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## **Dynamical Structure and Evolution of Stellar Systems**

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# CHAPTER 1

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## INTRODUCTION

### 1 STELLAR SYSTEMS

**T**HE universe formed in a 'Big Bang', after which it began expanding. Places with more dark matter than their surroundings collapsed under gravity and collected gas, from which the first stars were born. Depending on the distribution of the dark matter, these stars ended up in systems of different sizes and shapes. The stars inside these stellar systems evolve: most stars fade out at the end of their life, but the more massive stars explode. New stars can be formed from their debris. Also, the systems themselves evolve by interacting and merging. This leads to the question: Can we find out how the different stellar systems evolved from the Big Bang to the present day?

One way to answer this question is to observe objects at very large distances. Since light needs time to travel, by looking at objects very far away, we see how they were a long time ago. However, with increasing distance, these objects quickly become smaller and fainter, such that very large telescopes with the ability to make very sharp images are needed. Another approach is to study nearby stellar systems and try to uncover, like an archaeologist, the 'fossil record' of their formation and evolution. Because they are close, they are also brighter, and the motions and composition of the stars in these systems can be observed in great detail. We can try to reconstruct the three-dimensional stellar system by fitting theoretical models, based on Newton's law of gravity, to these observations. In this way, we can 'look' inside stellar systems and search for features, i.e. 'fossils', in their structure and internal motions related to their formation history.

The most suitable stellar systems to study the fossil record are those for which the stars are not hidden from sight by clouds of gas and dust, and which are not 'polluted' by recent star formation. Globular clusters are the cleanest stellar systems, containing of the order of a million very old stars, which formed from the same collapsing matter very soon after the Big Bang. In addition, they are simple, nearly spherical objects and we can observe them from nearby as they also surround our own Milky Way galaxy. We can resolve many of the individual stars in these clusters and observe their velocities along the line-of-sight and even in the plane of the sky ('proper motions') by measuring the small changes in their positions with time. Reliable kinematic measurements of individual stars are currently only possible for the nearest objects and for the stars inside the Milky Way.

In the beginning of the twentieth century it became clear that the Milky Way is just one of the 'island universes' that can be seen in the night sky. Photographic observations showed that these galaxies come in different flavors. This led Hubble (1936) to classify them into four distinct groups, according to their apparent shape. In the resulting Hubble sequence (or Hubble diagram or Hubble tuning fork) the Milky Way belongs to the group of disk-like galaxies which are called *spirals* according to

their prominent spiral arms. At the other end of the sequence, we find the *elliptical* galaxies that seemingly have little or no structure. *Lenticular* galaxies are placed in between, having a disk but no prominent spiral arms, and a spheroidal stellar distribution. The fourth group consists of galaxies without a regular shape, which appropriately were named *irregulars*. At that time, it was thought that the complex spirals were formed from the simple ellipticals. Although we now know that galaxy formation and evolution happens the other way around, the spirals are still called late-type galaxies, and ellipticals and lenticulars are known as early-type galaxies.

The late-type galaxies, including the Milky Way, contain significant amounts of gas and dust, and this material is converted into stars by continuous, and often intensive star formation, which makes it very hard to recover their formation history. Early-type galaxies do not contain much gas and dust, and consequently have no recent star formation, so that they are well suited to study galaxy formation and evolution. For the nearby ( $< 100$  Mpc) early-type galaxies, we can investigate the fossil record of their formation in detail. Although we are in general unable to resolve their individual stars, we can obtain accurate and spatially-resolved photometric and kinematic measurements from the integrated light of stars along the line-of-sight.

## 2 SURFACE BRIGHTNESS

To first order, the surface brightness of early-type galaxies is well described by a simple function of radius along elliptical isophotes. Although the photometry of early-type galaxies seems to be rather simple, this does not mean that their intrinsic dynamical structure can also be derived and described in a straightforward way.

