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Chapter 7

Determination of the model parameters

This chapter treats the determination of the model parameters on the basis of the measured input data obtained from the fundamental experiments. The determination procedure of the model parameters depends on the selected material model. As mentioned in chapter 5.2 in this thesis elasto-plastic material models, under the assumption of an isotropic hardening behavior, are applied. Thereby, the yield loci Hill '48, Barlat '89 and Barlat 2000 are investigated. The Hill '48 yield locus is obtained, if the exponent of the Barlat '89 yield locus is chosen to be equal to 2. For the investigation of the Bauschinger effect, the applied material model is also complemented by the Chaboche-Rousselier kinematic hardening model. However, this thesis is not focused on this effect and therefore a description of the calibration procedure of this model is omitted. Table 7.1 shows the parameters, which can be directly identified from the results of the fundamental experiments. This table is only valid for the considered material models.

Table 7.1: Directly identified model parameters.

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>Yield Locus</th>
<th>Hardening</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’ Modulus</td>
<td>-</td>
<td>Flow Curve</td>
</tr>
<tr>
<td>Poisson Ratio</td>
<td></td>
<td>Strain Rate Sensitivity m</td>
</tr>
</tbody>
</table>

As a consequence, only a calibration procedure for identifying the model parameters of the yield locus is needed. As mentioned in chapter 5.2, the model parameters are obtained by the minimization of the difference between the measured stress states and strain rate ratios (R values) of different experiments (fundamental experiments, chapter 6) and the predictions of the yield locus. First of all, the computation of the stress states and strain rate ratios based on the yield
locus are introduced.

7.1 Tensile test

As mentioned in chapter 6, the tensile test is characterized by an uniaxial stress state. The direction of the induced force $F$ defines the tensile axis of the experiment. $\alpha$ describes the angle between the tensile axis (coordinate system $\xi, \eta, \zeta$) and the axes of anisotropy (coordinate system $x, y, z$). The $x$-axis of the latter mentioned coordinate system corresponds to the rolling direction.

For the calibration of the yield locus, usually three types of specimens are investigated, differing regarding the angle (0°, 45° and 90°) between the tensile axis and the rolling direction. For each specimen the uniaxial stress and the $R$ values are determined. The application of the yield locus requires a stress tensor representation, whose basis is coincident with the axes of anisotropy (chapter 5.2).

The basis of the tensor $\sigma$ (7.1) is coincident with the coordinate system $\xi, \eta, \zeta$:

$$\sigma = \begin{pmatrix} \sigma_{\xi\xi} & \sigma_{\xi\eta} \\ \sigma_{\eta\xi} & \sigma_{\eta\eta} \end{pmatrix}, \quad \sigma_* = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}.$$  

(7.1)

The basis of the stress tensor $\sigma_*$ (7.1) is identical with the axes of anisotropy. The relation between both representations $\sigma, \sigma_*$ of the uniaxial stress state is given by (7.2):

$$\sigma = R(\alpha)^T \sigma_* R(\alpha), \quad R(\alpha) = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}.$$  

(7.2)

Thereby, $R(\alpha)$ is an orthogonal rotation tensor. Inserting $\sigma_*$ into (7.2) gives the equations [50]:

\[ \vspace{1cm} \]
7.1. TENSILE TEST

\[
\begin{align*}
\sigma_{\xi\xi} & = \sigma_{xx} \cos^2 \alpha + \sigma_{yy} \sin^2 \alpha + \sigma_{xy} \cos \alpha \sin \alpha, \\
\sigma_{\eta\eta} & = \sigma_{xx} \sin^2 \alpha + \sigma_{yy} \cos^2 \alpha - \sigma_{xy} \cos \alpha \sin \alpha, \\
\sigma_{\xi\eta} & = - (\sigma_{xx} - \sigma_{yy}) \cos \alpha \sin \alpha + \sigma_{xy} \left( \cos^2 \alpha - \sin^2 \alpha \right). 
\end{align*}
\] (7.3)

As the $\xi$-axis of the coordinate system $\xi, \eta, \zeta$ is coincident with the tensile axis and the stress state is uniaxial, the first eigenvector of $\sigma$ refers to the tensile axis. Therefore, $\sigma_{\xi\xi}$ is the principal value of the stress tensor and the remaining principal values are equal to 0.0. Consequently, a rearrangement of (7.3) in consideration of

\[
\sigma_{\text{ua}} = \sigma_{\xi\xi}, \quad \sigma_{\eta\eta} = 0, \quad \sigma_{\xi\eta} = 0
\] (7.4)

gives the stress tensor with respect to the axes of anisotropy

\[
\begin{align*}
\sigma_{xx} & = \sigma_{\text{ua}} \cos^2 \alpha, \\
\sigma_{yy} & = \sigma_{\text{ua}} \sin^2 \alpha, \\
\sigma_{xy} & = \sigma_{\text{ua}} \cos \alpha \sin \alpha. 
\end{align*}
\] (7.5)

Finally, the associated equivalent stress of the yield locus, depending on the measured stress state of the tensile test and the model parameters, is computed. If the model parameters of the yield locus are calibrated in the desired way, the deviation between the computed equivalent stress and the stress value of the flow curve, regarding the equivalent hardening state $Y(\dot{\epsilon}_{ref})$, is small. The quantification of this deviation is shown below. Expression (7.6) introduces an abbreviation for the equivalent stress under an uniaxial stress state:

\[
\bar{\sigma}_{\text{ua}} (P) = \sigma (\sigma_{\text{ua}}, \alpha), P).
\] (7.6)

Thereby, $\bar{\sigma}$ is an equivalent stress, obtained from an arbitrary yield locus. The vector $P$ comprises the parameters of the yield locus.

