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Bibliography

- [1] J. Alexander, R. Gardner, and C.K.R.T. Jones. A topological invariant arising in the stability analysis of travelling waves. *J. Reine Angew. Math.*, 410:167–212, 1990.
- [2] V.G. Allen, C. Batello, E.J. Berretta, J. Hodgson, M. Kothmann, X. Li, J. McIvor, J. Milne, C. Morris, A. Peeters, and M. Sanderson. A international terminology for grazing lands and grazing animals. *Grass and Forage Science*, 66:2–28, 2011.
- [3] H. Amann. Nonhomogeneous linear and quasilinear elliptic and parabolic boundary value problems. In *Function spaces, differential operators and nonlinear analysis*. Vieweg+Teubner Verlag, 1993.
- [4] H. Amann. *Linear and quasilinear parabolic problems. Volume I: Abstract linear theory*. Birkhäuser, 1995.
- [5] H. Amann. Nonlocal quasilinear parabolic equations. *Russian Math. Surveys*, 60:1021–1035, 2005.
- [6] H. Amann. Quasilinear parabolic problems via maximal regularity. *Advances Diff. Eqs.*, 10:1081–1110, 2005.
- [7] H. Amann, M. Hieber, and G. Simonett. Bounded h_∞ -calculus for elliptic operators. *Diff. Int. Eqs.*, 7:613–653, 1994.
- [8] S.B. Angenent. Nonlinear analytic semiflows. *Proc. R. Soc. Edinb. A*, 115:91–107, 1990.
- [9] R.A. Distel and A.I. Kröpfl. Degradation and recovery of semi-arid rangelands in the transitional zone between the caldenal and monte regions of argentina. In *Proceedings of the IX International Rangeland Congress*, 2011.
- [10] Millennium Ecosystem Assessment. *Ecosystems and Human Well-being; A Framework for Assessment*. Island Press, 2003.
- [11] M. Baldwin and H.W. Hawker. Soil survey of the fort lauderdale area, florida. Technical report, US Department of Agriculture. Bureau of Soils, 1915.
- [12] N. Barbier, P. Couteron, and V. Deblauwe. Case Study of Self-Organized Vegetation Patterning in Dryland Regions of Central Africa. In *Patterns of Land Degradation in Drylands: Understanding Self-Organised Ecogeomorphic Systems*. Springer, 2014.
- [13] A.P. Barrett, T.H. Painter, and C.C. Landry. Desert dust enhancement of mountain snowmelt. *Intermountain West Climate Summary*, 4(2), 2008.
- [14] J.T. Beale. Large-time regularity of viscous surface waves. *Arch. Rational Mech. Anal.*, 84:307–352, 1984.
- [15] M. Beck, B. Sandstede, and K. Zumbrun. Nonlinear stability of time-periodic shock waves. *Arch. Rat. Mechanics and Anal.*, 196:1011–1076, 2010.
- [16] G. Bel, A. Hagberg, and E. Meron. Gradual regime shifts in spatially extended ecosystems. *Theoretical Ecology*, 5:591–604, 2012.
- [17] A. Ben-Artzi and I. Gohberg. Dichotomy of systems and invertibility of linear ordinary differential operators. *Operator Theory: Advances and Applications*, 56:90–119, 1992.
- [18] J. Bergh and J. Löfström. *Interpolation Spaces: An Introduction*. Springer, 1976.
- [19] F. Borgogno, P. D’Odorico, F. Laio, and L. Ridolfi. Mathematical models of vegetation pattern formation in ecohydrology. *Rev. Geophys.*, 47, 2009.
- [20] B.M. Brown, M.S.P. Eastham, and K.M. Schmidt. *Periodic Differential Operators*. Springer

- Basel, 2013.
- [21] F. Busse. Non-linear properties of thermal convection. *Reports on Progress in Physics*, 41:1929–1967, 1978.
 - [22] J. Carballido-Landeira, P. Taboada, and A.P. Muñuzuri. Effect of electric field on turing patterns in a microemulsion. *Soft Matter*, 8, 2012.
 - [23] J.G. Charney. Dynamics of deserts and drought in the sahel. *Quarterly Journal of the Royal Meteorological Society*, 101(428):193–202, 1975.
 - [24] W. Chen and M.J. Ward. Oscillatory instabilities and dynamics of multi-spike patterns for the one-dimensional Gray-Scott model. *European J. Appl. Math.*, 20:187–214, 2009.
 - [25] W. Chen and M.J. Ward. The stability and dynamics of localized spot patterns in the two-dimensional Gray-Scott model. *SIAM J. Appl. Dyn. Sys.*, 10:582–666, 2011.
 - [26] C. Chicone and Y. Latushkin. *Evolution semigroups in dynamical systems and differential equations*. American Mathematical Society, 1999.
 - [27] J. Cholewa and T. Dlotko. *Global attractors in abstract parabolic problems*. Cambridge University Press, 2000.
 - [28] P. Clément and S. Li. Abstract parabolic quasilinear equations and applications to a ground-water flow problem. *Adv. Math. Sci. Appl.*, 3:17–32, 1994.
