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Depairing currents in superconducting films of Nb and amorphous MoGe

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We report on measuring the depairing current J_{dp} in thin superconducting films as a function of temperature. The main difficulties in such measurements are that heating has to be avoided, either due to contacts, or to vortex flow. The latter is almost unavoidable since the sample cross section is usually larger than the superconducting coherence length ξ_s and the magnetic field penetration depth λ_s . On the other hand, vortex flow is helpful since it homogenizes the distribution of the current across the sample. We used a pulsed current method, which allows us to overcome the difficulties caused by dissipation and measured the depairing current in films of thin polycrystalline Nb (low λ_s , low specific resistance ρ) and amorphous Mo_{0.7}Ge_{0.3} (high λ_s , high ρ), structured in the shape of bridges of various width. The experimental values of J_{dp} for different bridge dimensions are compared with theoretical predictions by Kupriyanov and Lukichev for dirty limit superconductors. For the smallest samples we find a very good agreement with theory, over essentially the whole temperature interval below the superconducting critical temperature.

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I. INTRODUCTION

The superconducting current density J_s is a unique feature of a superconducting material. It can be expressed as $J_s = en_s v_s$, where n_s and v_s are the density and velocity of the superconducting electrons, respectively, and e is the electron charge. Increasing J_s leads to increase of v_s but also to a reduction of the number of Cooper pairs. Finally, when J_s reaches the depairing current J_{dp} , the amount of carriers is not enough to support the supercurrent and the superconducting state collapses. For conventional superconductors the temperature dependence of J_{dp} near the critical temperature T_c is given by the classical Ginzburg-Landau (GL) expression $J_{dp}^{GL}(t) = J_{dp}^{GL}(0)(1-t)^{3/2}$, where $t = T/T_c$, and $J_{dp}^{GL}(0)$ is the depairing current at zero temperature. Early work on determining J_{dp} in Sn microbridges can be found in Refs. 1 and 2. The GL approach becomes invalid at lower temperatures, since the conditions $k^2 \gg 1 - T/T_c$ for clean limit superconductors (κ is Ginzburg-Landau parameter), or $(T_c - T) \ll T_c$ for dirty limit superconductors, are no longer fulfilled. A more complete and quantitative theory, valid for all temperatures and arbitrary mean free path, was developed by Kupriyanov and Lukichev (KL), who obtained the numerical solution of the Eilenberger equations for a superconductor carrying a current, with the velocity of the Cooper pairs proportional to a phase gradient of the superconducting order parameter Δ^3 . Notably, their theory gives the same expression for $J_{dp}(t)$ as GL theory for the temperature region close to T_c and also yields the correct expressions for $J_{dp}(0)$ in terms of the materials constants.

The amount of theoretical work done on depairing currents in conventional superconductors contrasts sharply with a lack of experimental observations, possibly because it is believed they would not yield new or relevant information. This may be so for simple superconductors, but for hybrid structures such data can provide very interesting information. For instance, in the case of ferromagnet/superconductor (F/S) combinations, well-known issues are the oscillatory

order parameter which can be induced in the F layer (the so-called π state) or the suppression of superconductivity by switching the magnetization of the F layers from antiparallel to parallel (the superconducting spin switch). In both cases, extensive use has been made of variations in T_c (for the π state, see, e.g., Refs. 4 and 5, for the spin switch see Refs. 6 and 7), but these are generally very small and prone to spurious effects. Using J_{dp} could give more unambiguous results, but would also allow to follow the state of the system below T_c and, for instance, detect a $0-\pi$ crossover. Another example is the case of spin-polarized quasiparticle injection. This presumably suppresses the order parameter, but the common use of an arbitrary voltage criterion does not allow to discern between this suppression or, for instance, vortex depinning.^{8,9} Before J_{dp} can be used for such purposes, it has to be shown that it can be measured reliably in different systems to far below T_c . Here we show this is possible for such different superconductors as Nb and amorphous Mo_{0.7}Ge_{0.3}.

