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Periodic pulse solutions to slowly nonlinear reaction-diffusion systems

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To my grandparents.

Contents

Abstract

Patterns arise frequently in reaction-diffusion systems with a strong spatial scale separation, which naturally leads to the question of their dynamic stability. The scale separation induces a slow-fast decomposition in both the existence and stability analyses, which reduces complexity. In the existence analysis, patterns can be obtained by concatenating orbit segments of *slow and fast reduced systems*. These patterns exhibit spatially localized fronts and pulses, while they vary slowly in between those localized interfaces – see also Figure 1. In the stability analysis, the slow-fast decomposition manifests itself through a complex-analytic determinant-type function: *the Evans function*, which vanishes on the spectrum of the linearization about the pattern. In many specific models it has been shown using geometric methods that the Evans function factorizes in accordance with the scale separation. This factorization corresponds to a decomposition of the spectrum into slow and fast components, which are explicitly determined by *slow and fast reduced eigenvalue problems* arising in an appropriate singular limit. Thus, the factorization method leads to asymptotic control over the spectrum.

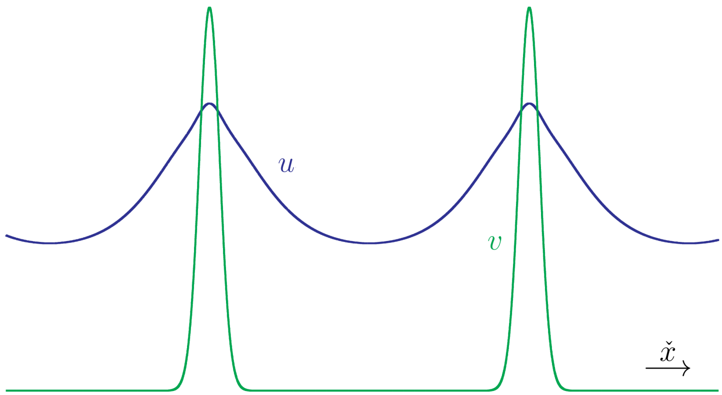

Figure 1: A periodic pulse solution in a reaction-diffusion system with two components. The v-component exhibits localized pulses and the u-component varies slowly.

The geometric factorization method has been developed in the context of *slowly linear* prototype models, in the sense that the associated slow reduced problems are linear. In the context of the periodic pulse solution shown in Figure 1, slow linearity entails that the dynamics of the slow u-component in between localized pulses is driven by linear equations. Recently, the geometric factorization procedure has been generalized to homoclinic pulse solutions in a general class of *slowly nonlinear* reaction-diffusion systems with two components. It has been shown that the dynamics in such systems differs fundamentally from their slowly linear counterparts. In this thesis we study *periodic* pulse solutions to a general class of *multi-component, slowly nonlinear* reaction-diffusion systems. At first sight this seems a straightforward extension of the homoclinic case. However, the geometric factorization method of the Evans function fails for periodic structures in slowly nonlinear systems. In addition, due to translational invariance of the pulse profile in space there is an entire curve of spectrum attached to the origin that shrinks to the origin in the singular limit, whereas for homoclinic pulse solutions there is only a simple eigenvalue residing at the origin. Therefore, in contrast to the homoclinic case, asymptotic control over the spectrum is insufficient to decide upon stability and a local higher-order analysis is required to determine the fine structure of the spectrum about the origin.

In this thesis we develop an alternative, analytic factorization method that does work for periodic structures in the slowly nonlinear regime, but applies more generally beyond this setting, i.e. it formalizes and generalizes the existing (geometric) factorization methods. We derive explicit formulas for the factors of the Evans function, which yield asymptotic control over the spectrum. Moreover, we obtain a leading-order expression for the critical spectral curve attached to origin. Together these spectral approximation results lead to explicit stability and instability criteria in terms of lower-dimensional reduced eigenvalue problems. Furthermore, the analytical grip on the spectrum provides insights into destabilization mechanisms of periodic pulse solutions, especially in the long-wavelength limit. Finally, we mention that, as a prerequisite for the stability analysis, we develop an existence theory for periodic pulse solutions with fine error estimates.