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Chaotic Dynamics in N-body systems

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Chaotic Dynamics in N-body Systems

Tjarda C. N. Boekholt

Chaotic Dynamics in N-body Systems

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Voor Pap en Mam

*“You must have chaos within you
to give birth to a dancing star.”*
Friedrich Nietzsche

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Preface

"This is how one tries to put the comet, which has caused so much fear, at the service of humour and of innocent children's play and it is clear that such things can count on a wide flow" — Leeuwarder Courant June 27, 1910

So ends the article in the Leeuwarder Courant June 27, 1910 entitled "The toy industry dedicated to Halley's Comet". And fear there was! The News of the Day for the Dutch East-Indies of June 7, 1910 reported a farmer from Eifeldorf-Geizenburg who threw her four month old baby in a pit out of fear of the destruction of the world by the arrival of Halley's Comet. The baby drowned. Also, the Algemeen Handelsblad of July 2, 1910 reported on numerous cases of suicide and insanity in Italy, and attacks of rage and suicide in France.

The close encounter of a comet with earth produces one of the most beautiful and dramatic sceneries we can observe, together with solar eclipses or the Aurora Borealis. Throughout history, the appearance of a comet was interpreted as a supernatural omen for good or bad things to come. As the examples above illustrate, the ignorance in the workings of nature and the belief in superstitions have caused harmless wonders of the universe to have horrific consequences. It is one of the tasks of the astronomer to understand the whereabouts of comets and other objects in the universe, and to explain them to the public.

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Leiden Observatory
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*"The Sun, with all those planets revolving around it
and dependent on it, can still ripen a bunch of grapes
as if it had nothing else in the Universe to do."*
Galileo Galilei

