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## **Better predictions when models are wrong or underspecified**

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# Better Predictions when Models are Wrong or Underspecified

Thijs van Ommen



# Better Predictions when Models are Wrong or Underspecified

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- Chapter 8 is based on work that is not currently available elsewhere.





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