The conversion from a surface brightness measured on the plane of the sky to an intrinsic luminous density is in general non-unique. This deprojection is unique for spherical objects, but only very few galaxies have a round appearance, and even then they need not be intrinsically spherical. In the case of flattened objects with axial symmetry, the deprojection is only unique for a viewing direction in the plane normal to the axis of symmetry, better known as an edge-on viewing direction and often described by an inclination angle  $i = 90^\circ$  (Rybicki 1987). However, a stellar system in equilibrium can also be of triaxial shape (Binney 1976). The deprojection then becomes highly non-unique, with the viewing direction described by two viewing angles. In contrast with axisymmetric objects, the orientation of the elliptical isophotes may vary with radius for triaxial shapes (Stark 1977). Very soon after this was realized, isophotal twists were indeed observed in real galaxies (e.g., Carter 1978; King 1978; Williams & Schwarzschild 1979; Leach 1981).

In the case of axisymmetric objects, the flattening might be in part caused by rotation, similar to the flattening of the Earth. Instead of this gravitational support by ordered motion, random motion, acting as a kind of pressure, can also prevent a stellar system against gravitational collapse. This random motion is measured from the mean velocity dispersion of the stars. The components in three orthogonal directions, often referred to as the semi-axis lengths of the velocity ellipsoid, can be different so that the stellar system is anisotropic, and can vary throughout the stellar system, even for an axisymmetric or spherical stellar system.

These dynamical properties cannot be inferred from photometry and require kinematic measurements, e.g., from spectroscopic observations. The development of telescopes and instruments in the mid-seventies and early eighties of the twentieth cen-

tury made it possible to measure the ordered-over-random motion ratios,  $V/\sigma$ , of early-type galaxies. From these kinematic measurements it became clear that these systems in general rotate too slowly for pure rotational support (e.g., Bertola & Cappaccioli 1975; Illingworth 1977). Further observations revealed that lenticulars and low-luminosity ellipticals have disk-like isophotes and show clear rotation, while giant ellipticals seem to have boxy isophotes and often hardly show any rotation (Davies et al. 1983; Bender 1988). The reason behind this dichotomy is ascribed to the different underlying dynamical structures, with faint early-type galaxies comparable to isotropic oblate rotators and luminous early-type galaxies consistent with anisotropic triaxial stellar systems (e.g., Davies et al. 1983; Bender & Nieto 1990; de Zeeuw & Franx 1991; Faber et al. 1997).

However, recent (N-body) simulations of merging galaxies seem to suggest the opposite concerning the degree of anisotropy, producing faint anisotropic and luminous isotropic early-type galaxies (Burkert & Naab 2005). Based on a detailed study of the orbital structure inferred from dynamical models of two dozen early-type galaxies, Cappellari et al. (2005a) come to the same conclusions. It is evident that such detailed simulations and dynamical models of galaxies are crucial to understand their formation history. At the same time, the improvement in the determination of the intrinsic dynamical structure would not have been possible without the aid of two-dimensional kinematic measurements and realistic dynamical modeling.

### 3 TWO-DIMENSIONAL KINEMATICS

Early-type galaxies can in general be assumed to be collisionless stellar systems in equilibrium. Only in the center can the stellar density become high enough for stars to significantly perturb each other's orbits; everywhere else the stellar system is collisionless. Except for the outskirts the dynamical time scale of the stars is short enough for the stellar system to have reached equilibrium in the time passed since its formation. These assumptions are also valid for many globular clusters, except for their cores, where two-body relaxation can play an important role. When a stellar system is collisionless and in equilibrium, its dynamical state is completely described by the (time-independent) distribution function (DF) of the stars in the six-dimensional phase space of positions and velocities.

For stars in the Milky Way and in nearby globular clusters, we can measure the line-of-sight velocity and proper motions as a function of position on the plane of the sky. The determination of the sixth dimension, the distance, is in general very difficult and relatively uncertain. Moreover, due to obscuration by gas and dust and limited spatial and spectral instrumental resolution, observations are not complete, although future space missions like *GAIA* are expected to provide a stereoscopic census of a significant part of the Milky-Way and its surroundings (Perryman et al. 2001).

