Multivariate and longitudinal data on growing children. Solutions using a three-mode principal component analysis and some comparison results with other approaches

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Section 1.7

Multivariate and longitudinal data on growing childre

SOLUTIONS USING A THREE-MODE PRINCIPAL COMPONENT ANALYSIS AND SOME COMPARISON RESULTS WITH THE OTHER APPROACHES

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Multivariate longitudinal data on the morphological development of young girls were analysed using three-mode principal component analysis. In particular, the deviations from the average growth curves were examined to investigate differential growth patterns of (groups of) girls. Results show girls to differ with respect to their growth in stoutness, skeletal length, and skeletal width. Most girls' growth patterns can succinctly be described as linear combinations of the average growth curves and three 'Girl Types'. The present analysis is compared with several other analyses with different techniques.

The present paper has two aims : the first and primary aim is to present an analysis of the multivariate longitudinal growth data collected by Sempé (1979, and this volume) employing three-mode principal component analysis. The secondary aim is to compare our analyses with the other analyses of the same data presented in this volume. In particular, comparisons are made with Lavit & Pernin's* work, and to a lesser extent with that of Lewi & Calomme, Pontier

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* References without a date refer to papers in this volume.

& Pernin, and Mineo. In the discussion of the method employed, a rough theoretical comparison between our method and STATIS (for references see Lavit & Pernin) is worked out. It should be stated at the outset that the present paper, in contrast with the oral presentation, was written with all other papers available. This implies that many of the ideas of other authors implicitly or explicitly have gone into shaping this paper, so that we are very much in debt to them. To bring some unity in the presentations it has been attempted to adopt the notation and terminology of other authors, especially that of Lavit & Pernin.

The growth data, yearly scores of 30 children on 8 morphological variables from their fourth to their fifteenth year, may be treated in two parts (1) a set of average growth curves for each of the variables, i.e. a 12 (years) by 8 (variables) matrix M with the averages of each of the variables at each point in time, and (2) the deviations from these average growth curves, i.e. a $30 \times 8 \times 12$ data block of girls by variables by years. Even though in an independent paper of these data one should provide a proper discussion of the trends and patterns in M, such an analysis is not presented here, as Lewi & Calomme have already done so (their Figures 1 and 2, and Table I), and to a lesser extent Mineo as well. The average growth curves themselves are portrayed as parts of Figures 4 and 5 in Lavit & Pernin. It should, however, be noted that in both Figures 1 and 2 of Lewi & Calomme not the averages themselves are displayed but a double-centred version of them (see their Introduction).

With respect to the second part, we will not analyse the deviations from the average growth curves per variable directly, but first scale these values such that the deviations across variables may be compared. The deviations themselves are not comparable without scaling because of different measurement scales. In practice there seem to be two options for this kind of scaling : either removing the scale (here defined as the sum of squares, and called variation) per variable per occasion, or removing the scale per variable over all occasions together. In connection with their global indices Pontier & Pernin (section 1.3) remark that the first kind of scaling eliminates the variation of the variables across time, and the second kind of scaling eliminates artifacts due to differences of unit of measurement or order of magnitude. In the present case we considered it undesirable to remove the increase or decrease in variation from the deviation scores as such variation is an essential feature of the transition to maturity, namely children start to diverge in their physical characteristics. To eliminate this process from the analysis by equating the sums of squares per variable per year seems rather artificial.

More formally, the data analysed with three-mode principal component analysis have the following form :

$$\hat{x}_{ijk} = (x_{ijk} - \bar{x}_{.jk})/s_{.j.} \qquad i = 1...n;\ j = 1...p;\ k = 1...K \qquad (1)$$

with $\bar{x}_{.jk} = (1/n)\,\Sigma_i\, x_{ijk}$, and $s^2_{.j.} = (1/nK)\,\Sigma_i\,\Sigma_k\,(x_{ijk} - \bar{x}_{.jk})^2$. Table 1 of Lewi & Calomme and Table 3 of Mineo give the $\bar{x}_{.jk}$, and the $s_{.j.}$ are 20.32 (Weight), 6.86 (Length), 5.18 (Crown-Rump Length), 3.53 (Head), 6.16 (Chest), 3.84 (Arm), 4.06 (Calf), and 3.16 (Pelvis). The $\hat{x}_{ijk}$ to be analysed represent the deviations from the average growth curves, or the curves representing the growth of an average girl. In other words, an average girl would have $\hat{x}_{ijk} = 0$ for all i, j, and k. Considering the way the sample was constituted (Sempé), i.e. of normal French girls, it is not unreasonable to accept the existence of such an average girl, and thus averaging over several growth curves can be accepted as a meaninful procedure. In fact, girl 5 is practically this average girl.

1. Three-mode principal component analysis

The present description of the technique owes much to Tucker (1963; 1966), Kroonenberg (1983), and Kroonenberg, Lammers, and

Stoop (1985). In order to make the presentation comparable to that of STATIS described by Lavit & Pernin, their notation will be used as much as possible, rather than that of the above references. The major exception is that we have eliminated the reference to their metric D (section II-2ff) as in this example it was chosen in such a way that it is irrelevant for the subtive outcomes of their (and our) analyses.

1.1. (Two-way or two-mode) principal component analysis

Let X be a (n × p) data matrix of p quantitative variables observed for n individuals. The singular value decomposition of X is defined as

$$X = T\Lambda^{\frac{1}{2}} U' = YU' = TZ', \tag{2}$$

where U is the eigenvector matrix of the sample scalar-product matrix for variables $V = X'X$, and T the eigenvector matrix of the scalar-product matrix for individuals $W = XX'$. If we define $Y = T\Lambda^{\frac{1}{2}}$, then the columns of Y contain the coordinates of the individuals on the columns of U, the principal axes of V. Similarly, $Z = U\Lambda^{\frac{1}{2}} = X'Y\Lambda^{-\frac{1}{2}}$ contains the coordinates of the variables on the columns of T, the principal axes of W.

1.2. Three-mode principal component analysis

Before describing the technique a few general remarks about STATIS are in order as we have placed the present technique within the STATIS framework, or rather terminology. The STATIS approach is characterised by the motto "Interstructure - Compromis - Intrastructure", in which the Interstructure describes the structure between the K matrices X_k, the Compromis describes a common component space for the individuals derived from a weighted average of the scalar-product matrices for individuals $W_k = X_k X_k'$, and finally the Intrastructure describes the variables loadings for the components of the Compromis, and the scores for individuals at each time point k around their barycentric or common solution. The three-mode analysis presented here bypasses the Interstructure. To investigate this structure one should use a different form of three-mode principal component analysis as described in Tucker (1966) or Kroonenberg and De Leeuw (1980), and many other authors.