 - [29] P. Clément and G. Simonett. Maximal regularity in continuous interpolation spaces and quasilinear parabolic problems. *J. Evol. Eqs.*, 1:39–67, 2001.
 - [30] P. Couteron and O. Lejeune. Periodic spotted patterns in semi-arid vegetation explained by a propagation-inhibition model. *Journal of Ecology*, 89:616–628, 2001.
 - [31] B.D. Dalziel, J.M. Morales, and J.M. Fryxell. Fitting probability distributions to animal movement trajectories: Using artificial neural networks to link distance, resources, and memory. *The American Naturalist*, 172(2):248–258, 2008.
 - [32] H.J. de Boer, E.I. Lammertsma, F. Wagner-Cremer, D.L. Dilcher, M.J. Wassen, and S.C. Dekker. Climate forcing due to optimization of maximal leaf conductance in subtropical vegetation under rising CO₂. *Proceedings of the National Academy of Sciences*, 108(10):4041–4046, 2011.
 - [33] L. de Simon. Un’applicazione della teoria degli integrali singolari allo studio delle equazioni differenziali lineari astratte del primo ordine. *Rendiconti del Seminario Matematico della Università di Padova*, 34:205–223, 1964.
 - [34] D.L. DeAngelis, W.M. Post, and C.C. Travis. *Positive Feedback in Natural Systems*. Springer-Verlag, 1980.
 - [35] V. Deblauwe, 2014. Personal communication.
 - [36] V. Deblauwe, N. Barbier, P. Couteron, O. Lejeune, and J. Bogaert. The global biogeography of semi-arid periodic vegetation patterns. *Global Ecology and Biogeography*, 17:715–723, 2008.
 - [37] V. Deblauwe, N. Barbier, P. Couteron, O. Lejeune, and J. Bogaert. The global biogeography of semi-arid periodic vegetation patterns. *Global Ecology and Biogeography*, 17:715–723, 2008.
 - [38] V. Deblauwe, P. Couteron, J. Bogaert, and N. Barbier. Determinants and dynamics of banded vegetation pattern migration in arid climates. *Ecological Monographs*, 82:3–21, 2012.
 - [39] V. Deblauwe, P. Couteron, O. Lejeune, J. Bogaert, and N. Barbier. Environmental modulation of self-organized periodic vegetation patterns in sudan. *Ecography*, 34:990–1001, 2011.
 - [40] S. Díaz, I. Noy-Meir, and M. Cabido. Can grazing response of herbaceous plants be predicted from simple vegetative traits? *Journal of Applied Ecology*, 38:497–508, 2001.
 - [41] E. Doedel. Auto: A program for the automatic bifurcation analysis of autonomous systems.

- Congressus Numerantium*, 30:265–284, 1981.
- [42] E.J. Doedel. Auto-07p: Continuation and bifurcation software for ordinary differential equations. <http://cmvl.cs.concordia.ca/auto>.
- [43] A. Doelman, W. Eckhaus, and T.J. Kaper. Slowly-modulated two-pulse solutions in the Gray–Scott model i: Asymptotic construction and stability. *SIAM J Appl. Math.*, 61:1080–1102, 2000.
- [44] A. Doelman, W. Eckhaus, and T.J. Kaper. Slowly-modulated two-pulse solutions in the Gray–Scott model ii: Geometric theory, bifurcations, and splitting dynamics. *SIAM J Appl. Math.*, 61:2036–2062, 2001.
- [45] A. Doelman, R.A. Gardner, and T.J. Kaper. Stability analysis of singular patterns in the 1-D Gray–Scott model: a matched asymptotics approach. *Physica D*, 122:1–36, 1998.
- [46] A. Doelman, R.A. Gardner, and T.J. Kaper. Large stable pulse solutions in reaction-diffusion equations. *Indiana Univ. Math. J.*, 50:443–507, 2001.
- [47] A. Doelman, R.A. Gardner, and T.J. Kaper. A stability index analysis of 1-d patterns of the Gray-Scott model. *Memoirs of the AMS*, 155(737), 2002.
- [48] A. Doelman and T.J. Kaper. Semistrong pulse interactions in a class of coupled reaction-diffusion equations. *SIADS*, 2:53–96, 2003.
- [49] A. Doelman, T.J. Kaper, and P. Zegelng. Pattern formation in the one-dimensional Gray-Scott model. *Nonlinearity*, 10(2):523–563, 1997.
- [50] A. Doelman, T.J. Kaper, and P. Zegelng. Pattern formation in the one-dimensional gray-scott model. *Nonlinearity*, 10:523–563, 1997.
- [51] A. Doelman, J.D.M. Rademacher, and S. van der Stelt. Hopf dances near the tips of busse balloons. *Discrete and Continuous Dynamical Systems*, 5:61–92, 2012.
- [52] A. Doelman, B. Sandstede, A. Scheel, and G. Schneider. *The dynamics of modulated wave trains*. American Mathematical Society, 2009.
- [53] A. Doelman and H. van der Ploeg. Homoclinic stripe patterns. *SIAM J. Appl. Dyn. Syst.*, 1(1):65–104, 2002.