Apart from the equivalent stress, also the computation of the $R_\alpha$ value (7.7) [2] on the basis of the yield locus is introduced:

\[
R_\alpha = \frac{\dot{\epsilon}_{\eta\eta}}{\dot{\epsilon}_{\xi\xi}}.
\] (7.7)

Figure 7.1 also illustrates the strain state of the tensile test in the $\xi, \eta$ plane. Figure 7.2 complements the visualization of the strain state of 7.1 by the thickness direction.
For the computation of $R_\alpha$ the relation between the strain rate tensor $\dot{\epsilon}$ and $\dot{\epsilon}_*$

$$\dot{\epsilon} = \begin{pmatrix} \epsilon_{\xi\xi} & \epsilon_{\eta\xi} \\ \epsilon_{\eta\xi} & \epsilon_{\eta\eta} \end{pmatrix}, \quad \dot{\epsilon}_* = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} \\ \epsilon_{yx} & \epsilon_{yy} \end{pmatrix},$$

(7.8)

is needed

$$\dot{\epsilon} = R(\alpha)^T \dot{\epsilon}_* R(\alpha), \quad R(\alpha) = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}. \quad (7.9)$$

The basis of the strain rate tensor $\dot{\epsilon}_*$ is the axes of anisotropy. The strain tensor $\dot{\epsilon}$ refers to the coordinate system $\xi, \eta, \zeta$. Inserting $\dot{\epsilon}_*$ into 7.9 and applying $R(\alpha)$ leads to

$$\dot{\epsilon}_{\xi\xi} = \dot{\epsilon}_{xx} \cos^2 \alpha + \dot{\epsilon}_{yy} \sin^2 \alpha + \dot{\epsilon}_{xy} 2 \cos \alpha \sin \alpha,$$

$$\dot{\epsilon}_{\eta\eta} = \dot{\epsilon}_{xx} \sin^2 \alpha + \dot{\epsilon}_{yy} \cos^2 \alpha - \dot{\epsilon}_{xy} 2 \cos \alpha \sin \alpha,$$

$$\dot{\epsilon}_{\xi\eta} = - (\dot{\epsilon}_{xx} - \dot{\epsilon}_{yy}) \cos \alpha \sin \alpha + \dot{\epsilon}_{xy} (\cos^2 \alpha - \sin^2 \alpha).$$

(7.10)

The strain rate $\dot{\epsilon}_{\zeta\zeta}$ is not affected by the transformation of the basis of the strain rate tensor as the rotation is performed in the $\xi, \eta$ plane. Provided the plastic incompressibility can be assumed, the expressions

$$\dot{\epsilon}_{\xi\xi} + \dot{\epsilon}_{\eta\eta} + \dot{\epsilon}_{\zeta\zeta} = 0$$

(7.11)

and

$$\dot{\epsilon}_{xx} + \dot{\epsilon}_{yy} + \dot{\epsilon}_{zz} = 0$$

(7.12)

hold. A rearrangement of 7.11, 7.12 and the equality $\dot{\epsilon}_{\zeta\zeta} = \dot{\epsilon}_{zz}$ leads to
\[
\dot{\epsilon}_{\xi\zeta} = \dot{\epsilon}_{zz} = - (\dot{\epsilon}_{\xi\xi} + \dot{\epsilon}_{\eta\eta}) = - (\dot{\epsilon}_{xx} + \dot{\epsilon}_{yy}).
\]

(7.13)

Consequently, it is possible to formulate \( R_\alpha \) depending on the strain rate tensor components of \( \dot{\epsilon}_* \):

\[
R_\alpha = - \frac{\dot{\epsilon}_{xx} \sin^2 \alpha + \dot{\epsilon}_{yy} \cos^2 \alpha - 2 \cos \alpha \sin \alpha \dot{\epsilon}_{xy}}{\dot{\epsilon}_{xx} + \dot{\epsilon}_{yy}}.
\]

(7.14)

If the introduced relation between the strain rate tensor and the stress tensor is applied (Expression (5.64)),

\[
\dot{\epsilon}_* = \lambda \frac{\partial f}{\partial \sigma}, \quad \dot{\epsilon}_{xx} = \lambda \frac{\partial f}{\partial \sigma_{xx}}, \quad \dot{\epsilon}_{yy} = \lambda \frac{\partial f}{\partial \sigma_{yy}}, \quad \dot{\epsilon}_{xy} = \lambda \frac{\partial f}{\partial \sigma_{xy}},
\]

(7.15)