Suppose the same population of individuals is observed on the same p variables at K points in time, then $X = (X_1, \ldots, X_k, \ldots, X_K)$ is the resulting set of $(n \times p)$ data matrices. Analogously to the singular value decomposition in the two-way case a generalized singular value decomposition for X may be defined, such that

$$X_k = TC_kU' \quad (k = 1 \ldots K) \tag{3}$$

The eigenvector matrices T and U may be interpreted as in the two-way case, and they are the 'compromise' solutions for the individuals and variables, respectively. The compromise solutions take the place of separate solutions T_k and U_k for each point in time k. The C_k contain the generalized singular values for occasion k, and generally they are not diagonal as they would be for the separate solutions. Furthermore, they do not necessarily contain non-negative numbers as in the two-way case. The c^2_{abk}, however, add up to the total sum of squares if a complete solution is obtained, and to the fitted sum of squares if an approximate solution is derived, and each c^2_{abk} may be interpreted as a variation or sum of squares accounted for as in the two-way case. The form of three-mode principal component analysis discussed here is based on Tucker (1972), rather than on Tucker (1966) in which a three-mode analysis is proposed which also contains a compromise solution for the third (or remaining) mode.

If we define the symmetric $(n \times n)$ matrix $W_k = X_kX_k'$ as the scalar-product matrix for individuals for the k-th point in time, then using (3) $W_k = TC_kU'UC_k'T' = TC_kC_k'T'$, and $W = \Sigma_k W_k = T(\Sigma_k C_kC_k')T'$. As $W = XX'$, T may be chosen to be the $(n \times n)$

eigenvector matrix of W, *and thus* $\Sigma_k C_k C_k' = \Lambda$ the diagonal eigenvalue matrix with decreasing values on its diagonal. Analogously to the two-way case a $\ddot{Y}$ may be defined as $\ddot{Y} = T\Lambda^{\frac{1}{2}}$, and $Z = (Z_1,\ldots,Z_k)$ with $Z_k = X_k'T = UC_k'T'T = UC_k'$, or equivalently $Z_k = X_k'\ddot{Y}\Lambda^{-\frac{1}{2}}$. The Z_k are the component scores, i.e. the columns of Z_k contain the coordinates of the variables on the compromise axes of the individuals.

Similarly, one may define the symmetric $(p \times p)$ matrix $V_k = X_k'X_k$ as the scalar-product matrix between variables for the k-th point in time. Then $V = \Sigma_k V_k = U(\Sigma_k C_k'C_k)U'$, and as $V = X'X$, U may be chosen to be the $(p \times p)$ eigenvector matrix of V with $M = \Sigma_k C_k'C_k$ the corresponding matrix with descending eigenvalues on the diagonal. When one defines $\ddot{Z} = UM^{\frac{1}{2}}$, and $Y = (Y_1,\ldots,Y_k)$ with $Y_k = X_kU = TC_kU'U = TC_k$, or equivalently $Y_k = T\ddot{Z}M^{-\frac{1}{2}}$ as the component scores, then the columns of Y_k contain the coordinates of the individuals at occasion k on the compromise axes for the variables.

In general, when not all n eigenvectors of W, and not all p eigenvectors of V are of interest but in both cases only a limited, not necessarily equal, number of eigenvectors are relevant, one has to solve the eigenvector-eigenvalue problems of W and V *simultaneously by iteration in order to find an optimal solution for* both compromises at the same time. This solution has the property that the variation accounted for by both compromises is the same and is due to the same part of the data. Such a solution may be found by minimizing the loss function

$$\Sigma_k \|X_k - TC_kU'\|^2 \tag{4}$$

over all orthonormal U *and* T, *and all arbitrary* $C = (C_1,\ldots,C_k)$. Kroonenberg and De Leeuw (1980; Kroonenberg, 1983) show that the solution to this problem is that $\hat{U}$ is the eigenvector matrix of $\hat{W} = \Sigma_k X_k\hat{T}\hat{T}'X_k'$, $\hat{T}$ is the eigenvector matrix of $\hat{V} = \Sigma_k X_k'\hat{U}\hat{U}'X_k$, and $\hat{C}_k = \hat{T}'X_k\hat{U}$ $(k = 1...K)$. Note that the solution of each mode takes into account the reduction over the other mode, and that when all eigenvectors are computed $\hat{V}$ and $\hat{W}$ reduce to V and W, and thus $\hat{T}$ and $\hat{U}$ to T and U. In the approximate solution we will define, unlike above, the component scores directly as $\hat{Z}_k = \hat{U}\hat{C}_k' = \hat{U}\hat{U}'X_k'\hat{T}$, and $\hat{Y}_k = \hat{T}\hat{C}_k = \hat{T}\hat{T}'X_k'\hat{U}$, again taking into account the reduction in dimension in the other mode. Also here $\hat{Z}_k$ and $\hat{Y}_k$ reduce to Z_k and Y_k when all components are computed.

What makes three-mode principal component analysis really different from techniques like STATIS is that information is available on two compromise solutions at the same time, and moreover that the relationships between the two compromise solutions at each point in time can be expressed numerically via the so-called 'core slices' or 'core planes' C_k with the generalized singular values. An element c_{abk} indicates the weight that is given (or importance that is attached) to the combination of the a-th column of T, and the b-th column of U in the description of a score x_{ijk} by the model (3), as can be seen from (3) in sum notation

$$x_{ijk} = \Sigma_a \Sigma_b c_{abk} t_{ia} u_{jb}. \tag{5}$$

Inspecting the vectors $(c_{ab1},...,c_{abK} \mid a = 1...A;\ b = 1...B;\ A \leqslant n;\ B \leqslant p)$ one can judge how the combination of the a-th component for the individuals and the b-th component for the variables change over time (see Figure 2).

In summary, after the (approximate) decomposition of the k-th slice of the three-mode data matrix X_k in TC_kU' (dropping the carets for convenience), one has available a compromise solution for individuals (T), a compromise solution for variables (U), for each occasion a matrix C_k with mutual weights for the combinations of axes from the two compromise solutions, furthermore component scores of the individuals on the variable axes (Y_k) for each time point, and similarly for the variables on the individual axes (Z_k).