- [54] T. Dohnal, J.D.M. Rademacher, H. Uecker, and D. Wetzl. pde2path 2.0: multi-parameter continuation and periodic domains. In *ENOC 2014 - Proceedings of 8th European Nonlinear Dynamics Conference*, 2014.
- [55] G. Dore. Maximal regularity for abstract cauchy problems. *Adv. Diff. Eqns.*, pages 293–322, 2000.
- [56] G.P. Drazin and W.H. Reid. *Hydrodynamic stability*. Cambridge Univ. P., 1982.
- [57] D. Dunkerley. Banded vegetation: development under uniform rainfall from a simple cellular automaton model. *Plant Ecology*, 129:103–111, 1997.
- [58] D.L. Dunkerley. Vegetation Mosaics of Arid Western New South Wales, Australia: Considerations of their Origin and Persistence. In *Patterns of Land Degradation in Drylands: Understanding Self-Organised Ecogeomorphic Systems*. Springer, 2013.
- [59] L. Edelstein-Keshet. *Mathematical models in biology*. McGraw-Hill, 1988.
- [60] K.-J. Engel and R. Nagel. *One-Parameter Semigroups for Linear Evolution Equations*. Springer Verlag, 2000.
- [61] M.B. Eppinga, P.C. de Ruiter, M.J. Wassen, and M. Rietkerk. Nutrients and hydrology indicate the driving mechanisms of peatland surface patterning. *The American naturalist*, 173:803–818, 2009.
- [62] J. Escher, J. Prüss, and G. Simonett. A new approach to the regularity of solutions for parabolic equations. In *Evolution Equations*. Taylor & Francis Inc, 2003.
- [63] J.W. Evans. Nerve axon equations. iv. the stable and the unstable impulse. *Indiana Univ. Math. J.*, 24:1169–1190, 1974/75.

Bibliography

- [64] S. Focardi and P. Marcellini and P. Montanaro. Do ungulates exhibit a food density threshold? a fieldstudy of optimal foraging and movement patterns. *Journal of Animal Ecology*, 65:606–620, 1996.
- [65] S.D. Fretwell and H.L. Lucas Jr. On territorial behavior and other factors influencing habitat distribution in birds. *Acta Biotheoretica*, 19(1):16–36, 1969.
- [66] J.M. Fryxell, M. Hazell, L. Brger, B.D. Dalziel, D.T. Haydon, J.M. Morales, T. McIntosh, and R.C. Rosatte. Multiple movement modes by large herbivores at multiple spatiotemporal scales. *PNAS*, 105(49):19114–19119, 2008.
- [67] R.A. Gardner. On the structure of the spectra of periodic travelling waves. *J. Math. Pures Appl.*, 9:415–439, 1993.
- [68] A. Gierer and H. Meinhardt. A theory of biological pattern formation. *Kybernetik*, 12:30–9, 1972.
- [69] E. Gilad, M. Shachak, and E. Meron. Dynamics and spatial organization of plant communities in water-limited systems. *Theoretical Population Biology*, 72(2):214–230, 2007.
- [70] E. Gilad, J. von Hardenberg, A. Provenzale, M. Shachak, and E. Meron. Ecosystem engineers: From pattern formation to habitat creation. *Physical Review Letters*, 93:1–4, 2004.
- [71] M. Golubitsky, J.W. Swift, and E. Knobloch. Symmetries and pattern selection in Rayleigh-Benard convection. *Physica D*, 10:249–276, 1984.
- [72] K. Gowda, H. Riecke, and M. Silber. Transitions between patterned states in vegetation models for semiarid ecosystems. *Physical Review E*, 89, 2014.
- [73] A. Grainger, M. Stafford Smith, V.R. Squires, and E.P. Glenn. Desertification and climate change: the case for greater convergence. *Mitigation and Adaptation Strategies for Global Change*, 5(4):361–377, 2000.
- [74] P. Gray and S.K. Scott. Autocatalytic reactions in the isothermal, continuous stirred tank reactor: oscillations and instabilities in the system $a + 2b \rightarrow 3b$, $b \rightarrow c$. *Chemical Engineering Science*, 39:1087–1097, 1984.
- [75] J.A. Griepentrog and L. Recke. Local existence, uniqueness, and smooth dependence for nonsmooth quasilinear parabolic problems. *J. Evol. Equ.*, 10:341–375, 2012.
- [76] L. Gunderson. Ecological resilience—in theory and application. *Annual Review of Ecology and Systematics*, 31, 2000.
- [77] T. Häcker, H. Uecker, and G. Schneider. Self-similar decay to the marginally stable ground state in a model for film flow over inclined wave bottoms. *Electronic Journal of Differential Equations*, pages 1–51, 2012.
- [78] D. Henry. *Geometric theory of semilinear parabolic equations*. Springer, 1981.
- [79] R. HilleRisLambers, M. Rietkerk, F.V.D. Bosch, H.H.T. Prins, and H.D. Kroon. Vegetation pattern formation in semi-arid grazing systems. *Ecology*, 82:50–61, 2001.