\( R_\alpha \) can be expressed on the basis of the partial derivatives of the yield locus with respect to the stress tensor \( \sigma_* \) [4]:

\[
R_\alpha (\sigma_* (\sigma_\alpha, \alpha), P) = - \frac{\sin^2 \alpha \frac{\partial f}{\partial \sigma_{xx}} + \cos^2 \alpha \frac{\partial f}{\partial \sigma_{yy}} - \sin 2\alpha \frac{\partial f}{\partial \sigma_{xy}}}{\frac{\partial f}{\partial \sigma_{xx}} + \frac{\partial f}{\partial \sigma_{xy}}} \bigg|_{\sigma_* (\sigma_\alpha, \alpha), P}.
\]

(7.16)

Expression (7.17) shows the application of a finite difference scheme for the numerical determination of the partial derivatives:

\[
\begin{align*}
\frac{\partial f}{\partial \sigma_{xx}} & \approx \frac{f (\sigma_{xx} + \Delta \sigma, \sigma_{yy}, \sigma_{xy}) - f (\sigma_{xx} - \Delta \sigma, \sigma_{yy}, \sigma_{xy})}{2 \Delta \sigma}, \\
\frac{\partial f}{\partial \sigma_{yy}} & \approx \frac{f (\sigma_{xx}, \sigma_{yy} + \Delta \sigma, \sigma_{xy}) - f (\sigma_{xx}, \sigma_{yy} - \Delta \sigma, \sigma_{xy})}{2 \Delta \sigma}, \\
\frac{\partial f}{\partial \sigma_{xy}} & \approx \frac{1}{2} \frac{f (\sigma_{xx}, \sigma_{yy}, \sigma_{xy} + \Delta \sigma) - f (\sigma_{xx}, \sigma_{yy}, \sigma_{xy} - \Delta \sigma)}{2 \Delta \sigma}.
\end{align*}
\]

(7.17)

The partial derivative with respect to \( \sigma_{xy} \) implies the factor 0.5, as the yield locus \( f \) includes \( \sigma_{xy} \) and \( \sigma_{yx} \) [63]. For the comparison of the experimentally determined value and the computed \( R_\alpha \) value the following abbreviation is introduced

\[
R_\alpha (P) = R_\alpha (\sigma_* (\sigma_{xx}, \alpha), P).
\]

(7.18)

7.2 Bulge test

As mentioned in chapter 6, at the apex of the bulge specimen an equibiaxial stress state is generated. According to
it is possible to extract the components $\sigma_b$ from the stress tensor. As a result, the stress state is equivalently expressed by a multiplication of the scalar value $\sigma_b$ and the identity tensor. As any vector remains unchanged by a multiplication with the identity tensor, every non-zero vector is an eigenvector of the identity matrix with eigenvalue of 1. Consequently, a transformation of the basis of the biaxial stress state does not affect the components of the stress tensor. Therefore, the biaxial stress state is directly inserted in the yield locus for the computation of the equivalent stress. Again, for the comparison of the obtained equivalent stress with the stress value of the flow curve, an abbreviation is introduced

$$\sigma_b(P) = \sigma_\ast(\sigma_b, P).$$  

(7.20)

The measurement of the strain rate ratio

$$R_b = \frac{\dot{\varepsilon}_{yy}}{\dot{\varepsilon}_{xx}}$$  

(7.21)

is an additional quantity for the calibration of the yield locus. Inserting (7.15) into (7.21) gives the $R_b$ value in dependency of the partial derivatives of the yield locus with respect to the stress state $\sigma_\ast$:

$$R_b(\sigma_\ast(\sigma_b), P) = \frac{\partial f}{\partial \sigma_{yy}} \bigg|_{\sigma_\ast(\sigma_b), P}. \quad (7.22)$$

As opposed to the $R_\alpha$ value, the $R_b$ value is defined for the strain state whose basis is identical with the axes of anisotropy. Expression (7.23) shows an abbreviation of the computed $R_b$ value:

$$R_b(P) = R_b(\sigma_\ast(\sigma_b), P).$$  

(7.23)

### 7.3 Shear test

Depending on the angle $\alpha$ of the shear axes with respect to the axes of anisotropy, a transformation of the basis of the stress tensor $\sigma$

$$\sigma = \begin{pmatrix} \sigma_{\xi\xi} & \sigma_{\xi\eta} \\ \sigma_{\eta\xi} & \sigma_{\eta\eta} \end{pmatrix} = \begin{pmatrix} \sigma_b & 0 \\ 0 & \sigma_b \end{pmatrix} = \sigma_b \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$  

(7.24)

has to be performed (figure 7.3). In consideration of an equivalent hardening state, the equivalent stress state is computed on the basis of the stress tensor, which is related to the axes of anisotropy. Expression (7.25) illustrates an abbreviation of the equivalent stress:
7.4 Calibration of the Barlat 2000 yield locus

Usually, the flow curve for modeling the isotropic hardening is derived from a tensile test whose tensile axis is coincident with the rolling direction. The Barlat 2000 yield locus does not necessarily reproduce the flow curve exactly for this experiment. For example, if a calibration of the yield locus is performed based on an optimization by equally weighting the deviations between all the measured and predicted quantities, a difference between $\sigma_{B2000}\left(\sigma^*, \alpha, a\right)$ and $Y(\epsilon_{\text{ref}})$ is expected. Provided the hardening of the material can be assumed to be isotropic and an exact reproduction of the flow curve is desired, a scaling of the yield locus is applied:

$$f(\sigma_s) = \gamma\sigma_{B2000}\left(\sigma^*, \alpha, a\right) - Y(\epsilon_{\text{ref}}) = 0.$$  \hspace{1cm} (7.26)