If a visual display is desired of the relationship between variables and individuals at each occasion within the framework of the compromise solutions, TC_kU' may be decomposed as

$$TC_kU' = TE_k\Gamma_kF_k'U' = (TE_k\Gamma_k^{\frac{1}{2}})(UF_k\Gamma_k^{\frac{1}{2}})' = \tilde{T}_k\tilde{U}'_k \qquad (6)$$

and the $\tilde{T}_k$ and $\tilde{U}_k$ may be simultaneously displayed in a single plot. ($E_k\Gamma_kF_k'$ *is the singular value decomposition of* C_k). Note that after the basic matrices T, U, and C have been derived all other information is directly based on them without referring back to the original data matrix.

When the special weighting of the W_k (and V_k) in STATIS is of relatively minor importance, i.e. $\Sigma_k\, \alpha_kW_k \simeq \Sigma_k\, \hat{W}_k$ (and $\Sigma_k\, \beta_kV_k \simeq \Sigma_k\, \hat{V}_k$), then the practical differences will be small. In the present data the α's are not very different as can be seen from Figure 1 of Lavit & Pernin. It should be noted that STATIS determines the Interstructure in a way, which has no direct equivalent in the present form of three-mode analysis, nor in Tucker's (1966) version be it that it can be shown that the third compromise solution is a special kind of Interstructure.

2. Results and interpretation

The analysis of the data will be discussed in several parts. First, we will present some statistics on the overall solution, then we will discuss the compromise solutions for individuals and variables, next we will portray how the relationships or mutual weights change over time, and give on the basis of these changes a general description how girls may deviate from the average growth partterns, and finally we will portray, and briefly discuss, the development of individual girls over time, both via 'differential growth curves' and via 'trajectories' in the compromise component space for variables.

2.1. Fit

In contrast with Lavit & Pernin's two-dimensional solution, we will examine a three-dimensional solution for both individuals and variables; in other cases different numbers of components may be necessary for the two compromise solutions, but three components for each seems adequate in the present case. The overall fitted sum of squares accounted for 77% of the total variation in the centred and scaled data. Due to the simultaneous derivation of the compromise solutions, both of them fit the same part of the data, they differ, however, in the way they divide the fitted sum of squares over their principal axes. The axes of the subject compromise solution account for 55, 14, and 7% of the variation in the data, respectively, while those of the variable compromise solution accounted for 57, 14, and 6% respectively. Note that these figures compare very well with the STATIS compromise solution for individuals of 57 and 13% for the first two axes.

As mentioned in the introduction the differences in variability over the years are still contained in the data set, and in Figure 1 this variability is shown in terms of the total and the fitted sums of squares. The figure shows that differences between girls measured over all variables increase until their thirteenth year, and diminish in the next two years available. The data allow, however, no extra-

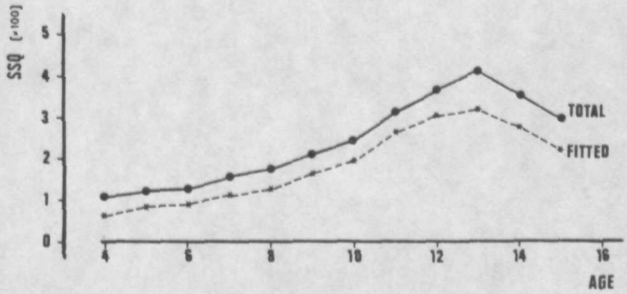

Fig.1. Total and fitted sums of squares per year

polation beyond the fifteenth year (see Mineo who attemps to fit logistic curves and to estimate upper asymptotes, which asymptotes can be taken as upper bounds for at least the averages beyond the fifteenth birthday. For two variables, head circumference and left upper arm circumference no asymptotes could be found). The increasing variability suggests that the onset and speed of growth is different over girls, such that slower growth catches up, at least partially, with faster growth. Whether the variability returns to its original level at the age of five cannot be judged from the present data, but it seems unlikely that it will.

2.2. Compromise solution for girls

In Figure 2 the compromise solution for girls is shown; the lengths of the axes are proportional to their explained variation. As the first axis (I) carries most weight the configuration has roughly the shape of a flattened elongated ellipsoid. No specific subgroups can be seen, and the second and third axis are largely dominated by a few girls; the second axis by 4, 6, 14(E), 18(I), 20(K), and 24(O), and the third axis by 14(E), 19(J), 22(M), and 30(U). (The letter between parentheses refers to the label in this and other figures). One way of using the 'girl space' is to consider the axes as (morphological) Types or ideal types, and describe each girl as a linear combination of such Types. One could also refer to average girls in the centre of the figure as a type (as in Mineo, cluster 2), but this seems not a proper thing to do. A Type is thus defined as a girl who loads exclusively on one single axis, and the loading of a real girl on such an axis indicates how relevant the axis is for that girl. As no distinct subgroups seem to be present, the Types defined here should not be taken as clusters with qualitative differences. In fact, Mineo's cluster analyses define clusters which can largely be recovered by cutting up the first principal axis into three parts. His first cluster consists of girls 4, 7, 21(L), and 27(R) on the positive side of the