Even at astronomically small distances it becomes no longer possible to resolve individual stars with current telescopes. We can still measure the projected surface brightness and (for the nearby galaxies) the line-of-sight velocity distribution of the integrated stellar light as a function of position on the plane of the sky. In the last two decades a major step forward has been made in the latter observations with the introduction of integral-field spectrographs. Via an array of lenses, a bundle of fibers or a set of adjacent slits, the integrated light from different positions on the plane of the sky is dispersed in the wavelength direction. In this way, integral-field

spectrographs provide a spectrum at each position within a two-dimensional area, from which we can simultaneously extract the kinematics of the stars and gas, as well as line-strength measurements, as a function of position on the plane of the sky.

Due to the high quality of modern spectroscopic observations, it is often possible to also measure the higher-order line-of-sight velocity moments of the DF, in addition to the mean velocity and velocity dispersion. These moments are often expressed in terms of the Gauss-Hermite moments, which are less sensitive to the noise in the wings than the true velocity moments (van der Marel & Franx 1993; Gerhard 1993). The measurement of these higher-order velocity moments are also important to break the so called mass-anisotropy degeneracy: a change in the observed line-of-sight velocity dispersion can be due to (a combination of) a change in the velocity ellipsoid, i.e., a change in anisotropy, or a change in mass. Since we cannot observe the velocity dispersion in the plane of the sky, we need the higher order line-of-sight velocity moments to constrain a possible change in the velocity ellipsoid. On the other hand, to measure a change in mass we need to know the mass-to-light ratio  $M/L$  to convert the observed surface brightness into a mass distribution. Unfortunately, we do not know the value of  $M/L$ , which moreover may vary throughout the galaxy due to a change in the properties of the underlying stellar populations, or due to the presence of non-luminous matter in the form of a central black hole and/or an extended dark halo. To overcome these problems, realistic and detailed dynamical models, which make full use of the information that is present in the photometric and (two-dimensional) kinematic observations, are crucial.

## 4 DYNAMICAL MODELS

Integral-field spectroscopy has (literally) added a new dimension to observations of nearby early-type galaxies. The resulting kinematic maps provide us with three-dimensional information on the DF. Still, taking into account the uncertainties in the maps due to inevitable noise in the observations, together with the unknown viewing direction,  $M/L$ , and possible dark matter contribution, it seems almost hopeless to recover the DF in the six-dimensional phase space. Fortunately, for stationary equilibrium stellar systems the DF depends in general on fewer than six parameters.

### 4.1 INTEGRALS OF MOTION

According to Jeans (1915) theorem the DF is a function of the isolating integrals of motion admitted by the potential (Lynden-Bell 1962b; Binney 1982). In a spherical symmetric potential these integrals of motion are the energy  $E$  and the three components of the angular momentum vector  $\mathbf{L}$ . In axisymmetric geometry orbits have two exact integrals of motion, the energy  $E$  and the angular momentum component  $L_z$  parallel to the symmetry  $z$ -axis. All regular orbits furthermore obey a third integral  $I_3$ , which in general is not known in closed form. In the triaxial case,  $E$  is conserved and all regular orbits have two additional integrals of motion,  $I_2$  and  $I_3$ , both of which in general are not known explicitly.

If, in addition to the potential, the DF itself is also spherically symmetric, it depends only on the magnitude  $L$  of the angular momentum vector and not on its direction, i.e.,  $f = f(E, L^2)$ . Such models have anisotropic velocity distributions, but if  $f = f(E)$ , the stars are in isotropic equilibrium. Eddington (1916) showed that in this case  $f(E)$  can uniquely be recovered from the intrinsic mass density  $\rho(r)$ . Although

anisotropic spherical models can sometimes be found by a similar analytic inversion (e.g., Dejonghe 1987), most of them are constructed by assumption of a special functional form for  $f(E, L^2)$  (e.g., Binney & Tremaine 1987). Well-known spherical models are for example those considered by Osipkov (1979) and Merritt (1985), with a DF of the form  $f(E \pm L^2/r_a^2)$ , where  $r_a$  is a constant scale length.