The scaling factor $\gamma$ is derived from equation

$$\gamma\sigma_{B2000}\left(\sigma^*, \alpha, a\right) = Y(\epsilon_{\text{ref}}).$$  \hspace{1cm} (7.27)

The vector $\alpha$ contains the model parameters $\alpha_i$ of the Barlat 2000 yield locus. Subsequently, a multiplication of the model parameters $\alpha_i$ by the factor $\beta$ is investigated. By inserting $\beta\alpha$ into the Barlat 2000 yield locus, 7.28 can be derived:

$$\sigma_{B2000}\left(\sigma^*, \beta\alpha, a\right) = \beta\sigma_{B2000}\left(\sigma^*, \alpha, a\right).$$  \hspace{1cm} (7.28)

**Figure 7.3:** Shear test; Stress state in the $\xi, \eta$-plane.
Consequently, the multiplication of the vector $\alpha$ by the scalar $\beta$ is equal to a scaling of the yield yield locus. Therefore, provided the yield locus is scaled according to (7.26), its shape is not affected by the factor $\beta$. As a result, one of the $\alpha_i$ parameters can be prescribed without loss of generality. As a consequence, the space of the model parameter is reduced by one dimension.

7.4.1 Objective function
Firstly, a multiplicative aggregation on the basis of Harrington desirability functions is shown. Secondly, the objectives are additively combined to a single scalar objective value [4]. For a compact presentation of the objective functions, the sets $A$ and $B$ are introduced

$$
A = \{ u0^\circ, u45^\circ, u90^\circ, b, s \},
B = \{ 0^\circ, 45^\circ, 90^\circ, b \},
$$

and the domain of the parameter space $\alpha_i$ is given by

$$
\alpha_i \in [L; U].
$$

7.4.2 Harrington desirability functions
By the application of the two-sided Harrington desirability function, the results of the equivalent stresses $\sigma_i^{B2000}$ ($I \in A$) and the R values $R_j^{B2000}$ ($J \in B$) are mapped onto an interval $[0, 1]$. For the formulation of the objective function, the results are multiplicatively aggregated as given by

$$
d = \prod_{I \in A} d_2(\sigma_i^{B2000}, U_I, L_I, n_I) \prod_{J \in B} d_2(R_j^{B2000}, U_J, L_J, n_J).
$$

The choice of the Parameters $U_I, L_I, n_I, U_J, L_J$ and $n_J$ is discussed below. The value of the optimum is a priori known, as the target of the calibration procedure is the minimization of the deviation between predicted and measured stress states and strain rate ratios. In this case, the Harrington desirability functions are especially suitable, as only the location of the optimum in the model parameter space is unknown.

The domain of the model parameters $\alpha_i$ (7.30) can be considered by applying the one-sided Harrington desirability functions. Originating from the limits of the domain, the parameters of the desirability functions are derived

$$
L \rightarrow b_{0L}, b_{1L}, U \rightarrow b_{0U}, b_{1U}.
$$

The parameter $b_1$ defines the slope within the gray zone between the states 0.0 and 1.0. Depending on the choice of $b_1$ the parameter $b_0$ determines the position
of the gray zone. As the domain of the model parameter space is a hypercube, for each dimension of the model parameter space two one-sided Harrington desirability functions have to be defined.

Expression (7.33) gives the desirability functions for the domain of the model parameter space:

\[ g = \prod_{i=1}^{7} d_1 (\alpha_i, b_{0,i}, b_{1,i}) \prod_{i=1}^{7} d_1 (\alpha_i, b_{0,i}, b_{1,i}). \] (7.33)

The objective function is given by

\[ F = 1 - dg. \] (7.34)

Subsequently, the term MAHDF (Multiplicative Aggregation based on Harrington Desirability Functions) is applied in order to refer to the presented aggregation of the objectives and the treatment of the constraints.

### 7.4.3 Additive aggregation of the objectives

Subsequently, another formulation of the objective function is discussed, without applying the Harrington desirability functions. Expression (7.35) shows an additive aggregation of the objectives to a single scalar value \( f \):

\[ f = \sum_{i \in A} \gamma_i \left( \frac{\sigma_{B2000} - Y(\epsilon_{ref})}{Y(\epsilon_{ref})} \right)^2 + \sum_{j \in B} \gamma_j \left( \frac{R_{B2000}^J - R_{ed}^J}{R_{ed}^J} \right)^2. \] (7.35)

Thereby, each measured quantity is compared with the prediction of the model and the result is normalized by the measured quantity. Finally, the normalized result is squared in order to penalize larger deviations disproportionate (ed: experimentally determined). In this case, the boundary condition with respect to the \( \alpha_i \) values is considered by a penalty function as given by

\[ g_U (x) = \begin{cases} 
(1 + |x - U|^n_k U) & \text{if } x > U \\
1 & \text{if } x \leq U
\end{cases}, \]

\[ g_L (x) = \begin{cases} 
(1 + |x - L|^n_k L) & \text{if } x < L \\
1 & \text{if } x \geq L
\end{cases}. \] (7.36)