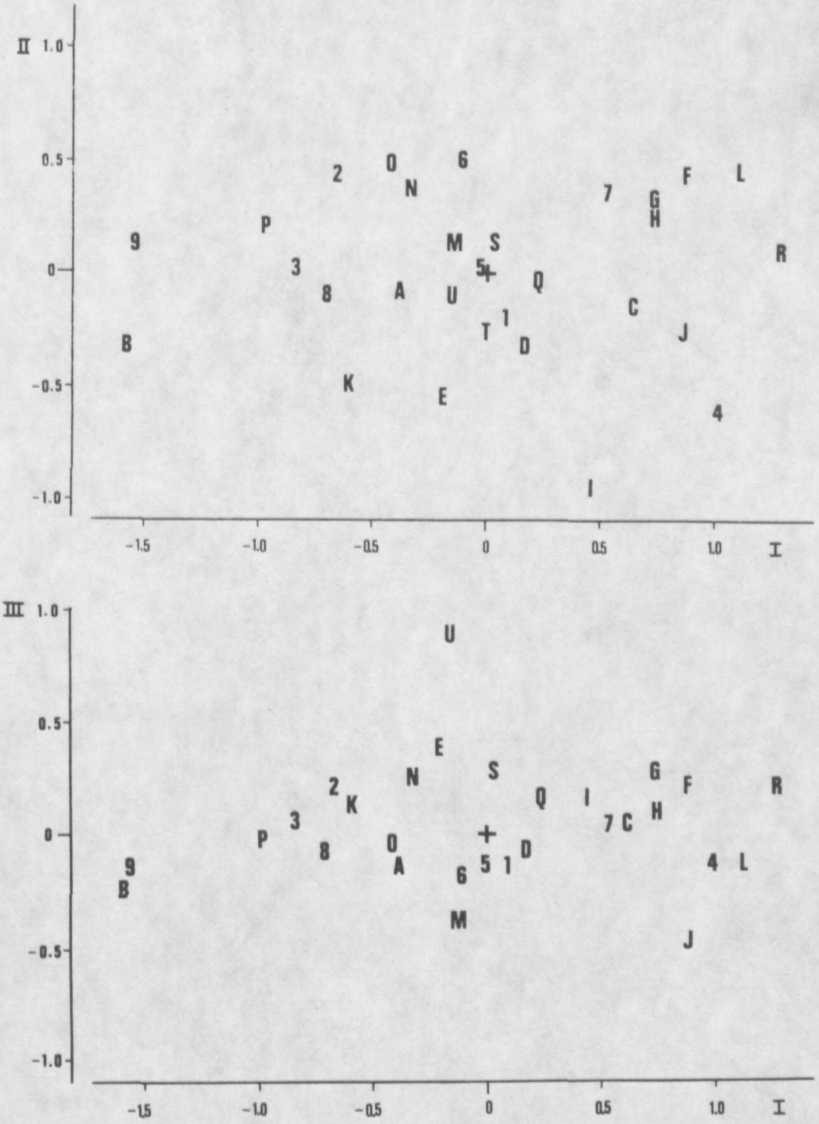

Fig.2. Compromise solution for girls

first axis, his second cluster consists of girls 1, 5, 10(A), and 13(D) located near the centre of Figure 2, and finally the third cluster consists of girls 3, 9, and 11(B) on the negative side of the first axis.

Even though there are three axes, we will, for convenience define six Girl Types, one for each side of an axis. Type I+ (Type I-) is defined by the positive (negative) side of the first axis, and thus 27(R) is approximately a Type I+ girl and 9 a Type I- girl, while 30(U) is a type III+ girl. Using the variables we will give more substantive descriptions of the Girl Types.

2.3. Compromise solution for variables

Instead of showing the principal components for the variable compromise solution, we will direct our attention to a varimax rotated version of them. The primary motivation is that the variables can be divided into three groups (see Lavit et Pernin, section 1.3) Stoutness (chest, calf, arm, weight), Skeletal Length (length, crown-rump length), and Skeletal Width (head, pelvis), and as Table 1 shows, the varimax transformed components pass roughly through the centroids of each of the groups of variables. By using the varimax axes we can describe the variable compromise solu-

Table 1. Variable compromise solution after varimax

Variables	Mnemonics[a]		Component[b]		
	K	L&P	1	2	3
Left Upper Arm Circumference	Arm	PB	.56	-.21	.04
Chest Circumference	Chest	PT	.52	.02	-.05
Left Calf Circumference	Calf	PJ	.44	.01	.03
Weight	Weight	TA	.43	.17	.08
Length	Length	PD	-.03	.76	-.06
Crown-Rump Length	CRLen	SS	.03	.57	.13
Head Circumference	Head	PC	-.11	-.10	.91
Maximum Pelvic Width	Pelvis	AB	.14	.17	.37
Percentage of Sum of Squares			35	23	19

[a] K=this paper; L&P=Lavit & Pernin. [b] Orthonormal components rotated with varimax

tion in terms of variable groups, rather than ratios of variable groups, such as Stoutness/Skeletal Length ratio, as Lewi & Calomme do explicitly, and Lavit & Pernin implicitly. The varimax transformation redistributes the variation accounted for per axis is such a way that Stoutness accounts for 35%, Skeletal Length for 23% and Skeletal Width for 19% of the deviations from the average growth curves.

2.4. Growth characteristics

Next we will investigate how the girl types defined above develop, in particular with respect to which variable groups this growth takes place. In order to discuss this we will first define some terms.

Positive (negative) growth curves lie above (below) the average growth curves, and are (approximately) parallel to them. Accelerating (decelerating) growth curves move away from (towards) the average growth curves, and positive (negative) crossing growth curves cross the average growth curves from above (below).

Figure 3 shows the relationships (or mutual weights) between the axes of the two compromise solutions for each point in time. The height of the curves is both influenced by the size of the deviation of girls from the average growth curves and by the number of girls showing such a pattern. Therefore, comparisons of absolute size are best made within girl type.

Girl Type I+. This girl type has accelerating growth curves uptil a certain age and decelerating growth curves thereafter. Stoutness accelerates until the eleventh year, levels off between 11 and 14 years old, and decelerates in the last two years. Thus this girl type is stouter to start with at the age of 4, and becomes increasingly more corpulent until her eleventh year, and relatively speaking only after fourteen years of age the Average Girl starts catching up, but probably only partly so. The acceleration for

Skeletal Length ends somewhat earlier (around 12), and the trend is reversed more suddenly. Furthermore, this girl type tends to take up roughly the same position in the end as in the beginning. Skeletal Width shows roughly the same pattern. In other words, Type I+ girls, like 27(R), have accelerating growth on all variable groups, indicating earlier growth than average, especially in the skeletal variables for which they end up about where they started. In the end they tend to be (overly) hefty compared to the Average Girl. The mirror image of this type of girl is the Type I- Girl (e.g. 9 and 11(B)), whose growth curves are the mirror images of the Type I+ Girl. They have decelerating growth curves on all variables, indicating later growth than average. In the end they regain their relative position with respect to the skeletal variables, but they remain on the slight side.

Girl Type II+. This girl type has a different set of growth patterns. From an average Stoutness she decelerates until 14 years of age, and thus becomes, relatively speaking, slighter all the time. Her relative Skeletal Width is slightly above average and remains so, on the other hand she keeps on growing in Skeletal Length all through the observation period. Thus a Type II+ Girl becomes a tall and skinny girl, and she gets more so all the time. Not so a Type II- Girl who becomes a more squat and compact girl all the time.

