

The effects of presenting multidigit mathematics problems in a realistic context on sixth graders' problem solving

Journal:	<i>Cognition and Instruction</i>
Manuscript ID:	HCGI-2012-0048
Manuscript Type:	Original Article
Keywords:	Assessment and Evaluation, Elementary math < Math Education, Quantitative Methodology, Cognitive Modeling

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SOLVING MULTIDIGIT MATHEMATICS PROBLEMS

The effects of presenting multidigit mathematics problems in a realistic context on sixth graders' problem solving

Date of submission 19-7-2012

*SOLVING MULTIDIGIT MATHEMATICS PROBLEMS***The effects of presenting multidigit mathematics problems in a realistic context on sixth graders' problem solving**

Mathematics education and assessments increasingly involve arithmetic problems presented in context: a realistic situation that requires mathematical modeling. The present study's aim was to assess the effects of such typical school mathematics contexts on two aspects of problem solving, performance and strategy use, with a special focus on the role of students' language level. 685 sixth graders from the Netherlands solved a set of multidigit arithmetic problems on addition, subtraction, multiplication, and division. These problems were presented in two conditions: with and without a realistic context. Regarding performance, item response theory (IRT) models showed first that the same (latent) ability dimension was involved in solving both types of problems. Second, the presence of a context increased the difficulty level of the division problems, but not of the other operations. Regarding strategy use, results showed that strategy choice and strategy accuracy were not affected by the presence of a problem context. More importantly, the absence of context effects on performance and on strategy use held for different subgroups of students, with respect to gender, home language, and language achievement scores. In sum, the present findings suggest that at the end of primary school the presence of a typical context in a multidigit mathematics problem had no marked effects on students' multidigit arithmetic problem solving behavior.

Keywords: word problems, mathematics education, solution strategy, multidimensional IRT, linear logistic test model (LLTM).

*SOLVING MULTIDIGIT MATHEMATICS PROBLEMS***Introduction**

Mathematics education has experienced a large international reform (e.g., Kilpatrick, Swafford, & Findell, 2001). A general characteristic of this reform is that instruction no longer focuses predominantly on decontextualized traditional mathematics skills, but that instead the process of solving realistic mathematics problems and doing mathematics are important educational goals (e.g., National Council of Teachers of Mathematics, 1989, 2000). Word problems or the broader category of contextual problems¹ – typically a mathematics structure in a realistic problem situation – serve a central role for several reasons (e.g., Verschaffel, Greer, & De Corte, 2000): they may have motivational potential, mathematical concepts and skills may be developed in a meaningful way, and children may develop knowledge of when and how to use mathematics in everyday-life situations. Furthermore, solving problems in context may ideally serve as a tool for mathematical modeling or mathematizing (e.g., Greer, 1997). As a consequence of this shift in educational goals, mathematics assessments include more and more contextual problems in their tests. For example, the PISA-2009 study (*Programme for International Student Assessment*; OECD, 2010) of students' mathematics performance included mainly problems presented in a real-world situation.

In the Netherlands, the reform is characterized by the principles of Realistic Mathematics Education (RME; Freudenthal, 1973, 1991; Treffers, 1993). In RME, contextual problems (defined as a problem that is experientially real to students) are central: they are the starting point for instruction, which is based on the principle of progressive schematization or mathematization by guided reinvention (Gravemeijer & Doorman, 1999). That is, contextual problems are expected to elicit informal or naive solution strategies which are progressively abbreviated and schematized, in a process guided by the teacher. In the last decades, RME has become the dominant instructional approach in mathematics curricula for Dutch primary education. The most recent national assessments in grade 3 and 6 showed that almost all elementary schools used a mathematics textbook based on RME principles (J. Janssen, Van der Schoot, & Hemker, 2005; Hop, 2012), although a return to more traditionally oriented mathematics textbooks has been observed

