



Universiteit
Leiden
The Netherlands

Radicals in arithmetic

Palenstijn, W.J.

Citation

Palenstijn, W. J. (2014, May 22). *Radicals in arithmetic*. Retrieved from <https://hdl.handle.net/1887/25833>

Version: Corrected Publisher's Version

License: [Licence agreement concerning inclusion of doctoral thesis in the Institutional Repository of the University of Leiden](#)

Downloaded from: <https://hdl.handle.net/1887/25833>

Note: To cite this publication please use the final published version (if applicable).

Cover Page



Universiteit Leiden



The handle <http://hdl.handle.net/1887/25833> holds various files of this Leiden University dissertation

Author: Palenstijn, Willem Jan

Title: Radicals in Arithmetic

Issue Date: 2014-05-22

Radicals in Arithmetic

Proefschrift

ter verkrijging van
de graad van Doctor aan de Universiteit Leiden,
op gezag van Rector Magnificus prof. mr. C.J.J.M. Stolker,
volgens besluit van het College voor Promoties
te verdedigen op donderdag 22 mei 2014
klokke 15:00 uur

door

Willem Jan Palenstijn

geboren te Leiden
in 1980

Promotiecommissie:

Promotor: Prof. dr. P. Stevenhagen

Copromotor: Dr. B. de Smit

Overige leden: Prof. dr. K.J. Batenburg
Prof. dr. H.W. Lenstra, Jr.
Dr. P. Moree
Prof. dr. R. Tijdeman
Prof. dr. J. Top

Radicals in Arithmetic

Copyright © Willem Jan Palenstijn, Leiden, 2014.

Printed by Ridderprint, Ridderkerk.

The research in this thesis has been financially supported by the Netherlands Organisation for Scientific Research (NWO), project 613.000.317, entitled Radicals in Arithmetic.

Contents

Preface	7
1 Primitive roots in number fields	11
1.1 Introduction	11
1.2 Entanglement	14
1.3 Computing the entanglement groups	19
1.4 Proof of main results	20
1.5 Explicit densities	23
2 Radical extensions of abelian groups	25
2.1 Introduction	25
2.2 Maximal radical extensions	26
2.3 Galois radical extensions	28
2.4 Kummer theory of radical extensions	31
2.5 Abelian radical extensions and Schinzel's theorem	36
2.6 The entanglement group	39
3 The absolute entanglement group	45
3.1 Introduction	45
3.2 Preliminaries	46
3.3 Main results	47
3.4 Positive characteristic	50
4 Computing radical field degrees	53
4.1 Introduction	53
4.2 Coprime bases	54
4.3 Entanglement	55
4.4 Field degrees	59
5 Near-primitive roots and higher rank	63
5.1 Introduction	63
5.2 Proof of main theorems	64
5.3 Explicit density computations	69

6	Artin for rank one tori	77
6.1	Introduction	77
6.2	Preliminaries	79
6.3	Proof of main theorems	81
6.4	Explicit density computations	83
7	Enumerating ABC triples	93
7.1	Introduction	93
7.2	Bounds	94
7.3	Enumeration algorithm	96
7.4	Implementation details	99
7.5	Data	101
	Bibliography	109
	Samenvatting	113
	Curriculum vitae	117