- [80] C.S. Holling. The components of predation as revealed by a study of small-mammal predation of the european pine sawfly. *The Canadian Entomologist*, 91(5):234–261, 1959.
- [81] C.S. Holling. Resilience and stability of ecological systems annual review of ecology and systematics. *Annual Review of Ecology and Systematics*, 4:1–23, 1973.
- [82] M. Holzer, A. Doelman, and T.J. Kaper. Existence and stability of traveling pulses in a reaction-diffusion-mechanics system. *J Nonlinear Sci*, 23:129–177, 2013.
- [83] W. Horsthemke and R. Lefever. *Noise-Induced Transitions. Theory and Applications in Physics, Chemistry, and Biology*. Springer-Verlag, 2 edition, 2006.
- [84] H.N. Le Houérou. Rain use-efficiency: a unifying concept in arid-land ecology. *Journal of Arid Environments*, 7:213–247, 1984.
- [85] R. Hoyle. *Pattern Formation: An Introduction to Methods*. Cambridge University Press, Cambridge, UK, 2006.

- [86] Working Group I. Climate change 2007: The physical science basis. Technical report, IPCC, 2007.
- [87] Working Group I. Climate change 2013: The physical science basis. Technical report, IPCC, 2013.
- [88] J.J.S. Jerome and J.-M. Chomaz. Extended Squire’s transformation and its consequences for transient growth in a confined shear flow. *J. Fluid Mech.*, 744:430–456, 2014.
- [89] F. Joos and R. Spahni. Rates of change in natural and anthropogenic radiative forcing over the past 20,000 years. *Proceedings of the National Academy of Sciences*, 105:1425–1430, 2008.
- [90] T. Kapitula and K. Promislow. Spectral and dynamical stability of nonlinear waves. *Applied Mathematical Sciences*, 185, 2013.
- [91] T. Kato. Quasi-linear equations of evolution, with applications to partial differential equations. *Springer-Verlag Lecture Notes in Mathematics*, 448:25–70, 1975.
- [92] T. Kato. *Perturbation theory for linear operators. Corrected Printing of the Second Edition.* Springer, 1980.
- [93] S. Kawashima. Systems of a hyperbolic-parabolic composite type with applications to the equations of magnetohydrodynamics. Master’s thesis, Kyoto University, 1983.
- [94] B.J. Kealy and D.J. Wollkind. A nonlinear stability analysis of vegetative turing pattern formation for an interaction-diffusion plant-surface water model system in an arid flat environment. *Bulletin of mathematical biology*, 74:803–833, 2012.
- [95] S. Kefi, M. Rietkerk, and G.G. Katul. Vegetation pattern shift as a result of rising atmospheric CO_2 in arid ecosystems. *Theoretical population biology*, 74:332–344, 2008.
- [96] S. Kinast, Y.R. Zelnik, G. Bel, and E. Meron. Interplay between turing mechanisms can increase pattern diversity. *Physical Review Letters*, 2014.
- [97] C.A. Klausmeier. Regular and irregular patterns in semi-arid vegetation. *Science*, 284:1826–1828, 1999.
- [98] A.Y. Kletter, J. von Hardenberg, E. Meron, and A. Provenzale. Patterned vegetation and rainfall intermittency. *Journal of Theoretical Biology*, 256:574–583, 2009.
- [99] M. Köhne, J. Prüss, and M. Wilke. On quasilinear parabolic evolution equations in weighted l_p -spaces. *J. Evol. Equ.*, 10:443–463, 2010.
- [100] T. Kolokolnikov, W. Sun, M.J. Ward, and J. Wei. The Stability of a Stripe for the Gierer-Meinhardt Model and the Effect of Saturation. *SIAM J. Appl. Dyn. Sys.*, 5(2):313–363, 2006.
- [101] T. Kolokolnikov, M.J. Ward, and J. Wei. The existence and stability of spike equilibria in the one-dimensional Gray–Scott model: the pulse-splitting regime. *Physica D*, 202:258–293, 2005.
- [102] T. Kolokolnikov, M.J. Ward, and J. Wei. The existence and stability of spike equilibria in the one-dimensional Gray–Scott model: the low feed rate regime. *Studies in Appl. Math.*, 115(1):21–71, 2005.
- [103] T. Kolokolnikov, M.J. Ward, and J. Wei. Zigzag and Breakup Instabilities of Stripes and Rings in the Two-Dimensional Gray-Scott Model. *Studies in Appl. Math.*, 116(1):35–95, 2006.
- [104] N. Kumar and W. Horsthemke. Turing bifurcation in a reaction-diffusion system with density-dependent dispersal. *Physica A*, 389:1812–1818, 2010.
- [105] O. Ladyzhenskaya, V. Solonnikov, and N. Ural’ceva. *Linear and quasi-linear equations of parabolic type.* American Mathematical Society, 1968.
- [106] Y. Latushkin, J. Prüss, and R. Schnaubelt. Stable and unstable manifolds for quasilinear parabolic systems with fully nonlinear boundary conditions. *J. Evol. Equ.*, 6:537–576, 2006.
