring resonators behaves as an effective medium near the resonance frequency. Image adopted from [19]

tics, as materials can be designed with spatial varying optical properties, in order to guide electromagnetic waves through the material as desired, making optical cloaking at visible wavelengths something to be achieved in the near future [18]!

1.2 Thermal Metamaterials

In a thermal metamaterials the flow of heat is controlled by a carefully designed micro structure of materials with different thermal conductivities, to build super coolers or thermal cloaks. The idea of a thermal cloak is to make an object thermally invisible. This means that when heating, e.g., a 2D sample homogeneously from the left side using a thermal bath, the heat flow distribution on the right side of the sample is similar of that of a homogenous plate (Fig. 1.3). Hence, iso-temperature curves on the right side of the metamaterial should be straight vertical lines (as would be the

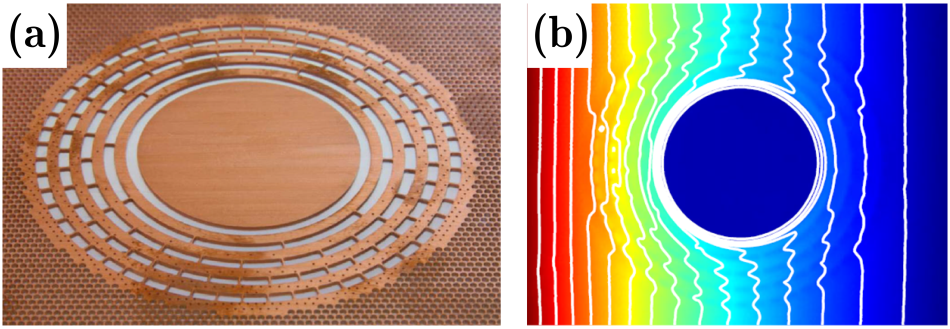

FIGURE 1.3: (a) A thermal cloak. The central copper cylindrical plate is made thermally invisible by shielding it with rings composed of PDMS and copper. (b) Thermal heat flows from left (hot, red) to right (blue, cool). The object remains cool and the iso-thermal curves (white lines), although distorted close to the object, are vertical lines further away from the object. Images adopted from [21]

case for a homogenous plate)(Fig. 1.3(b)). Note that cloaking is not the same as shielding, since a shielded object is visible as iso-thermal curves will not be straight lines when passing the shielded object [20–24]
Shown in Fig. 1.3(a) is an example of a thermal cloak [21]. Thermal cloaking is achieved by designing a metamaterial consisting of copper and PDMS (which has a heat conductivity 2600 times lower than that of copper) rings, around a central copper plate (the object that has to be made invisible). Each ring is designed such that the average radial heat conductivity is decreasing when the radius of the ring is decreasing while the azimuthal heat conductivity remains constant for all rings. When heating the sample from the left, using a thermal heat bath, iso-thermal curves are distorted close to the object. However, soon after passing the object, these lines become straight again. Hence, the copper plate is thermally invisible.

1.3 Mechanical Metamaterials

In mechanical metamaterials the materials response to a deformation is tuned by the precise design of the micro structure. Recent developments in computing, manufacturing (3D printing) and theoretical concepts have lead to the revolutionizing approach of bottom-up design of such materials, and designer matter in general [25].

1.3.1 Linear Material Properties

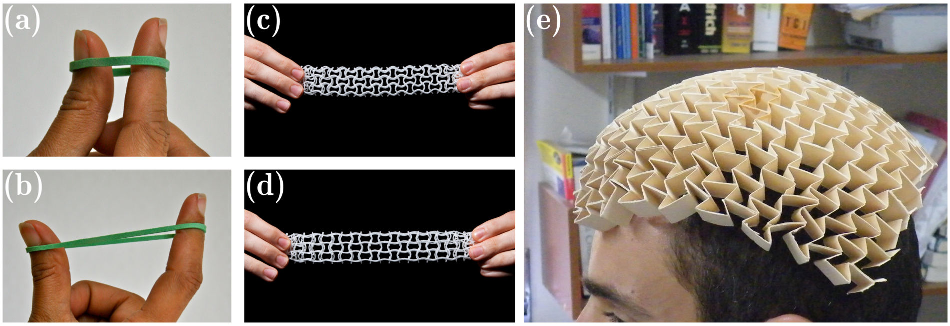

FIGURE 1.4: (a)-(b) A rubber ribbon (which has a Poissons ratio close to 0.5) shrinks in the transverse directions when stretched. Image courtesy of www.thatfirst.com. (c)-(d). An auxetic material (which has a Poissons ratio less then zero) expands in the transverse direction when stretched. Images courtesy of Bradley Rothenberg. (e) Due to the negative poissons ratio, auxetic materials have bending properties that make them curve perfectly around dome shaped objects. Image: courtesy of Auxetic Materials Research Group University of Malta