Girl Type III+. This girl type is yet again different. She has a decelerating growth curve for Stoutness until 13 years old, changing from a slightly above average girl to an under-average one. Realizing that these scores are deviations from the average curves, the deceleration implies a delayed growth, i.e. this girl type does not really start growing as her colleagues in Stoutness until thirteen years of age, but then in two years manages to make up for it entirely, as she ends up at about the same deviation above average as she started with. Her skeletal growth reflects the same pattern,

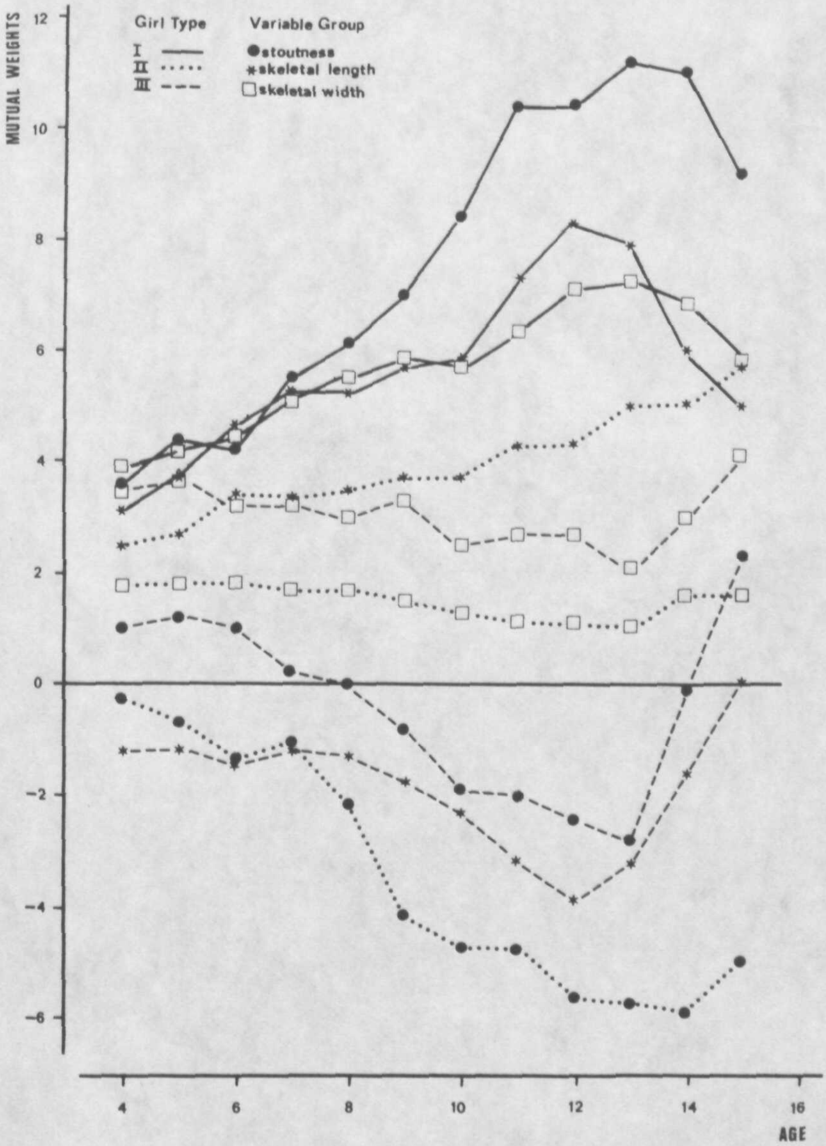

Fig.3. Time trends for mutual wieghts of axes of the variable and individual compromise solutions

but note that her Skeletal Width (i.e. her head circumference) ends up relatively speaking much larger than her Skeletal Length. Girl Type III- is, of course, again the mirror image of the Type III+ Girl.

In evaluating different girl types, it seems that it is not so much the appearance of the girls at a particular age that is important in distinguishing between them, but rather the different ways in which they develop. In other words, it is the dynamics of growth that differentiates, and not the static aspects of the situation.

2.5. Growth of individual girls : Time trends

If the interest was solely centred on morphological types and their development Figure 3 could serve this purpose admirably, if however one is also interested in inspecting the development of individual girls, then one may describe their growth on the variable groups by comparing them again with the Average Girl using the component scores (i.e. $Y_k = TC_k$) as defined above. One could also inspect the component scores of the variables on the girl compromise axes (i.e. $Z_k = UC_k'$) as Lavit & Pernin do (their Figure 2), but we will not display them here. Because the variables align rather closely with their components, their separate relationships with the girl types are already reflected in our Figure 3 by those of the components or variable groups.

In contrast with the variables, most girls are true linear combinations of girl types, and their growth curves with respect to, for instance, Stoutness are often influenced by more than one of the Stoutness curves in Figure 3. As an example, the growth curve for Stoutness of girl 4 is a linear combination of the Stoutness curves of all three girl types :

$$y_{41k} = t_{41}c_{11k} + t_{42}c_{21k} + t_{43}c_{31k} =$$

$$= .25c_{11k} - .31c_{21k} - .11c_{31k},$$

where c_{p1k} is the Stoutness growth curve (variable axis 1) for girl type p $(p = 1,2,3)$, and the t_{4p} are the loadings of girl 4 on the girl compromise axes. From Figure 3 we see that the signs of the t_{4p} orient all curves with their peaks upwards, so that girl 4 accelerates very rapidly (starts growing early and fast) in Stoutness. For girl 16(G) the Stoutness curves more or less cancel each other

$$y_{G1k} = .18c_{11k} + .17c_{21k} + .12c_{31k} ,$$

so that her Stoutness curve is nearly flat, and parallel to the average growth curve, just like that of girl 26(Q) for which no Stoutness curve is really important