- [107] Y. Latushkin, J. Prüss, and R. Schnaubelt. Center manifolds and dynamics near equilibria

Bibliography

- of quasilinear parabolic systems with fully nonlinear boundary conditions. *Discr. Cont. Dyn. Sys. Ser. B*, 9:595–633, 2008.
- [108] R. Lefever and O. Lejeune. On the origin of tiger bush. *Bulletin of Mathematical Biology*, 59:263–294, 1997.
- [109] O. Lejeune, P. Couteron, and R. Lefever. Short range co-operativity competing with long range inhibition explains vegetation patterns. *Acta Oecologica*, 20:171–183, 1999.
- [110] O. Lejeune and M. Tlidi. A model for the explanation of vegetation stripes (tiger bush). *Journal of Vegetation Science*, 10:201–208, 1999.
- [111] O. Lejeune, M. Tlidi, and P. Couteron. Localized vegetation patches: A self-organized response to resource scarcity. *Physical Review E*, 2002.
- [112] J.L. Lions and E. Magenes. *Non-Homogeneous Boundary Value Problems and Applications II*. Springer, 1972.
- [113] C.M. Luke and P.M. Cox. Soil carbon and climate change: from the Jenkinson effect to the compost-bomb instability. *European Journal of Soil Science*, 62:5–12, 2011.
- [114] A. Lunardi. *Analytic semigroups and optimal regularity for parabolic problems*. Birkhäuser, 1995.
- [115] A. Lunardi. *Interpolation theory*. Springer, 2009.
- [116] W. Macfadyen. Vegetation patterns in the semi-desert plains of british somaliland. *Geographical Journal*, 116:199–211, 1950.
- [117] P.K. Maini, M.R. Myerscough, J.D. Murray, and K.H. Winters. Bifurcating spatially heterogeneous solutions in a chemotaxis model for biological pattern formation. *Bul l. Math. Biol.*, 53:701–719, 1991.
- [118] C. Malmström. *Dege rö stormyr: en botanisk, hydrologisk och utvecklingshistorisk undersökning över ett nordsvenskt myrkomplex*. PhD thesis, Uppsala University, 1923.
- [119] Y. Mau, L. Haim, and E. Meron. Reversing desertification as a spatial resonance problem. *Physical Review E*, 91, 2015.
- [120] H. Meinhardt. *Models of biological pattern formation*. Academic Press, 1982.
- [121] H. Meinhardt. *The algorithmic beauty of sea shells*. Springer-Verlag, 1995.
- [122] J. Merkin, R. Satnoianu, and S. Scott. The development of spatial structure in an ionic chemical system induced by applied electric fields. *Dynamics and Stability of Systems: An International Journal*, 15(3):209–230, 2000.
- [123] J.H. Merkin, R.A. Satnoianu, and S.K. Scott. Spatiotemporal chaos in a differential flow reactor. *J. Chem. Soc., Faraday Trans.*, 94:1211–1216, 1998.
- [124] E. Meron. Pattern-formation approach to modelling spatially extended ecosystems. *Ecological Modelling*, 234:70–82, 2012.
- [125] E. Meron, E. Gilad, J. von Hardenberg, M. Shachak, and Y. Zarmi. Vegetation patterns along a rainfall gradient. *Chaos, Solitons & Fractals*, 19:367–376, 2004.
- [126] M. Meyries. Local well-posedness and instability of travelling waves in a chemotaxis model. *Adv. Diff. Equations*, 16:31–60, 2011.
- [127] M. Meyries, J.D.M. Rademacher, and E. Siero. Quasilinear parabolic reaction-diffusion systems: user’s guide to well-posedness, spectra and stability of travelling waves. *SIAM J. Appl. Dyn. Sys.*, 13:249–275, 2014.
- [128] A. Mielke. Instability and stability of rolls in the swift–hohenberg equation. *Comm. Math. Phys.*, 189:829–853, 1997.
- [129] A.S. Moretto and R.A. Distel. Effects of selective defoliation on the competitive interaction between palatable and unpalatable grasses. *Journal of Arid Environment*, 42:167–175, 1999.
- [130] D.S. Morgan, A. Doelman, and T.J. Kaper. Stationary periodic patterns in the 1d gray-scott model. *Meth. Appl. Anal.*, 7:105–150, 2000.

- [131] T. Nagaia and T. Ikeda. Traveling waves in a chemotactic model. *J. Math. Biol.*, pages 169–184, 1991.
- [132] R. Nagel. Towards a “matrix theory” for unbounded operator matrices. *Math. Z.*, 201:57–68, 1989.
- [133] W-M Ni. Diffusion, cross-diffusion, and their spike-layer steady states. *Notices of the AMS*, 45(1), 1998.
- [134] I. Noy-Meir. Stability of grazing systems: An application of predator-prey graphs. *Journal of Ecology*, 63(2):459–481, 1975.
- [135] T. Okayasu and Y. Aizawa. Systematic analysis of periodic vegetation patterns. *Progress of Theoretical Physics*, 106(4):705–720, 2001.