An example of a material property that can be pushed in a surprising direction by metamaterials is the Poisson's ratio. When an ordinary material, like rubber, is stretched (compressed) in the axial direction, it will contract (expand) in the transverse directions by (Fig. 1.4(a)-(b)). The negative ratio between these deformations is called the Poisson's ratio ν, $\nu = -\varepsilon_{trans}/\varepsilon_{axial}$, where ε_{trans} and ε_{axial} are the strains, which describe the deformations. For most materials, the Poisson's ratio is within the range $0 \leq \nu \leq 0.5$, with $\nu = 0.5$ for a perfect incompressible material and ν close to zero for cork (which is therefore used to seal bottles as it does not expands in the lateral directions when pushed in the neck of the bottle). By careful design of the materials micro structure, materials with a negative Poisson's ratio (also knows as *auxetic* or dilational materials) have been developed [26]. These materials expand (contract) in the transverse direction when stretched (compressed) in the axial direction (Fig. 1.4(c)-(d))!
Auxetic materials have bending properties such that they can be perfectly shaped around dome like (positive Gaussian curvature) objects [26] (Fig. 1.4(e) and have therefore applications in footwear, clothing and prosthet-

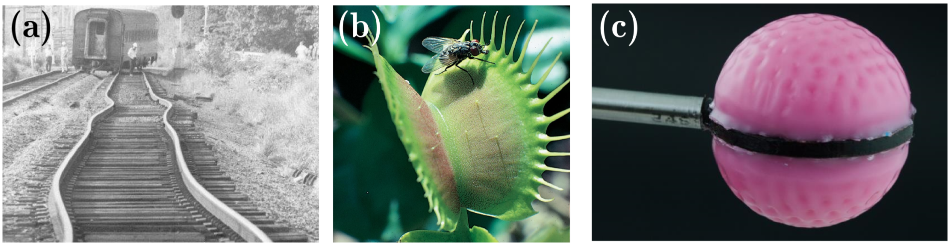

FIGURE 1.5: (a) Buckling as failure, sun kinks in railroad tracks. Image courtesy of AP (b) The rapid closing of the mouth of a venus fly trap is induced by a snap-buckling instability. Image courtesy of www.carolina.com (c) Wrinkling patterns at the surface activated by elastic instabilities. Image courtesy of MIT/P.Reis

ics. Furthermore, as the stiffness of these materials locally increases when the material undergoes indentation, auxetic materials are also useful for shock absorbing/damping applications [27]. Although the existence of these materials was both experimentally observed and theoretically understood in natural foam structures [26] since 1987, recent theoretical and experimental developments have lead to the design of many more such materials. Other important examples of (linear) mechanical metamaterials are materials with vanishing shear modulus [28–30]. These materials are hard to compress since they have a finite bulk modulus and behave similar to a liquid (easy to deform as the shear modulus approaches zero) when sheared. Finally, metamaterials with negative compressibility [31,32] have been designed.

1.3.2 Nonlinear Mechanical Metamaterials

Going beyond linear response, a range of metamaterials have recently been developed which harness geometric nonlinearities and elastic instabilities to obtain novel functionalities. Until recently, elastic instabilities, in particular buckling, were seen as a mode of failure [33]. When compressing a beam, at a force larger than the critical force, the energy of the compressed but straight beam is higher than that of the curved beam. The straight configuration of the beam then becomes unstable and the beam buckles. Elastic instabilities are undesired when designing buildings or railway tracks, Fig. 1.5(a), however, are used in nature by for example the venus fly trap to catch flies [34] (Fig. 1.5(b)). A small disturbance of the unstable state (open mouth) results into a large deformation, fed by a snap-buckling instability, towards a stable state (closing mouth). Elastic instabilities have recently been used to design *smorphs*, smart morphable surfaces, where for example the switching between the different wrinkling patterns on the surface, induced by the elastic instabilities [35, 36], is exploited to tune the drag reduction [37] (Fig. 1.5(c)). Moreover, small disturbances, in for example force or pressure, close to an elastic instability can result in large instantaneous material deformations. Therefore elastic instabilities are utilized to fabricate soft actuators [38, 39], and soft robotics [40, 41].

Collective buckling of beam like structures, such as observed in quasi 2D slabs perforated with a square array of holes, display a rapid pattern transformation [42, 44–47]. When compressed, these “holey sheets” undergo a buckling induced pattern switch, from circular holes to orthogonal ellipses (Fig. 1.6(a)-(b)). Before the pattern transformation the sample has a positive Poisson’s ratio, while after the pattern transformation, its (differential) Poisson’s ratio is negative (as the sample is compressed further, the lateral boundaries move inward, which is visible at the curved lateral boundaries in Fig. 1.6(b)). This can be explored to obtain switchable auxetics [46].
A 3-dimensional auxetic material has been created by pattering a soft spherical shell with circular dimples (Fig. 1.6(c)-(d)). When the air of this so called buckliball is extracted, using a syringe, the ligaments between the circular dimples collapse in a collective buckling cascade. The circular dimples become elliptical dimples, that eventually close, resulting in an overall reduction of the volume of the cylindrical shell by 54% [43, 48].
Expanding the idea of pattern transformation induced by collective buckling in the holey sheets, a full 3D solid material with negative Poissons ratio has been designed and fabricated using a stacking of a cuboid shaped unit cell [49]. There exist an extremely large amount of ways these unit cells can be tiled together (which grows as 2^{L+L^2}, where L is the cube size) and still demonstrate collective buckling (this in contrast to the 2D and quasi 3D cases (Fig. 1.6(a)-(d)), where there is only one way to tile space).