Generally, a major issue is the requirement with respect to sample dimensions. In principle, the sample width should not be larger than both the penetration depth λ_s , and the coherence length ξ_s . The first condition is needed to avoid current piling up at the edges, because of the Meissner effect.¹⁰ For a superconducting film λ_s is given by λ_b^2/d_s ($d_s \ll \lambda_b$), where λ_b is the bulk London penetration depth, d_s is film thickness, and the magnetic field is taken perpendicular to the film plane. At low temperatures in case of dirty superconductors it becomes $\lambda_b^2(\xi_0/\ell d_s)$, where ξ_0 is the BCS coherence length and ℓ is the elastic mean free path. A typical value of λ_b , for instance, for polycrystalline Nb, is 50 nm; for amorphous materials such as a -Mo_{0.7}Ge_{0.3}, λ_b is much larger, of the order of 0.5 μ m. The condition on ξ_s must be fulfilled when vortex nucleation and flow is to be prevented, which cause dissipation in sample before the J_{dp} is reached. Exact calculations made by Likharev¹¹ show that the smallest sample width below which no vortex can appear equals $4.4\xi_s(T)$, where $\xi_s(T)$ is the Ginzburg-Landau coherence length given

by $\xi_s(T) = 0.85 \xi_s(0) / \sqrt{1-t}$, with $\xi_s(0) = \sqrt{\xi_0 \ell}$. Typical values of $\xi_s(0)$ for our Nb and $\text{Mo}_{0.7}\text{Ge}_{0.3}$ are 12 nm (because of the small mean free path) and 5 nm, respectively. The only case where both conditions can be implemented is a thin aluminum film shaped in a form of a narrow (about 1 μm) bridge. The BCS coherence length for Al is of the order of 1.5 μm , while the penetration depth can be increased to a similar value by decreasing the film thickness. Romijn *et al.*¹² showed that for such system the experimental values of the depairing current density were in excellent agreement with KL theory for temperatures down to $0.2t$. In case of Nb and $\text{Mo}_{0.7}\text{Ge}_{0.3}$ films one would have to go to a bridge width not larger than 30 nm in order to prevent vortex appearance.

However, vortex motion also has an advantage, since it will homogenize the current distribution.¹³ The main problem then in determining J_{dp} is to avoid sample heating, either by dissipation due to vortex motion or, e.g., to heating in the contacts due to the relatively large currents. In this paper we demonstrate that the undesired sample heating can be avoided by using a pulsed current method. We use different superconductors, with widely different values of J_{dp} . Specifically, we use Nb with low λ_b and also relatively low specific resistance ρ (around 7 $\mu\Omega\text{ cm}$) and amorphous (*a*-) $\text{Mo}_{0.7}\text{Ge}_{0.3}$ with large λ_b and a large $\rho \approx 160 \mu\Omega\text{ cm}$. Especially, the large ρ easily leads to dissipation in the neighborhood of the transition to the normal metal state. Films of different thicknesses were patterned into bridges of different width w_s . The experimental values we obtain for the depairing current density $J_{dp}(t)$ are in very good agreement with the KL calculations, assuming that the current distribution across the samples is perfectly homogeneous.

II. EXPERIMENT

Nb single layer films were grown by dc magnetron sputtering in an ultra high vacuum system with a background pressure of about 10^{-10} mbar and an Ar sputtering pressure of 6×10^{-3} mbar. Films of *a*- $\text{Mo}_{0.7}\text{Ge}_{0.3}$ were deposited in a RF-diode sputtering system with a background pressure of 10^{-6} mbar in an Ar pressure of 8×10^{-3} mbar. Sputtering rates for Nb and *a*- $\text{Mo}_{0.7}\text{Ge}_{0.3}$ were 0.8 and 1.2 $\text{\AA}/\text{s}$, respectively. Both materials were grown on Si(100) substrates. The thickness of the films was determined during the deposition by a crystal thickness monitor, which was calibrated by low angle x-ray diffraction measurements and Rutherford backscattering. For the depairing current experiments, samples were structured in the shape of strips of different cross section by *e*-beam lithography and Ar-ion etching. The structure included the contacts. In the case of *a*- $\text{Mo}_{0.7}\text{Ge}_{0.3}$, samples were water-cooled during deposition and liquid nitrogen-cooled during etching, in order to prevent undesirable film crystallization. The typical geometry of the samples is shown in Fig. 1. In all cases the distance between voltage leads was $100 \pm 1 \mu\text{m}$. The width of resistive transition from the normal into the superconducting state was about 30 mK for all samples. An example for both materials is given in Fig. 2. Transport measurements in the normal state yielded an average value of specific resistance ρ of about 160 $\mu\Omega\text{ cm}$ for $\text{Mo}_{0.7}\text{Ge}_{0.3}$ and 7.2 $\mu\Omega\text{ cm}$ for Nb samples, respectively. For