For axisymmetric models with  $f = f(E, L_z)$ , inversion formulas have been known for a long time in the case where the density  $\rho(R, z)$  can be expressed explicitly in terms of the underlying gravitational potential  $V$  as  $\rho(R, V)$  (e.g., Lynden-Bell 1962; Hunter 1975; Dejonghe 1986). In spite of the latter limitation, many  $f(E, L_z)$  models have been derived in this way (e.g., de Zeeuw 1994), including for example the exact DF for the Kuzmin-Kutuzov (1962) model by Dejonghe & de Zeeuw (1988). With the method derived by Hunter & Qian (1993) it became possible to obtain the two-integral DF directly from  $\rho(R, z)$ . While  $\rho(R, z)$  constrains only the part of the DF that is even in the velocities, i.e.,  $f = f(E, L_z^2)$ , the odd part can be found once the mean azimuthal velocity field  $v_\phi(R, z)$  is known. Although these two-integral axisymmetric models have already significantly improved our understanding of the dynamical structure of stellar systems (e.g., Qian et al. 1995), for more realistic models we need to include the third integral of motion. How to do this is not evident because this third integral of motion is in general unknown. The construction of triaxial models with two non-classical integrals of motion is even more complex.

An exception is provided by the special family of models with a gravitational potential of Stäckel form, for which all three integrals of motion are exact and known explicitly. The associated densities have a large range of possible shapes, but they all have cores rather than central cusps, and hence are inadequate for describing the central parts of galaxies with massive black holes. Even so, their kinematic properties are as rich as those seen in the main body of early-type galaxies (Statler 1991, 1994a; Arnold et al. 1994). Several (numerical and analytic) DFs have been constructed for these separable models (e.g., Bishop 1986; Dejonghe & de Zeeuw 1988; Hunter & de Zeeuw 1992). These also include the Abel models, first introduced by Dejonghe & Laurent (1991) and extended by Mathieu & Dejonghe (1999), which generalize the spherical Osipkov-Merritt models and axisymmetric Kuzmin-Kutuzov models (Chapter 4).

## 4.2 VELOCITY MOMENTS

A way to avoid the unknown non-classical integrals of motion and even the DF is to solve the continuity equation and Jeans equations that follow by taking velocity moments of the collisionless Boltzmann equation. The continuity equation connects the first moments (mean streaming) and the Jeans equations connect the second moments (or the velocity dispersions, if the mean streaming is known) directly to the density and the gravitational potential, without the need to know the DF.

Unfortunately, in nearly all cases there are fewer equations than velocity moments, so that additional assumptions have to be made about the degree of anisotropy. The Jeans equations in the spherical case with a simple form for the anisotropy parameter (e.g., Binney & Tremaine 1987) are widely used to model a large variety of dynamical systems. Kinematic measurements of stellar systems have also been successfully fitted by using the solution of the Jeans equation in axisymmetric geometry with the DF assumed to be independent of the third integral of motion,  $f(E, L_z)$ , corresponding to isotropy in the meridional  $(R, z)$ -plane (e.g., Hunter 1977; Satoh 1980; Binney, Davies & Illingworth 1990; van der Marel 1991).

Such ad-hoc assumptions are not needed in the case of separable Stäckel models. For each orbit in a Stäckel potential, at most one component of the streaming motion is non-zero and all mixed second moments vanish in the coordinate system in which the equations of motion separate. Consequently, the continuity equation can be readily solved for the one non-vanishing first moment (Statler 1994a), and used to provide constraints on the intrinsic shapes of individual galaxies (e.g., Statler 1994b, 2001; Statler et al. 2004). The Jeans equations form a closed system with as many equations as non-vanishing second moments. The solution of these equations in axisymmetric geometry has been known for a while (e.g., Evans & Lynden-Bell 1989), and the solution for the triaxial case is presented in Chapter 5.

### 4.3 EQUATIONS OF MOTION

Although much has been learned about the dynamical structure of stellar systems by modeling their observed surface brightness and kinematics with solutions of the continuity equation and the Jeans equations (e.g., Binney & Tremaine 1987), the results need to be interpreted with care since the moment solutions may not correspond to a physical distribution function  $f \geq 0$ . A non-physical DF can be avoided, without actually specifying the DF, by solving directly the equations of motions in a given potential, and fitting the resulting density and velocity distribution to the observed surface brightness and kinematics. Analytically this is only possible for (very) special choices of the potential or in an approximate way by restricting to the lower-order (linear) terms in the equations of motions (e.g., Binney & Tremaine 1987; Chapter 3). Numerically, a very powerful tool is provided by Schwarzschild's (1979, 1982) orbit superposition method, originally designed to reproduce triaxial mass distributions.