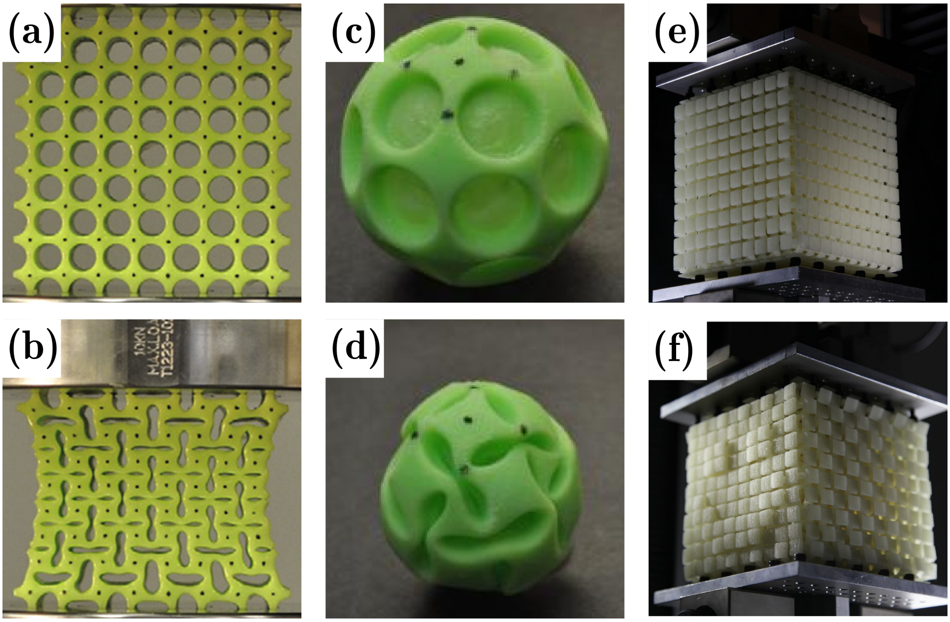

FIGURE 1.6: (a)-(b) When compressed, a quasi 2D slab of rubber perforated with a square array of holes undergoes a pattern transformation from circular holes to orthogonal ellipses. Images adopted from [42] (c)-(d) A similar patter change is observed in a spherical shell patterned with circular dimples, when air is extracted out. Images adopted from [43] (e)-(f) When a metacube, consiting of a specific stacking of cuboid shaped unit cells, is compressed, a smiley face texture is generated at one of the surfaces of the cube.

Designing the precise geometry of the tiling of the cuboid unit cells can then be used to create buckling induced surface textures on demand. Moreover, buckling induced pattern transformations, at the right dimensions, can be exploited to design switchable acoustics [50–52] and optics [53] as phononic and photonic properties are highly sensitive to geometrical parameters.

An important limitation common to all these metamaterials is that each mechanical functionality requires a different structure. (For example, for the case of the textured mechanical metamaterial shown in Fig. 1.6(e)-(f), every texture of the surface requires a different stacking of the constituent unit cells.) In this thesis we present a novel strategy to create programmable mechanical metamaterials, where the response of a single structure is determined and can be changed by lateral confinement. These materials consist of quasi 2D elastic sheets perforated by square arrays of holes of two different diameters. The different hole sizes break rotational symmetry which leads to highly nonlinear and tuneable coupling of deformations along the two primary axes (x and y) of the material. By controlling the boundary conditions of the material along x, the response to uniaxial compression along y can be varied over a wide range of behaviors, including monotonic, unstable, bistable and multi stable behavior.

1.4 This Thesis

This thesis starts with outlining the strategy to design a programmable mechanical metamaterial based on perforated sheets of rubber in Chapter 2. Then, in sections 2.2 and 2.3, the fabrication techniques, material properties and experimental setup for testing and making these sheets is described in detail. In section 2.4 we show for the first time that the mechanical response to uniaxial loading of a biholar sheet can be programmed by lateral confinement and exhibits four different mechanical regimes. Next, by performing numerical simulations on a unit cell with periodic boundaries, we show that this phenomenology is robust and is a bulk property, (section 2.5) and can also be captured in a simple model (2.6). In Chapter 3 we discuss the model in more detail and study the different bifurcation scenarios leading to the four different mechanical regimes. Moreover, a geometrical interpretation is presented that denotes the path to rational design of different types of (programmable) mechanical metamaterials. In chapter 4 we experimentally and numerically study the role of geometry for the mechanics of biholar metamaterials. We describe the behavior for the limiting cases where the relevant geometrical parameters become large, and discuss the new physics that arises there. Finally, in chapter 5 we show that the mechanical response and programmability of biholar metamaterials is sensitive to the configuration of the clamps. This leads to a complete new “barcode” programmability of our biholar metamaterials, where the location or absence of clamps at certain locations can be used to strongly influence the mechanical response of our materials to uniaxial compression.