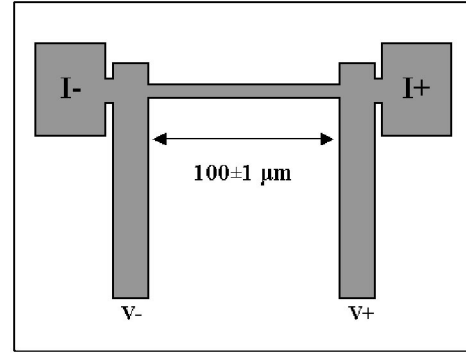


FIG. 1. Sample layout. The measurement procedure was performed with a classical four-point scheme. The massive current leads provide a good heat sink.

a- $\text{Mo}_{0.7}\text{Ge}_{0.3}$ the elastic mean free path ℓ is taken to be 0.4 nm,¹⁴ of the order of the interatomic distances and these samples are clearly in the dirty limit. For Nb, using the expressions of the free electron model with the product $\rho\ell = 3.75 \times 10^{-16} \Omega\text{ m}^2$ and the Fermi velocity $v_F = 5.6 \times 10^5 \text{ m/s}$ we find $\ell = 5.2 \text{ nm}$. Comparing this value to $\xi_0 = 39 \text{ nm}$ for Nb,¹⁵ it is seen that the dirty limit condition $\ell \ll \xi_0$ is also satisfied. The depairing currents measurements were performed in a ^4He cryostat shielded from external magnetic fields by a long permalloy ($\text{Ni}_{0.8}\text{Fe}_{0.2}$) screen annealed in hydrogen atmosphere. Hall probe measurements showed a constant magnetic field background less than 10^{-5} T . The samples were mounted on a massive brass holder with a resistive heater. In order to reduce possible errors in the temperature determination because of the temperature gradient along the sample holder, all samples were placed in immediate proximity to the thermometer. The temperature stability during the experiment was about 1 mK. For determination of the critical current value I_{dp} at different temperatures a pulsed current method was used, in which current pulses with a growing amplitude were sent through the sample. The average duration of a single pulse was about $3.00 \pm 0.05 \text{ ms}$. Each pulse was followed by a long pause of $7.0 \pm 0.1 \text{ s}$. The voltage response of the system was observed

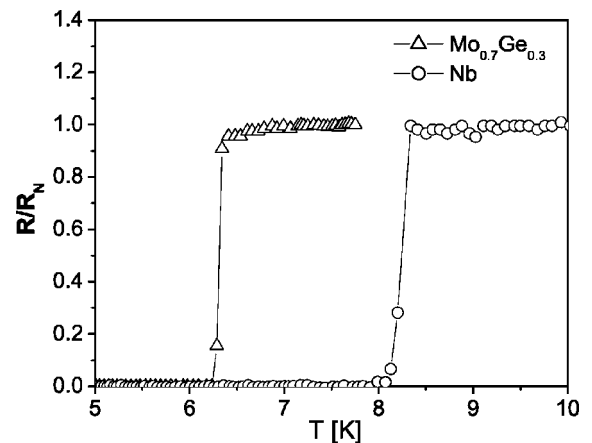


FIG. 2. Resistance normalized to its normal state value at 10 K as a function of temperature for a Nb bridge ($w_s = 1 \mu\text{m}$, $d_s = 20 \text{ nm}$) and an *a*- $\text{Mo}_{0.7}\text{Ge}_{0.3}$ bridge ($w_s = 2 \mu\text{m}$, $d_s = 64 \text{ nm}$).