Schwarzschild's method allows for an arbitrary gravitational potential, with possible contributions from dark components. The equations of motion are integrated for a representative library of orbits, and then the orbital weights are determined for which the combined and projected density and higher order velocity moments of the orbits best fit the observed surface brightness and (two-dimensional) kinematics. The resulting best-fit distribution of (positive) orbital weights represents the DF (cf. Vandervoort 1984), which is thus guaranteed to be everywhere non-negative.

A number of groups have developed independent numerical implementations of Schwarzschild's method in axisymmetric geometry and determined black hole masses, mass-to-light ratios, dark matter profiles as well as the DF of early-type galaxies by fitting in detail their projected surface brightness and line-of-sight velocity distributions (see § 1 of Chapter 4 for an overview and references). By including proper motion measurements the distance and dynamical structure of nearby globular clusters can be determined (Chapter 2; van den Bosch et al. 2005). The non-trivial extension of Schwarzschild's method to triaxial geometry (Chapter 4; van den Bosch et al. 2006) allows the modeling of giant ellipticals with significant features of triaxiality both in their observed photometry (isophotal twist) and in their observed kinematics (kinematic misalignment, kinematically decoupled components, etc.).

## 5 DYNAMICAL STRUCTURE AND EVOLUTION

Above we presented three different approaches to model stellar systems: analytically computing the DF specified as a function of the known integrals of motion; solving the continuity equation and Jeans equations for the velocity moments; and integrating

the equations of motion. In this order, the approaches show an increase in freedom and flexibility, but at the same time an increase in complexity and a corresponding increase in (computational) effort to find the best-fit dynamical model. For triaxial geometries in particular, the first two approaches can be very useful to constrain the large parameter space before applying the more general but computational expensive Schwarzschild method. Such a combination of modeling techniques applied to two-dimensional observations provides a very powerful tool to investigate the fossil record of formation in nearby globular clusters and early-type galaxies.

The gravitational potential forms the basis of all dynamical models, and in general is inferred from the observed surface brightness. This involves a deprojection and a conversion from light to mass, for given viewing angle(s) and mass-to-light ratio  $M/L$ , which enter the model as free parameters. The deprojection is nearly always non-unique and mass does not have to follow light, because of varying properties of the underlying stellar population or the presence of dark matter, so that  $M/L$  does not have to be constant. Although the inferred gravitational potential might thus be different from the true one, various tests seem to suggest that the parameters as well as the DF are recovered well, as long as there are enough accurate photometric and kinematic constraints (Chapters 2 and 4).

A unique way to get a more direct handle on the gravitational potential is via strong gravitational lensing. The mass of a foreground galaxy bends the light of a distant quasar behind it, resulting in multiple images. From the separation and relative fluxes of the images the total mass distribution (including possible dark matter) of the lens galaxy, and hence the potential, can be constrained. Next, by constructing a dynamical model of the lens galaxy that fits the observed surface brightness and kinematics, the dark matter distribution in the lens galaxy can be studied. Only very few of the known lens galaxies are close enough to obtain sufficient photometric and (two-dimensional) kinematic measurements for a detailed dynamical study (Chapter 6).

At higher redshift, measurements of stellar systems are limited to their global properties. Often only photometric properties such as luminosity, color and size are readily accessible, because kinematic measurements from spectra become very challenging due to the dimming of the light. Strong gravitational lensing provides a way out here: since the velocity dispersion of the lens galaxy is related to its mass, the (central) velocity dispersion can be estimated from the separation of the quasar images (e.g., Schneider et al. 1992). Once the global properties of several stellar systems are known, these stellar systems can be linked and their evolution investigated by means of scaling relations such as the Fundamental Plane. The change with redshift of the latter tight relation between the structural parameters and velocity dispersion of early-type galaxies, provides a measurement of the  $M/L$  evolution (Chapter 7). Comparing such measurements of the change in the global (dynamical) properties of early-type galaxies with time, with the detailed determinations of the (dynamical) properties of nearby early-type galaxies, allows a better understanding of the dynamical structure and evolution of stellar systems from the Big Bang to the present day.