$$y_{Q1k} = .05c_{11k} - .01c_{21k} + .06c_{31k} ,$$

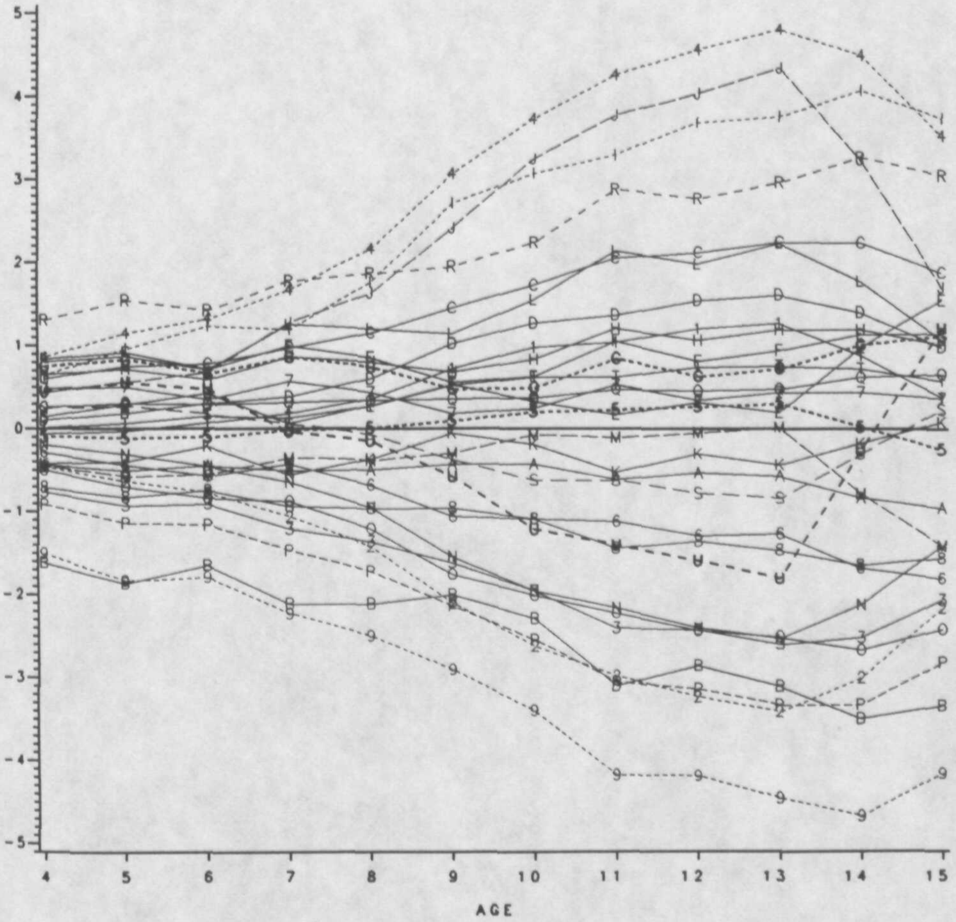

Fig.4. Individual trends for Stoutness

In Figures 4, 5, 6 the individual trends are shown for Stoutness, Skeletal Length, and Skeletal Width, respectively. These figures, of course, confirm the conclusions from Figure 3, but as each girl is now shown separately, number of girls and size of deviation are no longer confounded, so that it is easier to make statements about the absolute size.

Stoutness. Differentiation between girls occurs especially in Stoutness. As expected from the above discussion, the general trend is that stout girls above average stay above average, and the stouter the quicker the acceleration in Stoutness until around 11 to 13 years of age, while after that the Stoutness decelerates or stabilizes, with the reverse pattern for slight girls. There are notable exceptions, for instance, after her thirteenth year girl 19(J) virtually stopped growing allowing the Average Girl to catch up in Stoutness. Girl 14(E) suddenly started getting fleshier in her thirteenth year. The most noticeable deviant pattern is of girl 30(U) who provides the only example of a substantial crossing curve. Around her thirteenth birthday she was relatively slight, but started growing very rapidly thereafter, to end up considerably above the average growth curve two years later. She is probably largely responsible for the form of the Stoutness curve of the Type III Girl.

With respect to Skeletal Length, it is clear that the overall differences are less. The differences at the age of four between girls are comparable to those for Stoutness, but Skeletal Length increases less, and the differentiation in growth curves is not as marked. The overall trend is, however, comparable with very few crossing curves, with taller (shorter) girls tending to become taller (shorter) until their 12th to 13th birthday. For some girls Stoutness and Skeletal Length increase hand in hand (e.g. 19(J)), but this is not true for all; for instance, girl 9 makes up for her length deficit but not her wieght deficit. Girl 18(I) is very peculiar in that the acceleration of her Stoutness growth is considerable and only seems to be

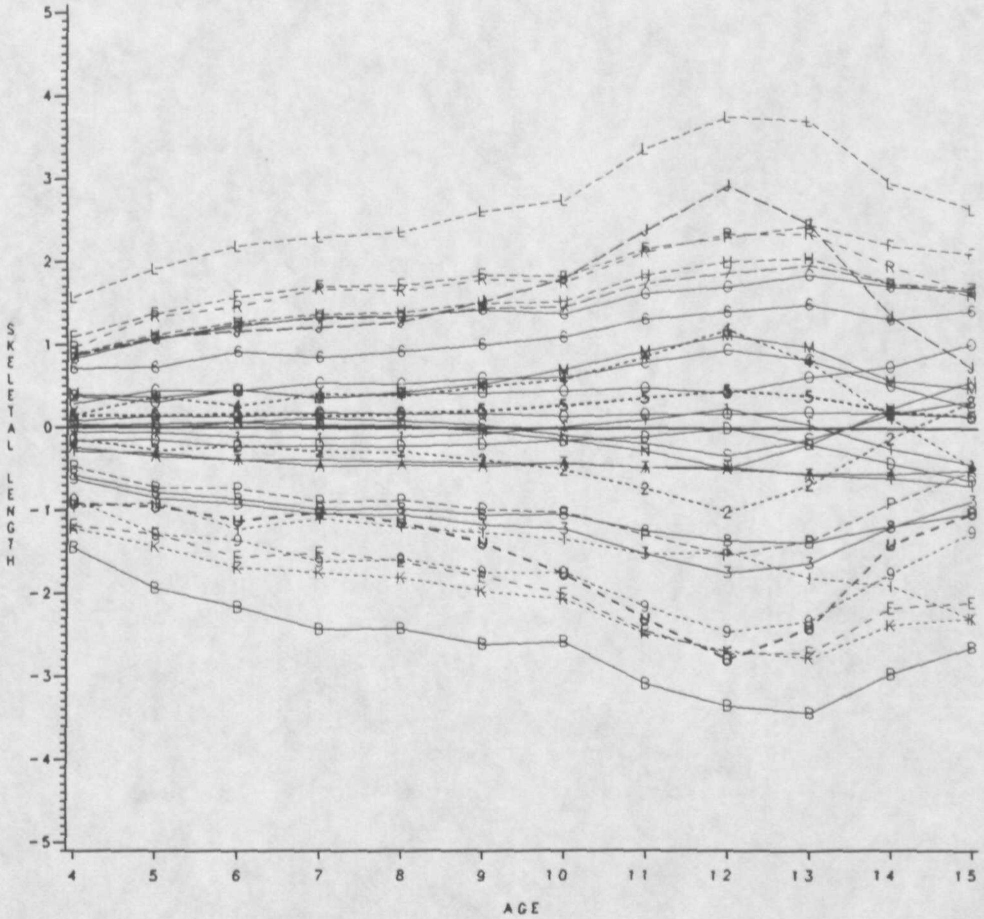

Fig.5. Individual trends for Skeletal Length

levelling off at the age of 15, but she is still decelerating in Skeletal Length in her 15th year. The relationships between the various groups for individual girls can however be better examined via the trajectories shown below.