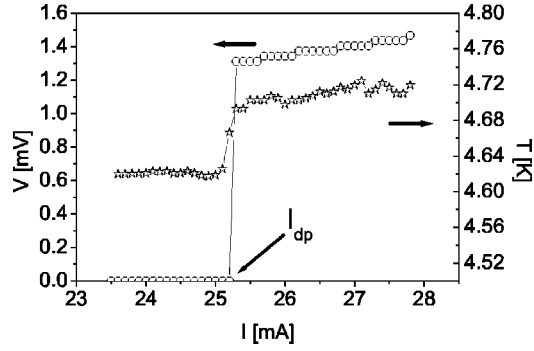


FIG. 3. Typical dependence of voltage V (open circles) and temperature T (open stars) on current I , measured on a $2\ \mu\text{m}$ wide $a\text{-Mo}_{0.7}\text{Ge}_{0.3}$ bridge.

on an oscilloscope triggered for the time of a single pulse. To improve the signal resolution a differential amplifier was used, combined with low-noise band filters. A typical current- I -voltage (V) characteristic for $a\text{-Mo}_{0.7}\text{Ge}_{0.3}$ at a reduced temperature of $t=0.74$ is shown in Fig. 3. One can see a clear jump from the superconducting to the normal state at I_{dp} . For temperatures close to T_c a small onset of voltage was observed in all samples, probably because of vortex motion. In order to make certain that this effect has no influence on the determination of I_{dp} , the temperature was monitored during every current pulse. Measurable differences were found very close to I_{dp} , as shown in Fig. 3. We conclude that a short pulse in a combination with a long pause does not cause sample heating and keeps the system in temperature equilibrium until the dissipation related to the normal state occurs.

III. RESULTS AND DISCUSSION

To illustrate the raw data, experimentally determined values of J_{dp} as a function of reduced temperature t for two bridges of Nb ($d_s=20\ \text{nm}$, $w_s=1\ \mu\text{m}$) and $a\text{-Mo}_{0.7}\text{Ge}_{0.3}$ ($d_s=64\ \text{nm}$, $w_s=2\ \mu\text{m}$) are shown in Fig. 4. Between $t=1$ and $t=0.85$ both curves show a clear upturn, which indicates the

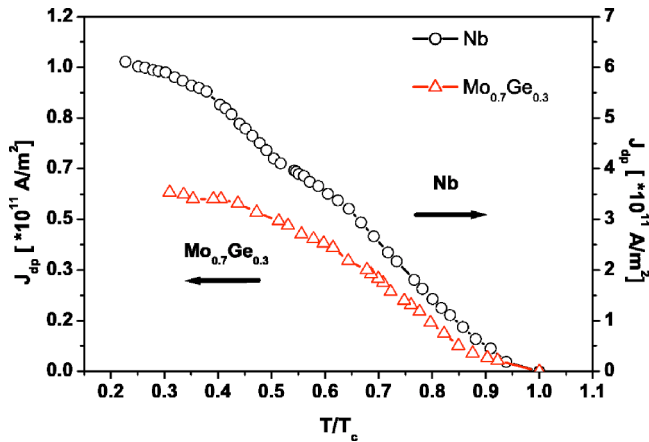


FIG. 4. Experimental results for pair-braking current J_{dp} as a function of reduced temperature for a Nb bridge ($d_s=20\ \text{nm}$, $w_s=1\ \mu\text{m}$) and an $a\text{-Mo}_{0.7}\text{Ge}_{0.3}$ bridge ($d_s=64\ \text{nm}$, $w_s=2\ \mu\text{m}$).

TABLE I. Transport and superconducting properties of the Nb and $\text{Mo}_{0.7}\text{Ge}_{0.3}$ samples. Here d_s and w_s are the film thickness and bridge width, respectively, T_c is the sample critical temperature, ρ is the measured specific resistance, $J_{\text{dp}}(0)$ and $J_{\text{dp}}^{\text{GL}}(0)$ are extrapolated and calculated critical current density at zero temperature.

Sample	d_s [nm]	w_s [μm]	T_c [K]	ρ [$\mu\Omega\ \text{cm}$]	$J_{\text{dp}}(0)$ $10^{11}[\text{A}/\text{m}^2]$	$J_{\text{dp}}^{\text{GL}}(0)$ $10^{11}[\text{A}/\text{m}^2]$
Nb	20	1.0	8.3	7.25	17	15
Nb	40	2.0	9.0	7.24	16	17
Nb	53	2.5	9.0	7.24	19	17
Nb	53	5.0	9.0	7.24	20	17
MoGe	64	2.0	6.25	160	2.0	1.6
MoGe	64	5.0	6.25	160	2.1	1.6
MoGe	64	7.0	6.25	160	2.0	1.6