Thereby, for both, the lower \( g_L \) and the upper bound \( g_U \), a penalty function is defined. The penalty functions (7.36) are multiplicatively aggregated as given by expression

\[ g = \prod_{i=1}^{7} g_U (\alpha_i) g_L (\alpha_i). \] (7.37)
Finally, the additively aggregated objectives and the penalty functions are combined multiplicatively to a single scalar objective value

$$F = fg.$$ (7.38)

For this type of aggregation the term AAOMAC (Additive Aggregation of the Objectives and Multiplicative Aggregation of the Constraints) is introduced.

### 7.4.4 Termination of the optimization

The termination criterion for the application of the evolutionary strategies is twofold. On the one hand, the maximum number of objective function evaluations is limited by the parameter $n_{\text{max}}$. On the other hand, a criterion based on the model parameter space is applied. For the latter mentioned criterion, the difference between the $\alpha_i$ values of current and the previous fitness function evaluation is computed for each dimension

$$\Delta \alpha_i^{n+1} = |\alpha_i^{n+1} - \alpha_i^n|.$$ (7.39)

The obtained value $\Delta \alpha_i^{n+1}$ is accumulated according to

$$\Delta \beta_i^{n+1} = \Delta \beta_i^n (1 - c) + \Delta \alpha_i^{n+1} c.$$ (7.40)

The accumulation concept is taken from [33]. The significance, defined by $c$, of the previous $\Delta \alpha_i^{(n+1)}$ values decreases exponentially. If the accumulated value $\Delta \beta_i^{n+1}$ of all $i$ is smaller than the threshold $\epsilon$, the optimization is aborted:

$$\left( \forall i \in \{1, \ldots, 7\} : \Delta \beta_i^{(n+1)} < \epsilon \right) \lor n > n_{\text{max}}.$$ (7.41)

In this thesis, $c$ is chosen to be equal to 0.8 and for $\epsilon$ a value of 0.0001 is applied.

### 7.4.5 Weighting of the objectives

Basically, the yield locus should give a good prediction of the stress states and the strain rate ratios of the fundamental experiments. However, over-fitting effects have to be definitely avoided. Oscillations of the yield locus shape could lead to a non-physical response of the material model, as the Clausius-Duhem inequality might be violated (chapter 5.2). Considering the Barlat 2000 yield locus, the anisotropy of the sheet metal is introduced by two linear transformations, which avoids oscillations and the yield locus is even convex. However, depending on the material, the Barlat 2000 yield locus might not be able to predict all the measured results of the fundamental experiments in the same quality. Therefore, the calibration procedure has to deal with the definition of compromises. For a high quality of the prediction of some stress states and strain rate ratios, a worse representation of the remaining states has to be accepted. In this chapter, the
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underlying prioritization is performed based on experience. The lowest priority for the calibration of the investigated DX54 steel grade is given to the shear stress. All the subsequent analyses are performed on the basis of this steel grade.

The expressions (7.42) give the relation between the parameters of the desirability functions and the constants, defined in table 7.2:

\[
U_I = Y(\epsilon_{ref}) + Y(\epsilon_{ref}) c_I, \quad I \in A, \\
L_I = Y(\epsilon_{ref}) - Y(\epsilon_{ref}) c_I, \\
U_J = R_{J}^{ed} + R_{J}^{ed} c_J, \quad J \in B, \\
L_J = R_{J}^{ed} - R_{J}^{ed} c_J.
\]

The priority of a stress state or a strain rate ratio can be increased by choosing a lower value of the exponent \( n \). Also the parameters \( U \) and \( L \) can be modified in order to change the priority of an objective. For the subsequent investigations, only the exponent of the shear stress is increased in order to reduce its priority in comparison to the other objectives.

Table 7.2: Weights of the multiplicative aggregation.

<table>
<thead>
<tr>
<th>( c_I )</th>
<th>( c_J )</th>
<th>( n_{u0^\circ} = n_{u45^\circ} = n_{u90^\circ} = n_b = 3; ) ( n_s = 10 )</th>
<th>( I \in A, ) ( J \in B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>( I \in A, ) ( J \in B )</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.3 summarizes the weights, applied for the additive aggregation of the objectives. Again, the priority of the objective related to the shear stress is reduced. In this case, the weight \( \gamma_s \) is decreased. The other objectives are equally weighted. Furthermore, the choice of the parameters regarding the penalty function are given by table 7.3.

Table 7.3: Weights of the additive aggregation and the parameters of the penalty functions.