<u>Skeletal Width</u> (Figure 6) is different in the sense that virtually all girls have curves roughly parallel to the average growth curves, showing that Skeletal Width, especially head circumference has a regular and stable growth pattern, and that generally differences between grils are maintained troughout the observation period.

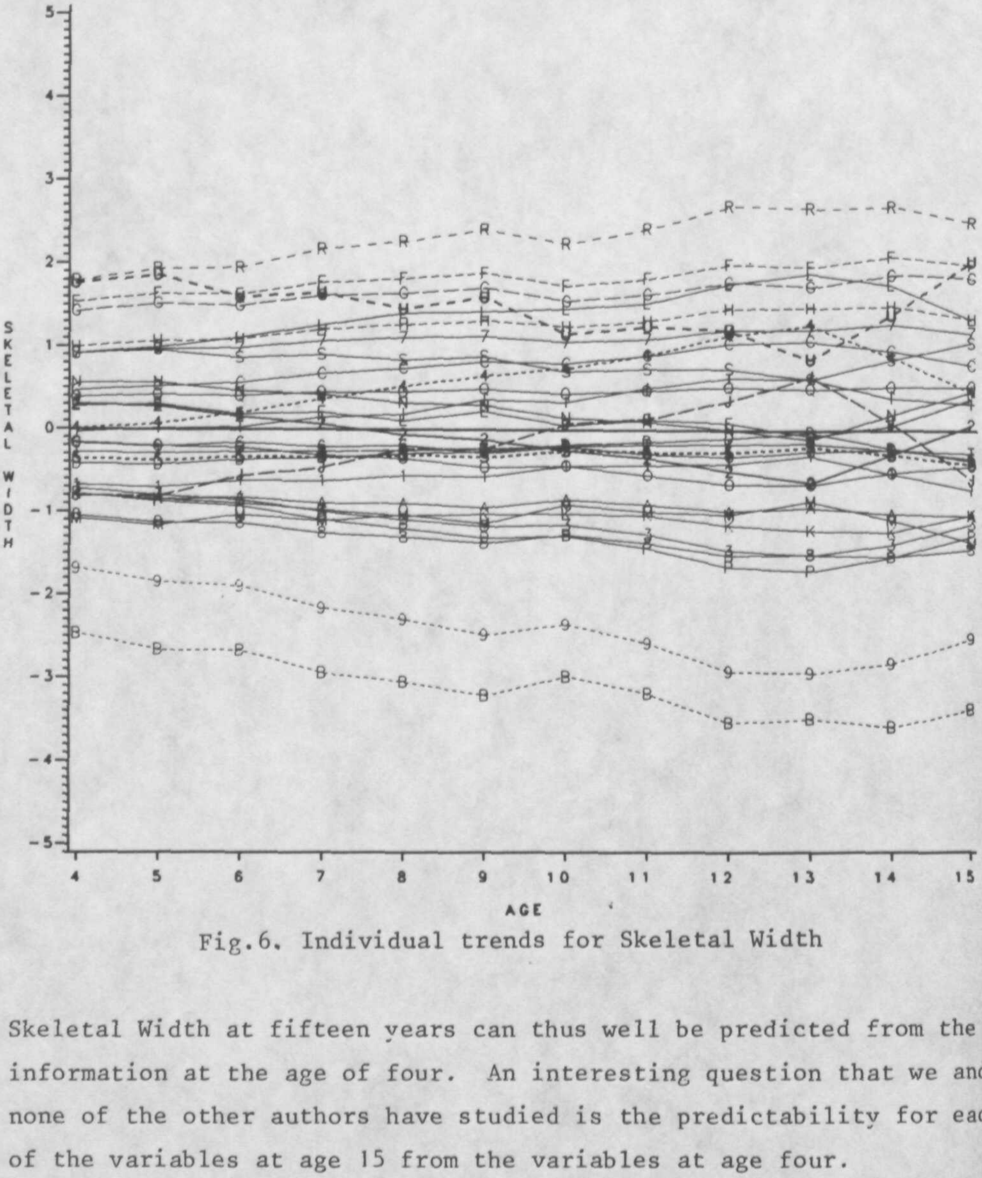

Fig.6. Individual trends for Skeletal Width

Skeletal Width at fifteen years can thus well be predicted from the information at the age of four. An interesting question that we and none of the other authors have studied is the predictability for each of the variables at age 15 from the variables at age four.

2.6. Growth of individual girls : Trajectories in variable space

By pairwise plotting component scores on the variable components, one obtains plots with 'trajectories' for each of the girls showing how the relative importance of the components change over time (see also Lavit & Pernin, and Pontiers & Pernin). Note that Lavit & Pernin in their Figure 3 show the trajectories of the girls in the compro-