expected GL behavior. Plotting $J_{\text{dp}}^{2/3}$ as a function of t in this temperature region results in a straight line, which can be used to extrapolate $J_{\text{dp}}(t)$ to zero temperature. Table I shows the values of $J_{\text{dp}}(0)$ for all samples investigated. It can also be used to obtain the normalized temperature dependence $[J_{\text{dp}}(t)/J_{\text{dp}}(0)]^{2/3}$, which has a universal form in KL theory. Plots of this quantity for samples with different bridge widths are shown in Fig. 5 for Nb and in Fig. 6 for $a\text{-Mo}_{0.7}\text{Ge}_{0.3}$. Both the absolute values of $J_{\text{dp}}(0)$ and the temperature dependence can be directly compared to the KL results, which we now briefly reiterate.

Close to T_c the depairing current density can be written as follows:

$$J_{\text{dp}}^{\text{GL}}(t) = 1.93\chi^{1/2}(\rho)eN(0)v_Fk_BT_c(1 - T/T_c)^{3/2}, \quad (1)$$

where $\chi(\rho)$ is the Gor'kov function controlled by a dimensionless parameter characterizing the amount of electron

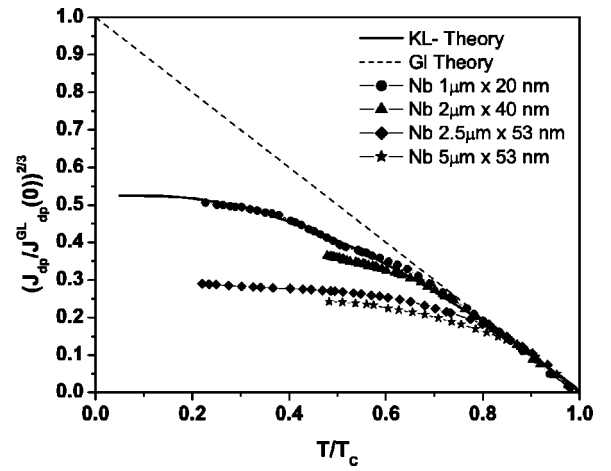


FIG. 5. Experimental results for the pair-braking current density J_{dp} normalized to its extrapolated value $J_{\text{dp}}(0)$ as a function of reduced temperature in Nb bridges of different width and thickness as denoted. The black solid and dashed lines indicate KL and GL results, respectively.

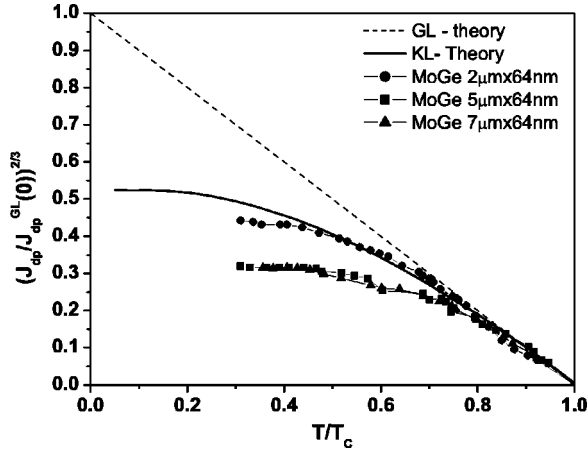


FIG. 6. Experimental results for the pair-braking current density J_{dp} normalized to extrapolated value $J_{dp}^GL(0)$ as a function of reduced temperature in $\text{Mo}_{0.7}\text{Ge}_{0.3}$ bridges of different width and thickness as denoted. The black solid and dashed lines indicate KL and GL results, respectively.