<table>
<thead>
<tr>
<th>( \gamma_{u0^\circ} = \gamma_{u45^\circ} = \gamma_{u90^\circ} = \gamma_b = 1; ) ( \gamma_s = 0.01 )</th>
<th>( J \in B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_{u0^\circ} = \gamma_{u45^\circ} = \gamma_{u90^\circ} = \gamma_b = 1; ) ( \gamma_s = 0.01 )</td>
<td>( J \in B )</td>
</tr>
<tr>
<td>( n_U = n_L = 5 )</td>
<td>( J \in B )</td>
</tr>
<tr>
<td>( k_U = k_L = 5 )</td>
<td>( J \in B )</td>
</tr>
</tbody>
</table>
7.4.6 Investigation of the calibration procedure

In this section, the calibration of the Barlat 2000 yield locus on the basis of different optimization algorithms ((µ/µ, λ)-CMA-ES, (1 + 1)-CMA-ES, (1, λ)-DR-ES and SQP) is investigated. Additionally, the introduced procedures for aggregating the objectives are compared. In this section, the exponent of the Barlat 2000 yield locus is chosen to be equal to 5.0. It is assumed that the subsequently presented results are invariant with respect to the choice of this exponent.

Generally, calibration problems can be multimodal [13]. Therefore, a global optimization algorithm could be advantageous for the calibration of the yield locus. For the subsequent investigation, the initial values of the parameters $\alpha_i$ are chosen to be equal to 1, as recommended in [3]. The domain of the parameter space is given by

$$\alpha_i \in [0; 2].$$

(7.43)

Firstly, the results of a yield locus calibration resulting from a standard $(\mu/\mu, \lambda)$-CMA-ES and an additive aggregation (AAOMAC) are presented. The performance of evolutionary strategies can differ between optimizations of the same problem, as these algorithms are based on statistical methods. As a consequence, the calibrations of the yield locus are repeated 100 times in order to evaluate the performance of the optimization. Figure 7.4 (left) shows the number of objective function evaluations, needed for finding an optimum for each optimization. The $(\mu/\mu, \lambda)$-CMA-ES algorithm in combination with the mentioned aggregation of the objectives is able to find the optimum within 1500 and 3000 computations of the objective function value. The mean value of the needed objective function evaluations is 2398.

For the analysis of the progression of the objective function value, optimization 21 is chosen (figure 7.4 (right)), which represents the mean in terms of the needed objective function values for finding the optimum. Figure 7.4 (right) is based on a logarithmic scale. This figure confirms the above suggested stopping criterion.

Figure 7.5 shows the results of optimizations, performed under the application of a $(\mu/\mu, \lambda)$-CMA-ES in combination with a multiplicative aggregation (MAHDF). The results are evaluated in the same way as described above. The analysis of the results show that the MAHDF is advantageous regarding the convergence.

Apart from the $(\mu/\mu, \lambda)$-CMA-ES also a $(1 + 1)$-CMA-ES is analyzed. Igel [20] reported a better performance of the $(1 + 1)$-CMA-ES in the case of a unimodal optimization problem. According to figure 7.6 (left), the performance of the $(1 + 1)$-CMA-ES is better than the standard $(\mu/\mu, \lambda)$-CMA-ES. This result indicates a possible unimodality of the Barlat 2000 yield locus calibration problem.

By applying the MAHDF approach in combination with the CMA-ES-(1+1) the performance of the optimization improves (figure 7.7 (left)).
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Figure 7.4: This investigation is based on the \((\mu/\mu, \lambda)\)-CMA-ES algorithm and the AAOMAC is applied; Left: Analysis of the number of the needed objective function evaluations for finding the optimum; Right: Propagation of a single optimization (Optimization 21).

Figure 7.5: This investigation is based on the \((\mu/\mu, \lambda)\)-CMA-ES algorithm and the MAHDF is applied; Left: Analysis of the number of the needed objective function evaluations for finding the optimum; Right: Propagation of a single optimization (Optimization 27).
Figure 7.6: This investigation is based on the (1 + 1)-CMA-ES algorithm and the AAOMAC is applied; Left: Analysis of the number of the needed objective function evaluations for finding the optimum; Right: Propagation of a single optimization (Optimization 30).

Figure 7.7: This investigation is based on the (1 + 1)-CMA-ES algorithm and the MAHDF is applied; Left: Analysis of the number of the needed objective function evaluations for finding the optimum; Right: Propagation of a single optimization (Optimization 93).
7.4. CALIBRATION OF THE BARLAT 2000 YIELD LOCUS

Figure 7.8: This investigation is based on the \((1, \lambda)\)-DR-ES algorithm and the AAOMAC is applied; Left: Analysis of the number of the needed objective function evaluations for finding the optimum; Right: Propagation of a single optimization (Optimization 4).

The \((1, \lambda)\)-DR-ES shows a worse performance as the \((\mu/\mu, \lambda)\)-CMA-ES algorithms (figures 7.8, 7.9) and in this case, the AAOMAC is advantageous in comparison with the MAHDF.

Figure 7.9: This investigation is based on the \((1, \lambda)\)-DR-ES algorithm and the MAHDF is applied; Left: Analysis of the number of the needed objective function evaluations for finding the optimum; Right: Propagation of a single optimization (Optimization 79).

Finally, the SQP algorithm in combination with the AAOMAC shows the best performance (figure 7.10). This result is again an indication that the investigated problem could be, at least within the investigated domain, unimodal. Both optimizations based on the SQP algorithm are stopped, when the same value of the objective function is reached as obtained by applying the \((\mu/\mu, \lambda)\)-CMA-ES.
Figure 7.10: This investigation is based on the SQP algorithm; Left: Propagation of the optimization using the AAOMAC; Right: Propagation of the optimization using the MAHDF.

