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Inverse problems for universal deformation rings of group representations

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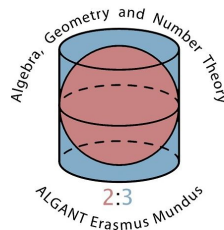
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Basic notation and conventions

All considered commutative rings are assumed to have an identity element. The morphisms between them are required to preserve the identity elements.

Given a positive integer n , we will denote by $[n]$ the set $\{1, 2, \dots, n\}$.

For a positive integer n and an abelian group A the additive group of $n \times n$ matrices over A will be denoted by $M_n(A)$. We adopt the convention of writing $M(i, j)$ for the (i, j) -entry of $M \in M_n(A)$.

We reserve the symbol I_n for the $n \times n$ identity matrix. Given $i, j \in [n]$ we denote by e_{ij} the $n \times n$ matrix having only one non-zero entry, which is 1 at the (i, j) -th place.

Let F, G be functors between categories $\mathcal{D}$ and $\mathcal{E}$. If $\Phi : F \rightarrow G$ is a natural transformation then for $A \in \text{Ob}(\mathcal{D})$ we denote the induced map $F(A) \rightarrow G(A)$ by Φ_A . Moreover, in case $\mathcal{E} = \text{Sets}$ we say that Φ is injective (surjective) if and only if Φ_A is injective (surjective) for every $A \in \text{Ob}(\mathcal{D})$.

If G, H are topological groups then $\text{CHom}(G, H) := \{f \in \text{Hom}(G, H) \mid f \text{ continuous}\}$.