## 6 THIS THESIS

In CHAPTER TWO, we determine the dynamical distance  $D$ , inclination  $i$ , mass-to-light ratio  $M/L$  and intrinsic orbital structure of the Milky Way globular cluster  $\omega$  Centauri, by fitting axisymmetric dynamical models to the ground-based proper motions of van



Leeuwen et al. (2000) and line-of-sight velocities from four independent data-sets. We correct the observed velocities for perspective rotation caused by the space motion of the cluster, and show that the residual solid-body rotation component in the proper motions can be taken out without any modeling other than assuming axisymmetry. This also provides a tight constraint on  $D \tan i$ . The corrected mean velocity fields are consistent with regular rotation, and the velocity dispersion fields display significant deviations from isotropy.

We model  $\omega$  Centauri with an axisymmetric implementation of Schwarzschild’s orbit superposition method. We bin the individual measurements on the plane of the sky to search efficiently through the parameter space of the models. Tests on an analytic model demonstrate that this approach is capable of measuring the cluster distance to an accuracy of about 6 per cent. Application to  $\omega$  Centauri reveals no dynamical evidence for a significant radial dependence of  $M/L$ , in harmony with the relatively long relaxation time of the cluster. The best-fit dynamical model has a stellar  $V$ -band mass-to-light ratio  $M/L_V = 2.5 \pm 0.1 M_\odot/L_\odot$  and an inclination  $i = 50^\circ \pm 4^\circ$ , which corresponds to an average intrinsic axial ratio of  $0.78 \pm 0.03$ . The best-fit dynamical distance  $D = 4.8 \pm 0.3$  kpc (distance modulus  $13.75 \pm 0.13$  mag) is significantly larger than obtained by means of simple spherical or constant-anisotropy axisymmetric dynamical models, and is consistent with the canonical value  $5.0 \pm 0.2$  kpc obtained by photometric methods. The total mass of the cluster is  $(2.5 \pm 0.3) \times 10^6 M_\odot$ . The best-fit model is close to isotropic inside a radius of about 10 arcmin and becomes increasingly tangentially anisotropic in the outer region, which displays significant mean rotation. This phase-space structure may well be caused by the effects of the tidal field of the Milky Way. The cluster contains a separate disk-like component in the radial range between 1 and 3 arcmin, contributing about 4% to the total mass.

In CHAPTER THREE, we analyze spatially resolved SAURON kinematic maps of the inner kpc of the nearby early-type barred spiral galaxy NGC 5448. The observed morphology and kinematics of the emission-line gas are patchy and perturbed, indicating clear departures from circular motion. The kinematics of the stars are more regular, and display a small inner disk-like system embedded in a large-scale rotating structure. We focus on the [O III] gas, and use a harmonic decomposition formalism to analyze the gas velocity field. The higher-order harmonic terms and the main kinematic features of the observed data are consistent with a simple bar model. We construct a bar model by solving the linearized equations of motion, considering an  $m = 2$  perturbation mode, and with parameters which are consistent with the large-scale bar detected via imaging. Optical and near infra-red images reveal asymmetric extinction in NGC 5448, and we recognize that some of the deviations between the data and the analytical bar model may be due to these complex dust features. Our study illustrates how the harmonic decomposition formalism can be used as a powerful tool to quantify non-circular motions in observed gas velocity fields.