Table 7.4: Calibration of the Barlat 2000 yield locus using the AAOMAC.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>ED</th>
<th>$\frac{\mu}{\mu', \lambda}$</th>
<th>(1 + 1)</th>
<th>(1, $\lambda$)</th>
<th>SQP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{N}^{0}$</td>
<td>1.000000</td>
<td>1.000000</td>
<td>1.000000</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>$\sigma_{N}^{M}$</td>
<td>1.032511</td>
<td>1.000002</td>
<td>0.999951</td>
<td>0.998167</td>
<td>0.999970</td>
</tr>
<tr>
<td>$\sigma_{N}^{B}$</td>
<td>0.996696</td>
<td>1.000506</td>
<td>1.000513</td>
<td>1.025667</td>
<td>1.000575</td>
</tr>
<tr>
<td>$\sigma_{N}^{b}$</td>
<td>1.164572</td>
<td>1.164636</td>
<td>1.164670</td>
<td>1.165610</td>
<td>1.164549</td>
</tr>
<tr>
<td>$\sigma_{N}^{s}$</td>
<td>0.655851</td>
<td>0.532718</td>
<td>0.532719</td>
<td>0.538683</td>
<td>0.532729</td>
</tr>
<tr>
<td>$R_{Q}^{N}$</td>
<td>1.000000</td>
<td>1.000000</td>
<td>1.000000</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>$R_{Q}^{B}$</td>
<td>0.780172</td>
<td>0.780297</td>
<td>0.780203</td>
<td>0.777518</td>
<td>0.780189</td>
</tr>
<tr>
<td>$R_{Q}^{b}$</td>
<td>1.271552</td>
<td>1.271703</td>
<td>1.271600</td>
<td>1.263591</td>
<td>1.271696</td>
</tr>
<tr>
<td>$R_{Q}^{s}$</td>
<td>0.431034</td>
<td>0.431082</td>
<td>0.431052</td>
<td>0.428901</td>
<td>0.431043</td>
</tr>
<tr>
<td>$F$</td>
<td>9.98E-05</td>
<td>9.98E-05</td>
<td>7.29E-05</td>
<td>7.29E-05</td>
<td>1.00E-04</td>
</tr>
</tbody>
</table>

The tables 7.4 and 7.5 summarize the predictions of the Barlat 2000 yield locus depending on the used optimization algorithm for the calibration of the model parameters (ED: experimentally determined). The depicted stress states are normalized with respect to the stress state of the tensile test, whose tensile axis is coincident with the rolling direction (N: Normalized):

$$\sigma_{I}^{N} = \frac{\sigma_{I}}{\sigma_{U0}^{0}}, \; I \in A. \quad (7.44)$$

Also the strain rate ratios are transformed as given by

$$R_{J}^{N} = \frac{R_{J}}{R_{0}}, \; J \in B. \quad (7.45)$$
The algorithms \((\mu/\mu, \lambda)\)-CMA-ES, \((1 + 1)\)-CMA-ES and SQP lead to similar results independent of the applied method for the aggregation of the objectives. The \((1, \lambda)\)-DR-ES algorithm does not reach the same level regarding the quality of the calibration. Neither the AAOMAC nor the MAHDF approach generally leads to a better performance of the optimization. For the industrial application, the SQP algorithm combined with the AAOMAC seems to be a good choice.

Table 7.5: Calibration of the Barlat 2000 yield locus using the MAHDF.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>((\mu/\mu, \lambda))-CMA-ES</th>
<th>((1 + 1))-CMA-ES</th>
<th>((1, \lambda))-DR-ES</th>
<th>SQP</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_{N0})</td>
<td>1.000000</td>
<td>1.000000</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>(\sigma_{N45})</td>
<td>1.003333</td>
<td>1.003666</td>
<td>0.995789</td>
<td>1.000884</td>
</tr>
<tr>
<td>(\sigma_{N90})</td>
<td>1.001217</td>
<td>1.001835</td>
<td>0.986455</td>
<td>1.003796</td>
</tr>
<tr>
<td>(\sigma_N^b)</td>
<td>1.161177</td>
<td>1.161204</td>
<td>1.161819</td>
<td>1.160623</td>
</tr>
<tr>
<td>(R_N^c)</td>
<td>0.532888</td>
<td>0.532892</td>
<td>0.529737</td>
<td>0.533360</td>
</tr>
<tr>
<td>(R_N^c)</td>
<td>1.000000</td>
<td>1.000000</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>(R_N^c)</td>
<td>0.777034</td>
<td>0.777314</td>
<td>0.792740</td>
<td>0.774955</td>
</tr>
<tr>
<td>(R_N^c)</td>
<td>1.261597</td>
<td>1.261531</td>
<td>1.307770</td>
<td>1.258105</td>
</tr>
<tr>
<td>(R_N^b)</td>
<td>0.427429</td>
<td>0.427425</td>
<td>0.442515</td>
<td>0.429197</td>
</tr>
<tr>
<td>(F)</td>
<td>9.16E-07</td>
<td>9.16E-07</td>
<td>7.75E-06</td>
<td>1.00E-06</td>
</tr>
</tbody>
</table>

7.5 Calibration of the Barlat ‘89 yield locus

For the identification of the parameters \(a, c, h\) and \(p\) of the Barlat ‘89 yield locus, the same procedure based on the minimization of the introduced objective functions could be applied. However, another approach exists for the determination of the Barlat ‘89 yield locus parameter, which has been suggested by Barlat and is shown below [2].