- [136] T. Okayasu, T. Okuro, U. Jamsran, and K. Takeuchi. Degraded rangeland dominated by unpalatable forbs exhibits large-scale spatial heterogeneity. *Plant Ecology*, 213(4):625–635, 2012.
- [137] K.J. Palmer. Exponential dichotomies and transversal homoclinic points. *J. Diff. Eqns.*, 55:225–256, 1984.
- [138] K.J. Palmer. Exponential dichotomies and fredholm operators. *Proceedings of the American Mathematical Society*, 104:149–156, 1988.
- [139] J.E. Pearson. Complex patterns in a simple system. *Science*, 261:189–192, 1993.
- [140] B. Peña, C. Pérez-García, A. Sanz-Anchelergues, D.G. Míguez, and A.P. Muñuzuri. Transverse instabilities in chemical Turing patterns of stripes. *Phys. Rev. E*, 68, 2003.
- [141] A.J. Perumpanani, J.A. Sherratt, and P.K. Maini. Phase differences in reaction-diffusion-advection systems and applications to morphogenesis. *IMA Journal of Applied Mathematics*, 55:19–33, 1995.
- [142] S.D. Prince, E. Brown de Colstoun, and L.L. Kravitz. Evidence from rain-use efficiencies does not indicate extensive sahelian desertification. *Global Change Biology*, 4:359–374, 1998.
- [143] J. Prüss. Maximal regularity for evolution equations in l_p -spaces. *Conferenze del Seminario di Matematica dell’Università di Bari*, 285:1–39, 2003.
- [144] J. Prüss, G. Simonett, and R. Zacher. On convergence of solutions to equilibria for quasi-linear parabolic problems. *J. Diff. Eq.*, 246:3902–3931, 2009.
- [145] J. Prüss, G. Simonett, and R. Zacher. On normal stability for nonlinear parabolic problems. *Discrete Contin. Dyn. Syst. Supplement*, pages 612–621, 2009.
- [146] F. Rübiger and R. Schnaubelt. The spectral mapping theorem for evolution semigroups on spaces of vector-valued functions. *Semigroup Forum*, 52:225–239, 1996.
- [147] J.D.M. Rademacher, B. Sandstede, and A. Scheel. Computing absolute and essential spectra using continuation. *Physica D*, 229:166–183, 2007.
- [148] M. Rietkerk, M.C. Boerlijst, F. van Langevelde, R. Hillerislambers, J. van de Koppel, L. Kumar, H.H.T. Prins, and A.M. de Roos. Self-Organization of Vegetation in Arid Ecosystems. *The American naturalist*, 160:524–530, 2002.
- [149] M. Rietkerk, S.C. Dekker, P.C. de Ruiter, and J. van de Koppel. Self-organized patchiness and catastrophic shifts in ecosystems. *Science*, 305:1926–1929, 2004.
- [150] M. Rietkerk, P. Ketner, J. Burger, B. Hoorens, and H. Olf. Multiscale soil and vegetation patchiness along a gradient of herbivore impact in a semi-arid grazing system in west africa. *Plant Ecology*, 148:207–224, 2000.
- [151] M. Rietkerk and J. van de Koppel. Regular pattern formation in real ecosystems. *Trends in ecology & evolution*, 23:169–75, 2008.
- [152] C.J. Roussel and M.R. Roussel. Reaction–diffusion models of development with state-dependent chemical diffusion coefficients. *Progress in Biophysics & Molecular Biology*, 86:113–160, 2004.

Bibliography

- [153] M.R. Roussel and J. Wang. Transition from self-replicating behavior to stationary patterns induced by concentration-dependent diffusivities. *PRL*, 87:188–302, 2001.
- [154] A.B. Rovinsky and M. Menzinger. Chemical instability induced by a differential flow. *Phys. Rev. Lett.*, 69, 1992.
- [155] A.B. Rovinsky and M. Menzinger. Self-organization induced by the differential-flow of activator and inhibitor. *Phys. Rev. Lett.*, 70:778–781, 1993.
- [156] P.M. Saco, G.R. Willgoose, and G.R. Hancock. Eco-geomorphology of banded vegetation patterns in arid and semi-arid regions. *Hydrol. Earth Syst. Sci.*, 11:1717–1730, 2007.
- [157] B. Sandstede. Stability of travelling waves. In *Handbook of Dynamical Systems, II*, pages 983–1055. Elsevier, 2002.
- [158] B. Sandstede and A. Scheel. On the structure of spectra of modulated travelling waves. *Math. Nachr.*, 232:39–93, 2001.
- [159] B. Sandstede and A. Scheel. Defects in oscillatory media: toward a classification. *SIAM J. Appl. Dyn. Syst.*, 3:1–68, 2004.
- [160] B. Sandstede and A. Scheel. Relative morse indices, fredholm indices, and group velocities. *Discrete and Continuous Dynamical Systems A*, 20:139–158, 2008.
- [161] R.A. Satnoianu, J.H. Merkin, and S.K. Scott. Spatio-temporal structures in a differential flow reactor with cubic autocatalator kinetics. *Physica D*, 124:345–367, 1998.