In CHAPTER FOUR, we construct axisymmetric and triaxial galaxy models with a phase-space distribution function that depends on linear combinations of the three exact integrals of motion for a separable potential. For these Abel models the density and higher velocity moments can be calculated efficiently, and they capture much of the rich internal dynamics of early-type galaxies. We use these models to mimic the two-dimensional kinematics obtained with integral-field spectrographs such as SAURON. We fit the simulated observations with axisymmetric and triaxial dynamical models obtained with our numerical implementation of Schwarzschild’s orbit-

superposition method, while varying the viewing direction and the mass-to-light ratio. We find that Schwarzschild’s method is able to recover the internal dynamical structure of early-type galaxies and to accurately determine the mass-to-light ratio, but additional information is needed to constrain better the viewing direction.

In CHAPTER FIVE, we continue our analysis of galaxy models with separable potentials and derive the general solution of the Jeans equations. The Jeans equations relate the second-order velocity moments to the density and potential of a stellar system, without making any assumptions about the distribution function. For general three-dimensional stellar systems, there are three equations and six independent moments. By assuming that the potential is triaxial and of separable Stäckel form, the mixed moments vanish in confocal ellipsoidal coordinates. Consequently, the three Jeans equations and three remaining non-vanishing moments form a closed system of three highly-symmetric coupled first-order partial differential equations in three variables. These equations were first derived by Lynden-Bell (1960), but have resisted solution by standard methods for a long time. We present the general solution here.

We consider the two-dimensional limiting cases first. We solve their Jeans equations by a new method which superposes singular solutions. The resulting solutions of the Jeans equations give the second moments throughout the system in terms of prescribed boundary values of certain second moments. The two-dimensional solutions are applied to non-axisymmetric disks, oblate and prolate spheroids, and also to the scale-free triaxial limit. We then extend the method of singular solutions to the triaxial case, and obtain a full solution, again in terms of prescribed boundary values of second moments. The general solution can be expressed in terms of complete (hyper)elliptic integrals which can be evaluated in a straightforward way, and provides the full set of second moments which can support a triaxial density distribution in a separable triaxial potential.

In CHAPTER SIX, we investigate the total mass distribution in the inner parts of the strong gravitational lens system QSO 2237+0305, well-known as the Einstein Cross. In this system, a distant quasar is lensed by the bulge of an early-type spiral at a redshift  $z \sim 0.04$  (i.e., at a distance of about 160 Mpc). We obtain a realistic luminosity density of the lens galaxy by deprojecting its observed surface brightness, and we construct a lens model that accurately fits the positions and relative fluxes of the four quasar images. We combine both to build axisymmetric dynamical models that fit preliminary two-dimensional stellar kinematics derived from recent observations with the integral-field spectrograph GMOS. We find that the stellar velocity dispersion measurements with a mean value of  $167 \pm 10 \text{ km s}^{-1}$  within the Einstein radius  $R_E = 0.90''$ , are in agreement with predictions from our and previous lens models. From the best-fit dynamical model, with  $I$ -band mass-to-light ratio  $M/L = 3.6 M_\odot/L_\odot$ , the Einstein mass is consistent with  $M_E = 1.60 \times 10^{10} M_\odot$  from our lens model. The shapes of the density inferred from the lens model and from the surface brightness are very similar, but further improvement on the preliminary kinematic data is needed, before firm conclusions on the total mass distribution can be drawn.

In CHAPTER SEVEN, we consider in addition to the Einstein Cross twenty-five strong gravitational lens galaxies with redshifts up to  $z \sim 1$ . At such large distances, we are limited to the global properties of these lens galaxies, which effectively form a mass-selected sample of early-type galaxies in environments of relatively low density. We analyze their Fundamental Plane and use it, under the assumption that early-type galaxies are a homologous family, to measure the  $M/L$  ratio evolution.