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3 recently (Royal Netherlands Academy of Arts and Sciences, 2009). These RME-based
4 textbooks contain many contextual problems, although there are substantial
5 differences in this respect between the different textbooks. In order to
6 accommodate these developments, Dutch mathematics assessments (J. Janssen et
7 al.; Hop) and commonly used student monitoring tests also contain predominantly
8 contextual problems. Therefore, today's Dutch primary school students'
9 mathematics education and assessment consists for a large part of problems in
10 realistic contexts.
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18 The growing importance of contextual problems in mathematics education
19 and assessments necessitates that we increase our understanding of the impact of
20 typical school mathematics contexts, both on a theoretical level (what aspects of
21 mathematical cognition are involved?) as well as from a practical educational
22 perspective (what are the implications regarding testing and instruction practices?).
23 Therefore, the main question asked in the current study is: What is the effect on
24 problem solving of presenting a multidigit arithmetic problem in a realistic context
25 typical for school mathematics for students in sixth grade? Two aspects of problem
26 solving are addressed: *performance* (i.e., accuracy) and *solution strategy use*.
27 Furthermore, because a necessary condition for solving the mathematics problem is
28 that the student understands the usually verbal context, it is likely that students'
29 language level plays a role. Therefore, the role of language is an important point of
30 focus in the current study.
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42 Many older studies were carried out in the field of word problems,
43 particularly in the domain of addition and subtraction with young children (e.g., see
44 Carpenter, Moser, & Romberg, 1982 and for a recent overview see Verschaffel,
45 Greer, & De Corte, 2007). *Word problems* can be considered a subcategory of the
46 broader class of mathematics problems presented in a realistic context. This word
47 problem research has shown that problem solving behavior is influenced by
48 characteristics of the students (e.g., how advanced they are) and characteristics of
49 the problems (e.g., particular semantic features), and the interaction between the
50 two. Therefore, it is important to specify the population of students and type of
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3 problems addressed. The major objective of the current study was to investigate
4 students and problems typical for the regular school setting, in order to obtain high
5 ecological validity of the conclusions and implications for educational and testing
6 practices. To that end, contextual multidigit arithmetic problems involving one of
7 the four basic operations (addition, subtraction, multiplication, and division) were
8 selected from national assessment studies. These problems, involving one-step
9 arithmetic operations, were considered typical for contextual problems in school
10 mathematics. Regarding the population of students, we focused on students at the
11 end of primary school (i.e., sixth grade) for at least two reasons. First, by
12 investigating the influence of contexts at a time point where students have received
13 many years of formal schooling in solving mathematics problems with and without a
14 context, we aimed to extend existing research mainly focusing on young children.
15 Second, students at the end of primary school are often subjected to high-stakes
16 educational tests, of which the mathematics subtests increasingly rely on problems
17 presented in a context. Therefore, it is important to investigate the possible effects
18 of such contexts in mathematics tests specifically for this group of students.
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Solving numerical and contextual arithmetic problems

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36 Solving numerical and contextual problems (sometimes referred to as
37 'computations' and 'applications', respectively) is likely to involve different aspects
38 of mathematical cognition. At least two different perspectives exist. In one
39 perspective, it is stressed that solving contextual problems involves a complex
40 process consisting of several cognitive processes or phases (e.g., Fuchs et al., 2006,
41 2008; Wu & Adams, 2006). Specifically, first an accurate situational and
42 mathematical model of the problem situation has to be formed (a process called
43 mathematization), before computational skill – and carefulness therein – comes into
44 play. Therefore, factors other than 'pure' computational skills are likely to
45 contribute to success in solving contextual problems. This perspective yields the
46 expectation that contextual problems are more difficult to solve than numerical
47 problems, as supported by early research findings of Cummins et al. (1988) in
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3 simple addition and subtraction word problems.
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6 By contrast, in the other perspective on the effect of problem representation,
7 it is argued that contextual problems can also be easier to solve than numerical
8 problems. That is, the realistic context may activate real-world knowledge which
9 can aid problem-solving by eliciting other strategic approaches (e.g. Gravemeijer,
10 1997). In the domain of algebra, Koedinger and Nathan (2004) and Koedinger,
11 Alibali, and Nathan (2008) found that students were more successful in solving so-
12 called grounded representations (story problems) than in solving algebra equations
13 presented symbolically, because they used intuitive, informal strategies more often
14 to solve the story problems. However, this advantage of story problems held only for
15 simple problems and not for more complex ones, a pattern the authors called the
16 representation-complexity trade-off. They argued that both grounded and abstract
17 representations each have specific advantages based on their different properties:
18 grounded representations are expected to be more familiar and less error-prone,
19 while abstract representation are more concise and put fewer demands on working
20 memory (Koedinger et al., 2008).
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33 Both perspectives have in common, however, that there are at least two ways
34 in which the problem format (contextual versus numerical) may affect performance:
35 different ability dimensions may be involved in solving problems with and without a
36 realistic context, or the context may affect the difficulty level of a problem.
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40 Recently, a few studies empirically investigated to what extent different
41 abilities were involved in solving mathematic problems of both formats in American
42 third graders (Fuchs et al., 2006, 2008) and in Dutch first to third graders
43 (Hickendorff, in press). These studies showed that solving numerical mathematics
44 problems and solving contextual problems involved two highly related but distinct
45 ability dimensions, as evidenced by a less than perfect correlation between
46 performance measures and by different cognitive correlates for the two measures.
47 However, these studies did not allow a direct investigation of how a realistic context
48 affects the difficulty level of a problem, because the numerical characteristics of
49 contextual and numerical problems were not matched. In contrast, Vermeer,
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3 Boekaerts, and Seegers' (2000) study on sixth graders' arithmetic problem solving
4 involved matched computation (i.e., numerical) problems and application (i.e.,
5 contextual) problems, thereby potentially allowing for direct comparisons.
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7 Regrettably, direct tests comparing performance on the two types of problems were
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9 not reported, although the proportion correct was found to be slightly higher on the
10 application problems than those involving computation. Since theoretical
11 hypotheses and empirical results are inconclusive on the effects of problem format
12 on performance, further systematic study is needed.
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18 The present study extends the previously discussed studies in three ways.
19 First, the effect of problem format (with or without a context) on problem solving is
20 investigated in a systematic test design, consisting of problem pairs in which one
21 problem was presented with a realistic context and the matched parallel problem
22 without such a context. Second, a more complete account of problem solving was
23 taken: not only performance but also strategy use is researched. Finally, given the
24 verbal nature of contextual problems, the role of students' language level is
25 addressed.
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Solution strategies