Provided the flow curve is derived from a tensile test, whose tensile axis is coincident with the rolling direction, the parameters \(a\) and \(c\) are coupled:

\[
a = 2 - c.
\]

This relation is derived by inserting this stress state \(\sigma_{xx} = Y(\epsilon_{ref}) ; \sigma_{yy} = 0; \sigma_{xy} = 0\) into the Barlat ‘89 yield locus. Expression

\[
\sigma_{yy}Y = Y(\epsilon_{ref})
\]

is derived by inserting the stress state of an tensile test, whose tensile axis is perpendicular to the rolling direction \(\sigma_{xx} = 0 \; \sigma_{xy} = 0\). The formulation of the
CHAPTER 7. DETERMINATION OF THE MODEL PARAMETERS

Barlat ’89 yield locus enables the analytical determination of $R_\alpha$, as an analytical solution of the derivative of $f$ with respect to $\sigma_\alpha$ exists [2]:

$$
\dot{\epsilon}_{xx} = \dot{\lambda} \frac{\partial f}{\partial \sigma_{xx}} = \dot{\lambda} m \left\{ a (K_1 - K_2) |K_1 - K_2|^{m-2} \left( \frac{1}{2} - \frac{\sigma_{xx} - h \sigma_{yy}}{4K_2} \right) + a (K_1 + K_2) |K_1 + K_2|^{m-2} \left( \frac{1}{2} + \frac{\sigma_{xx} - h \sigma_{yy}}{4K_2} \right) + 2^m c K_2^{m-1} \frac{\sigma_{xx} - h \sigma_{yy}}{4K_2} \right\},
$$

(7.48)

$$
\dot{\epsilon}_{yy} = \dot{\lambda} \frac{\partial f}{\partial \sigma_{yy}} = \dot{\lambda} m \left\{ a (K_1 - K_2) |K_1 - K_2|^{m-2} \left( \frac{1}{2} + \frac{h \sigma_{xx} - h \sigma_{yy}}{4K_2} \right) + a (K_1 + K_2) |K_1 + K_2|^{m-2} \left( \frac{1}{2} - \frac{h \sigma_{xx} - h \sigma_{yy}}{4K_2} \right) - 2^m c K_2^{m-1} \frac{h \sigma_{xx} - h \sigma_{yy}}{4K_2} \right\},
$$

(7.49)

$$
\dot{\epsilon}_{xy} = \dot{\lambda} \frac{\partial f}{\partial \sigma_{xy}} = \dot{\lambda} m \left\{ a (K_1 + K_2) |K_1 + K_2|^{m-2} - a (K_1 - K_2) |K_1 - K_2|^{m-2} + 2^m c K_2^{m-1} \right\} \frac{\sigma_{xy}}{2K_2}.
$$

(7.50)

For this yield locus Barlat derived an alternative expression for the computation of $R_\alpha$:

$$
R_\alpha (a, c, h, p) = \frac{2mY (\epsilon_{ref})^m}{\left( \frac{\partial f}{\partial \sigma_{xx}} + \frac{\partial f}{\partial \sigma_{yy}} \right) \sigma_{xx}} - 1.
$$

(7.51)

If $R_{00}^\circ$ and $R_{90}^\circ$ are represented under the application of (7.48),(7.49) and (7.51), two expressions are obtained depending on $c$ and $h$. Both expressions are independent of $p$ as the shear stress $\sigma_{xy}$ vanishes by transforming the related uniaxial stress tensors of $R_{00}^\circ$ and $R_{90}^\circ$ to the axes of anisotropy. A rearrangement of the obtained relations leads to the expressions

$$
a = 2 - c = 2 - 2 \sqrt{\frac{R_{00}^\circ - R_{90}^\circ}{1 + R_{00}^\circ - 1 + R_{90}^\circ}}.
$$

(7.52)
7.6. SUMMARY

and

\[ h = \sqrt{\frac{R_{0^\circ}}{1 + R_{90^\circ}}} \frac{1 + R_{90^\circ}}{R_{90^\circ}}. \]  

Finally, \( p \) can be numerically determined by equating the experimentally determined \( R_{45^\circ}^{cd} \) value with the computed \( R_{45^\circ}^{BS}(a, c, h, p) \) value according to

\[ R_{45^\circ}^{BS}(a, c, h, p) = R_{45^\circ}^{cd}. \]  

7.6 Summary

In this chapter, calibrations of the yield loci are introduced, which are applied in the subsequent sections. Within the domain of the investigated parameter space, the calibration of the Barlat 2000 yield locus seems to be an unimodal problem. Therefore, a SQP algorithm in combination with an additive aggregation of the deviations between the predictions of the model and the measured quantities is recommended for this task.