scattering $\rho = (\hbar v_F) / (2\pi k_B T_c \ell)$, with ℓ the elastic mean free path and $N(0)$ the density of states at the Fermi level for each spin direction. For $\ell \ll \xi_0$ (dirty limit) $\rho \rightarrow \infty$, which yields for $\chi(\rho) \rightarrow 1.33\ell / \xi_0$. Thus, at zero temperature the extrapolated depairing current density $J_{dp}^{GL}(0)$ becomes

$$J_{dp}^{GL}(0) = 1.26eN(0)v_F\Delta(0)\sqrt{\frac{\ell}{\xi_0}}. \quad (2)$$

Because of the small mean free path in both types of samples, we may assume applicability of the free-electron model, so the density of states $N(0)$ can be expressed as

$$N(0) = \left(\frac{2}{3} e^2 v_F \rho \ell \right)^{-1}. \quad (3)$$

Substituting this formula in Eq. (2) with $\xi_0 = \hbar v_F / \pi \Delta(0)$ and $\Delta(0) = 1.76 k_B T_c$ we obtain

$$J_{dp}^{GL}(0) = 244 \left[\frac{(T_c)^3}{v_F(\rho\ell)\rho} \right]^{1/2}. \quad (4)$$

This result is similar to the one obtained in Refs. 12 and 13. Equation (4) contains only experimental quantities and the $\rho\ell$ product, which is known for both materials from literature.^{14–16} Looking now at Figs. 5 and 6, all curves follow GL behavior down to about $t=0.85$. The values of $J_{dp}(0)$ extrapolated from this region can be compared to the values calculated from Eq. (4) for $J_{dp}^{GL}(0)$. This comparison is made in Table I which gives all relevant parameters for the different samples. Basically, we find quite good agreement for all sample widths. In the case of Nb, the most serious deviation is found for the 5 μm bridge, which is presumably due to contact heating as a result of the larger current. It is interesting to note that the extrapolated values are the same as found

by Geers *et al.*¹³ who used continuous currents and larger bridge widths. The differences are in the extent of the GL regime, which was only found down to $t=0.93$ in the earlier experiments, and also in the temperature dependence below the GL regime. There, the temperature dependence is described by the full KL calculation, which was also performed in Ref. 13. For a single superconducting film, the results for Nb are shown in Fig. 5 by the solid line. The smallest sample ($d=20$ nm, $w=1$ μm) follows the KL theoretical curve down to $t=0.2$ without significant deviations. Wider bridges show a suppression of $J_{dp}(t)$ with respect to the calculated value, again in agreement with earlier results.¹³ Presumably, sample heating via contacts and vortex flow occurs even for the short time of a current pulse. It appears therefore that using low (pulsed) currents, $J_{dp}(t)$ can be determined correctly over the full temperature range for other materials than Al. Circumstances can be somewhat less favorable, however, as shown by the measurements on $\alpha\text{-Mo}_{0.7}\text{Ge}_{0.3}$. These were performed only for a film thickness of 64 nm. In the GL regime the difference between measured and calculated values of $J_{dp}(0)$ is somewhat larger than for Nb (see Table I), with the measured values larger than the calculated ones. It will be clear that this cannot be due to pile up of current at the samples edges, which would yield the opposite effect. Moreover, for amorphous materials this should be less of a problem, since the penetration depths are very large and actually of the order of the smallest bridge width. The difficulty rather lies in the correct determination of $J_{dp}(t)$ close to T_c , with more scatter in the individual points. One reason for this may be the very low vortex pinning which is characteristic of amorphous materials.^{17,18} Another may be that the processing of the film during the structuring process may lead to changes in the material. For instance, the specific resistance we find for the bridges is about 10% lower than for wider structures.¹⁹ Also, thinner films showed increasing ρ and decreasing T_c , which in this thickness regime cannot be well explained by the onset of localization effects.¹⁴ Since amorphous materials are very sensitive to recrystallization, this may be playing a role. Still, the difference between $J_{dp}(0)$ and $J_{dp}^{GL}(0)$ is only 20%, which may still be considered very good. For the temperature dependence (Fig. 6) the result is also similar to Nb. For the smallest bridge, the experimental curve shows good agreement with the theoretical prediction, while for wider bridges the values remain too low.

In summary, we have shown that measurements of depairing currents in conventional type-II superconductors with cross sections larger than their characteristic lengths ξ_s and λ_s is well possible by using a pulsed current method. Using two different superconductors with quite different values of their depairing current, we found good agreement between experiments and theory with respect to both the absolute values and the temperature dependence, over essentially the full range of temperatures. Such an unambiguous determination of a quantity which directly measures the superconducting order parameter should find use in problems posed by systems where the order parameter varies in a nontrivial way, as in mesoscopic superconductor/ferromagnet hybrids.

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