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A recipe for desert : analysis of an extended Klausmeier model

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A recipe for desert

Analysis of an extended Klausmeier model

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