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**Deciphering fermionic matter: from holography to field theory**  
Meszéna, B.

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**Deciphering Fermionic Matter:  
from Holography to Field Theory**

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