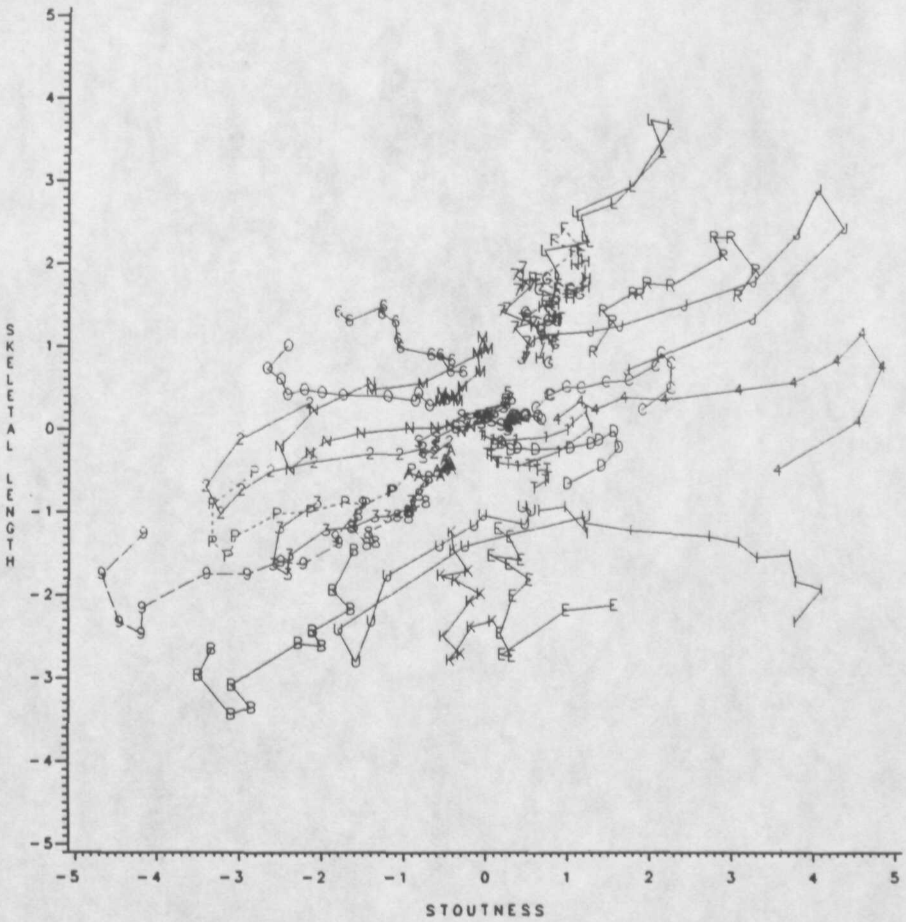

Fig.7. Stoutness versus Skeletal Length

mise solution for individuals, while here in Figures 7 and 8 they are shown in the compromise solution for variables. Lavit & Pernin derived their coordinates per occasion via $\ddot{Y}_k = W_k Y \Lambda^{-1}$ (see their section 1.3).

In Figure 7 Stoutness versus Skeletal Length is displayed by plotting the vertical coordinates of Figure 4 and 5 against one another. The elongated pattern shows the correlation between the scores on the two variable groups, and the longest axis roughly corresponds to the first principal component. Trajectories at an angle of the main axis

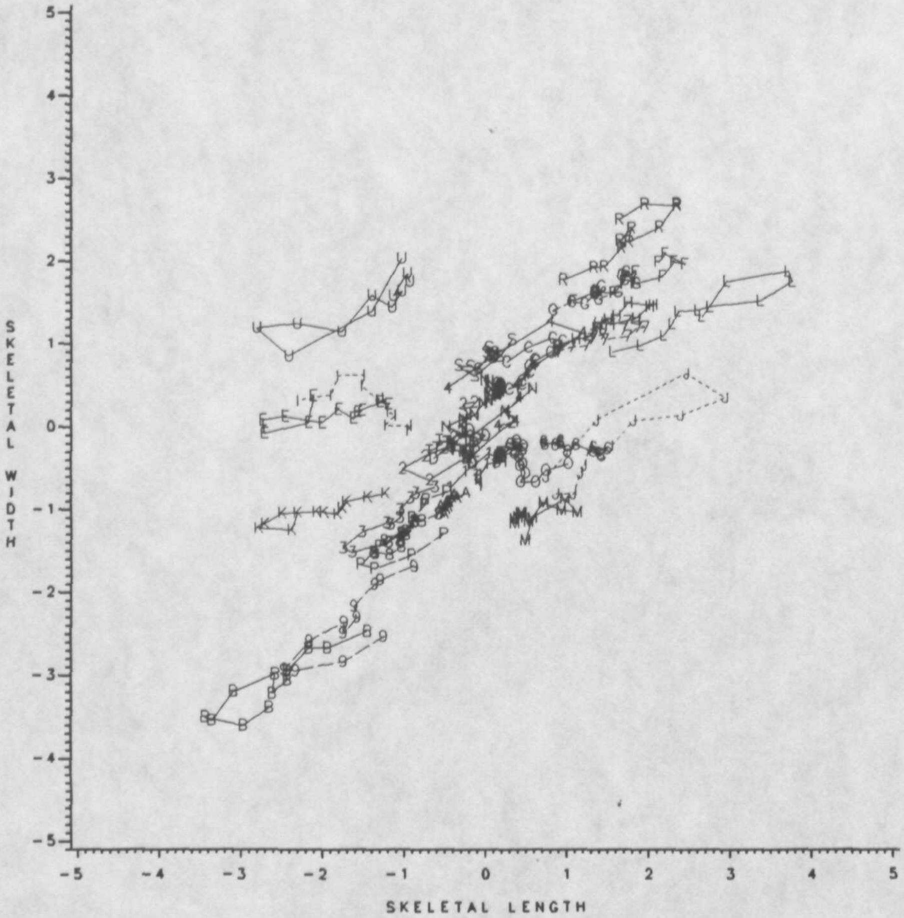

Fig.8. Skeletal Length versus Skeletal Width

of the ellipse show 'imbalances' in growth, i.e. growing stouter but not taller, and vice versa. The centre of the plot represents the Average Girl, or at least her growth pattern. All clearly visible trajectories start more or less outwards, indicating an acceleration away from the average, and most trajectories turn inward again somewhere between the eleventh and thirteenth year, showing that they tend to become more like the Average Girl later on. The turning points of the trajectories indicates changes in the relative rate of growth between the Variable Groups defining the axes.

Figure 8 gives an impression of differential skeletal growth. Except for nine girls, 6, 14(E), 18(I), 19(J), 20(K), 21(L), 22(M), and 24(0), there is a near perfect correlation between differential growth in Skeletal Length and Skeletal Width.

3. Conclusion

In this paper we have tried to give a fairly detailed description of both large and small scale patterns in the morphological growth of girls between 4 and 15 years of age. Different types of girls could be described, be it that some types originate primarily due to a few individuals. The types differ in the way they grow on specific groups of variables. The latter could be grouped into variables indicating Stoutness, Skeletal Length, and Skeletal Width. Several interesting details emerged, for instance, that heavier (lighter) girls tend to grow heavier (lighter) at a faster rate and earlier on, that differences in growth speed between girls are concentrated in the Stoutness variables, and that Skeletal Width (especially head circumference) shows differences in size between the girls but hardly in growth rate, with Skeletal Length taking an intermediate position. Also noteworthy is that with very few exceptions, girls do not crossover to a substantial degree, i.e. as a rule stouter, longer, and/or wider than average girls do not become below average on these variables, but stay above average, with a similar pattern for the slighter, smaller and/or slimmer girls.

Thus the overall impression from the entire analysis is that even though there are differences in growth rates, and the variables in which this becomes manifest, there is little evidence for large scale qualitatively deviating differential growth patterns for the thirty girls. Thirty is, however, not a large sample to base general conclusions on about the deviations from the average growth patterns for all French girls. Some deviating individuals might be part of a larger group of girls with qualitatively different growth patterns, but from the present data alone this cannot be inferred.

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