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## **Deciphering fermionic matter: from holography to field theory**

Meszéna, B.

### **Citation**

Meszéna, B. (2016, December 21). *Deciphering fermionic matter: from holography to field theory*. *Casimir PhD Series*. Retrieved from <https://hdl.handle.net/1887/45226>

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**Author:** Meszéna, B.

**Title:** Deciphering fermionic matter: from holography to field theory

**Issue Date:** 2016-12-21

**Deciphering Fermionic Matter:  
from Holography to Field Theory**

PROEFSCHRIFT

TER VERKRIJGING VAN  
DE GRAAD VAN DOCTOR AAN DE UNIVERSITEIT LEIDEN,  
OP GEZAG VAN RECTOR MAGNIFICUS  
PROF. MR. C.J.J.M. STOLKER,  
VOLGENS BESLUIT VAN HET COLLEGE VOOR PROMOTIES  
TE VERDEDIGEN OP WOENSDAG 21 DECEMBER 2016  
KLOKKE 13.45 UUR

DOOR

Balazs Meszena

GEBOREN TE BOEDAPEST, HONGARIJE IN 1989

Promotor: Prof. dr K. E. Schalm  
Promotiecommissie: Prof. dr S. S. Lee (McMaster University, Hamilton,  
Canada)  
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Casimir PhD series, Delft-Leiden 2016-35

ISBN 978-90-8593-228-9

An electronic version of this thesis can be found at

<https://openaccess.leidenuniv.nl>

Cover design: Proefschriftmaken.nl — Uitgeverij BOXPress

Printed & Lay Out by: Proefschriftmaken.nl — Uitgeverij BOXPress

The research presented in this thesis was founded by Leiden University through a Huygens fellowship.





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