- [162] M. Scheffer. *Critical Transitions in Nature and Society*. Princeton University Press, 2009.
- [163] M. Scheffer, J. Bascompte, W.A. Brock, V. Brovkin, S.R. Carpenter, V. Dakos, H. Held, E.H. van Nes, M. Rietkerk, and G. Sugihara. Early-warning signals for critical transitions. *Nature*, 461:53–59, 2009.
- [164] M. Scheffer, S. Carpenter, J. Foley, C. Folke, and B. Walker. Catastrophic shifts in ecosystems. *Nature*, 413:591–596, 2001.
- [165] M. Scheffer, E.H. Nes, M. Holmgren, and T. Hughes. Pulse-driven loss of top-down control: The critical-rate hypothesis. *Ecosystems*, 11:226–237, 2008.
- [166] G. Schneider. Nonlinear diffusive stability of spatially periodic solutions— abstract theorem and higher space dimensions. In *Proceedings of the International Conference on Asymptotics in Nonlinear Diffusive Systems (Sendai, 1997)*, pages 159–167, 1998.
- [167] L. Sewalt and A. Doelman. Spatially periodic multi-pulse patterns in a generalized Klausmeier–Gray–Scott model. In preparation.
- [168] J. Shatah and W. Strauss. Spectral condition for instability. *Contemp. Math.*, 255:189–198, 2000.
- [169] E. Sheffer, H. Yizhaq, M. Shachak, and E. Meron. Mechanisms of vegetation-ring formation in water-limited systems. *Journal of Theoretical Biology*, 273:138–146, 2011.
- [170] J.A. Sherratt. An Analysis of Vegetation Stripe Formation in Semi-Arid Landscapes. *J. Math. Biol.*, 51:183–197, 2005.
- [171] J.A. Sherratt. Numerical continuation of boundaries in parameter space between stable and unstable periodic travelling wave (wavetrain) solutions of partial differential equations. *Advances in Computational Mathematics*, 2012.
- [172] J.A. Sherratt. History-dependent patterns of whole ecosystems. *Ecological Complexity*, 14:8–20, 2013.
- [173] J.A. Sherratt. Pattern solutions of the klausmeier model for banded vegetation in semi-arid environments V: the transition from patterns to desert. *SIAM J. Appl. Math.*, 73:1347–1367, 2013.
- [174] J.A. Sherratt and G.J. Lord. Nonlinear dynamics and pattern bifurcations in a model for vegetation stripes in semi-arid environments. *Theoretical population biology*, 71:1–11, 2007.
- [175] N.M. Shnerb, P. Sarah, H. Lavee, and S. Solomon. Reactive glass and vegetation patterns.

- Physical Review Letters*, 90(3), 2003.
- [176] E. Siero, A. Doelman, M.B. Eppinga, J.D.M. Rademacher, M. Rietkerk, and K. Siteur. Striped pattern selection by advective reaction-diffusion systems: Resilience of banded vegetation on slopes. *Chaos*, 25, 2015.
- [177] G. Simonett. Center manifolds for quasilinear reaction-diffusion systems. *Differential Integral Equations*, 8:753–796, 1995.
- [178] K. Siteur, M.B. Eppinga, A. Doelman, E. Siero, and M.G. Rietkerk. Ecosystems off track: Rate-induced critical transitions in ecological models. Preprint.
- [179] K. Siteur, M.B. Eppinga, D. Karssenber, M. Baudena, M.F.P. Bierkens, and M. Rietkerk. How will increases in rainfall intensity affect semiarid ecosystems? *Water Resources Research*, 50(7):5980–6001, 2014.
- [180] K. Siteur, E. Siero, M.B. Eppinga, J.D.M. Rademacher, A. Doelman, and M.G. Rietkerk. Beyond Turing: the response of patterned ecosystems to environmental change. *Ecological Complexity*, 20:81–96, 2014.
- [181] M.E. Solomon. The natural control of animal populations. *Journal of Animal Ecology*, 18(1):1–35, 1949.
- [182] D.E. Spalinger and N.T. Hobbs. Mechanisms of foraging in mammalian herbivores: New models of functional response. *The American Naturalist*, 140(2):325–348, 1992.
- [183] H.B. Squire. On the stability of 3D disturbances of viscous flow between parallel walls. *Proc. R. Soc. Lond. A*, 142:621–628, 1933.
- [184] W. Sun, M.J. Ward, and R. Russell. The slow dynamics of two-spike solutions for the Gray–Scott and Gierer–Meinhardt systems: Competition and oscillatory instabilities. *SIADS*, 4:904–953, 2005.
- [185] S. Thompson, G. Katul, and S.M. McMahon. Role of biomass spread in vegetation pattern formation within arid ecosystems. *Water Resources Research*, 44, 2008.
- [186] S.E. Thompson, C.J. Harman, P. Heine, and G.G. Katul. Vegetation-infiltration relationships across climatic and soil type gradients. *Journal of Geophysical Research*, 2010.