If we assume that the  $M/L$  ratios of field early-type galaxies evolve as power-laws, we find for the lens galaxies an evolution rate  $d \log(M/L)/dz = -0.62 \pm 0.13$  in the rest-frame  $B$ -band for a flat cosmology with  $\Omega_M = 0.3$  and  $\Omega_\Lambda = 0.7$ . For a Salpeter (1955) Initial Mass Function and Solar metallicity, these results correspond to a mean stellar formation redshift of  $\langle z_* \rangle = 1.8_{-0.5}^{+1.4}$ . After correction for maximum progenitor bias, van Dokkum & Franx (2001) find for cluster galaxies  $\langle z_*^{cl} \rangle = 2.0_{-0.2}^{+0.3}$ , which is not significantly different from that found for the lens galaxies. However, without progenitor bias correction and imposing the constraint that lens and cluster galaxies that are of the same age have equal  $M/L$  ratios, the difference is significant and the stellar populations of the lens galaxies are 10–15% younger than those of the cluster galaxies. Furthermore, we find that both the  $M/L$  ratios as well as the restframe colors of the lens galaxies show significant scatter. About half of the lens galaxies are consistent with an old cluster-like stellar population, but the other galaxies are bluer and best fit by single burst models with stellar formation redshifts as low as  $z_* \sim 1$ . Moreover, the scatter in color is correlated with the scatter in  $M/L$  ratio. We interpret this as evidence of a significant age spread among the stellar populations of lens galaxies, whereas those in cluster galaxies are well approximated by a single formation epoch.

## 7 FUTURE PROSPECTS

An important part of the work presented in this thesis concerns the extension of dynamical modeling to triaxial geometry. This is in particular important for the giant ellipticals, many of which show clear signatures of non-axisymmetry in their kinematics as observed with integral-field spectrograph such as SAURON (Emsellem et al. 2004). Triaxial models of these giant ellipticals, together with axisymmetric models of other ellipticals and lenticulars (Cappellari et al. 2005b), will allow us to study in detail the clean fossil record of their formation.

Because SAURON typically observes the bright inner parts of galaxies, we need additional information to investigate the extended dark matter distribution predicted by current theories of galaxy formation (e.g., Kauffmann & van den Bosch 2002). We saw that strong gravitational lensing can place constraints on the dark matter, but only very few lens galaxies are close enough for detailed dynamical modeling. Currently we are investigating the use of the large field-of-view of SAURON to obtain stellar kinematic measurements in the faint outer parts. Further kinematic constraints are provided by HI and X-ray observations as well as velocities of globular clusters and planetary nebulae at large radii. We have started extending our modeling software to allow the inclusion of both integrated and discrete kinematics. This is also important for the dynamical modeling of stars and stellar systems in the Milky Way.

For nearby globular clusters such as  $\omega$  Centauri, such discrete modeling software will enable us to fit directly the three-dimensional velocity measurements of the individual stars, and even incorporate measurements of their age and metallicity. By fitting an orbit-based model simultaneously to all these observations, different stellar populations can be separated in phase-space, after which their structure and dynamics can be studied separately. This will be important for solving the puzzle of the multiple stellar populations in  $\omega$  Centauri (e.g., Freeman & Rodgers 1975; Bedin et al. 2004) and to reveal its formation history. Moreover, fitting directly the proper motion measurements in the very center of globular clusters, provided by observations with the Hubble Space Telescope (e.g., King & Anderson 2002), will allow us to investigate

the presence of a possible intermediate-mass black hole.

The modeling of the stars in the Milky Way is complicated by dust extinction and the presence of a rotating bar, which requires a non-trivial extension of our existing steady-state modeling software. In a preliminary study (Habing et al. 2005), we use the very accurate line-of-sight velocities of more than a thousand OH/IR and SiO masers to show that with such an extension we can model the dynamical structure in the inner Milky Way and provide direct evidence for the existence of a bar. Moreover, this extension will make it possible to model other rotating and barred galaxies, including the early-type and late-type spirals observed with SAURON (Falcón-Barroso et al. 2005; Ganda et al. 2005), and link the stellar and gas kinematics.

The large amount of already available photometric and kinematic data will grow rapidly with existing and future instruments and space missions, such as RAVE, GAIA and SIM, which will provide data for millions of stars, as well as VIMOS, SINFONI, MUSE and other integral-field spectrographs, which will provide two-dimensional data for many nearby galaxies. At the same time, the rapid increase of telescope size and instrument sensitivity will allow an ever deeper look into the universe, with a direct view on the evolution and even formation of stellar systems. The work presented in this thesis provides a step forward in the development and application of dynamical models to deduce from this wealth of data how the different stellar systems evolved from the Big Bang to the present day.

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