- [187] A. Toth, D. Horvath, and W. van Saarloos. Lateral instabilities of cubic autocatalytic reaction fronts in constant electric field. *J. Chem. Phys.*, 111:10964–10968, 1999.
- [188] H. Triebel. *Interpolation theory, function spaces, differential operators*. North-Holland, 1978.
- [189] H. Triebel. *Theory of function spaces II*. Birkhäuser, 1983.
- [190] A. Turing. The chemical basis of morphogenesis. *Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences*, 237:37–72, 1952.
- [191] H. Uecker. Self-similar decay of spatially localized perturbations of the nusselt solution for the inclined film problem. *Arch. Rational Mech. Anal.*, 184:401–447, 2007.
- [192] H. Uecker, D. Wetzel, and J.D.M. Rademacher. pde2path - A Matlab package for continuation and bifurcation in 2D elliptic systems. *Num. Math.: Th. Meth. Appl.*, 7:58–106, 2014.
- [193] C. Valentin, J.M. d’Herbès, and J. Poesen. Soil and water components of banded vegetation patterns. *CATENA*, 37:1–24, 1999.
- [194] J. van de Koppel and M. Rietkerk. Herbivore regulation and irreversible vegetation change in semi-arid grazing systems. *OIKOS*, 90:253–260, 2000.
- [195] J. van de Koppel, M. Rietkerk, N. Dankers, and P.M.J. Herman. Scale-dependent feedback and regular spatial patterns in young mussel beds. *The American naturalist*, 165(3):E66–77, 2005.
- [196] J. van de Koppel, M. Rietkerk, F. van Langevelde, L. Kumar, C.A. Klausmeier, J.M. Fryxell, J.W. Hearne, J. van Andel, N. de Ridder, A. Skidmore, L. Stroosnijder, and H.H.T. Prins. Spatial heterogeneity and irreversible vegetation change in semiarid grazing systems. *The*

Bibliography

- American Naturalist*, 159(2):209–218, 2002.
- [197] H. van der Ploeg and A. Doelman. Stability of spatially periodic pulse patterns in a class of singularly perturbed reaction-diffusion equations. *Indiana Univ. Math. J.*, 54:1219–1302, 2005.
- [198] S. van der Stelt. *Rise and Fall of Periodic Patterns in a Generalized Klausmeier-Gray-Scott Model*. PhD thesis, Universiteit Leiden, 2012.
- [199] S. van der Stelt, A. Doelman, G. Hek, and J.D.M. Rademacher. Rise and fall of periodic patterns for a Generalized Klausmeier-Gray-Scott model. *J. Nonl. Sc.*, 23:39–95, 2013.
- [200] J.L. Vázquez. *The Porous Medium Equation*. Oxford University Press, 2007.
- [201] B.Z. Virany, A. Szommer, A. Toth, and D. Horvath. Lateral instability controlled by constant electric field in an acid-catalyzed reaction. *Phys. Chem. Chem. Phys.*, 6:3396–3401, 2004.
- [202] J. von Hardenberg, E. Meron, M. Shachak, and Y. Zarmi. Diversity of Vegetation Patterns and Desertification. *Physical Review Letters*, 87:3–6, 2001.
- [203] R.-H. Wang, Q.-X. Liu, G.-Q. Sun, Z. Jin, and J. van de Koppel. Nonlinear dynamic and pattern bifurcations in a model for spatial patterns in young mussel beds. *J. R. Soc. Interface*, 6:705–718, 2009.
- [204] M. Westoby, B. Walker, and I. Noy-Meir. Opportunistic management for rangelands not at equilibrium. *Journal of Range Management*, 42:266–274, 1989.
- [205] X. Zhao Y. Wu. The existence and stability of travelling waves with transition layers for some singular cross-diffusion systems. *Physica D*, 200:325–358, 2005.
- [206] A. Yagi. *Abstract Parabolic Evolution Equations and their Applications*. Springer, 2010.
- [207] H. Yizhaq, E. Gilad, and E. Meron. Banded vegetation: biological productivity and resilience. *Physica A: Statistical Mechanics and its Applications*, 356:139–144, 2005.
- [208] Y.R. Zelnik, S. Kinast, H. Yizhaq, G. Bel, and E. Meron. Regime shifts in models of dryland vegetation. *Philosophical Transactions of the Royal Society A*, 371, 2013.
- [209] S. Zhao, J. Ovadia, X. Liu, Y.-T. Zhang, and Q. Nie. Operator splitting implicit integration factor methods for stiff reaction-diffusion-advection systems. *J Comput Phys.*, 230(15):5996–6009, 2011.
- [210] K. Zumbrun. Center stable manifolds for quasilinear parabolic pde and conditional stability of nonclassical viscous shock waves. Preprint, available as arXiv:0811.2788.
- [211] K. Zumbrun. Planar stability criteria for viscous shock waves of systems with real viscosity. In *Hyperbolic systems of balance laws*. Springer, 2007.
- [212] K. Zumbrun and P. Howard. Pointwise semigroup methods and stability of viscous shock waves. *Indiana Univ. Math. J.*, 47:741–872, 1998.