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36 Performance or accuracy is probably the most salient aspect of problem
37 solving, and many studies into solving problems with and without a realistic context
38 focused only on that aspect (Fuchs et al., 2006, 2008; Hickendorff, in press; Vermeer
39 et al., 2000). However, another important aspect of problem solving is strategic
40 competence. In cognitive psychology it is well-established that adults and children
41 know and use multiple solution strategies to solve mathematics problems (e.g.,
42 Lemaire & Siegler; 1995 Siegler, 1988a). Furthermore, solution strategies are also
43 important from the perspective of mathematics education in at least two ways. First,
44 the didactics for solving complex arithmetic problems have changed, from
45 instructing standard written algorithms to building on children's informal or naive
46 strategies (Freudenthal, 1973; Treffers, 1987, 1993), and mental arithmetic has
47 become very important (Blöte, Van der Burg, & Klein, 2001). Second, mathematics
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3 education reform aims at attaining adaptive expertise instead of routine expertise:
4 instruction should foster the ability to solve mathematics problems efficiently,
5 creatively, and flexibly, with a diversity of strategies (Baroody & Dowker, 2003;
6 Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009).
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11 Lemaire & Siegler (1995) distinguished four aspects of strategic competence:
12 strategy repertoire, strategy choice, strategy performance (such as accuracy), and
13 strategy adaptivity. The current study focuses on the first three of these aspects, on
14 the domain of multidigit arithmetic. In the domain of elementary or simple
15 arithmetic, strategy use has been studied extensively: in elementary addition and
16 subtraction (e.g., Carr & Jessup, 1997; Carr & Davis, 2001; Torbeyns, Verschaffel, &
17 Ghesquière, 2004, 2005), in elementary multiplication (e.g., Anghileri, 1989; Imbo &
18 Vandierendonck, 2007; Lemaire & Siegler, 1995; Mabbott & Bisanz, 2003; Mulligan
19 & Mitchelmore, 1997; Sherin & Fuson, 2005; Siegler, 1988b), and in elementary
20 division (e.g., Robinson et al., 2006). By contrast, research on solution strategies in
21 complex or multidigit arithmetic problems is less extensive, but there is a growing
22 body of studies in multidigit addition and subtraction (e.g., Beishuizen, 1993;
23 Beishuizen, Van Putten, & Van Mulken, 1997; Blöte et al., 2001; Torbeyns, Verschaffel,
24 & Ghesquière, 2006) and in multidigit multiplication and division (e.g., Ambrose,
25 Baek, & Carpenter, 2003; Buijs, 2008; Hickendorff, Heiser, Van Putten, & Verhelst,
26 2009; Hickendorff & Van Putten, 2012; Hickendorff, Van Putten, Verhelst, & Heiser,
27 2010; Van Putten, Van den Brom-Snijders, & Beishuizen, 2005).
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42 The current study addressed multidigit arithmetic involving the four basic
43 operations. Based on the solution strategies reported in the aforementioned studies
44 in multidigit addition, subtraction, multiplication, and division (see also a recent
45 review by Verschaffel et al., 2007), a classification scheme of written solution
46 strategies was developed (i.e., the strategy repertoire). For each of the four
47 operations, a basic distinction can be made (see also Selter, 2001) between the
48 *traditional standard algorithm* that proceeds digit-wise, *non-traditional procedures*
49 that are morer informal, dealing with whole numbers, and *answers without written*
50 *working*, most likely mental strategies. A subcategory of the non-traditional
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3 strategies are the RME approaches (labeled 'columnwise arithmetic' by the
4 developers, see Van den Heuvel-Panhuizen, Buys, & Treffers, 2001, and Treffers,
5 1987). These can be considered transitory between informal approaches and the
6 traditional algorithm: they work with whole numbers instead of single-digits (like
7 informal strategies), but they proceed in a more or less standard way (like the
8 written algorithm). More details are given in the Method-section.

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14 Based on the literature, we had the following expectations regarding the
15 effects of problem format (contextual or numerical) on strategy use. Studies on
16 elementary word problem solving with young children showed that different
17 semantic structures of word problems elicited different strategies (for a review, see
18 Verschaffel et al, 2007). Extending these findings leads to the expectation that
19 contextual and numerical problems on multidigit arithmetic elicit different
20 strategies. In particular, the theory behind the RME didactical approach yields the
21 expectation that problems in a realistic context are more likely to elicit more
22 informal, less structured strategies (i.e., non-traditional or mental solution
23 strategies), while numerical problems would elicit more use of traditional
24 algorithms, as hypothesized by Van den Heuvel-Panhuizen, Robitzsch, Treffers, &
25 Köller (2009). This pattern was indeed found in Koedinger and Nathan's (2004) and
26 Koedinger et al.'s (2008) study on algebra problem solving. In contrast, Van Putten
27 et al. (2005) investigated Dutch fourth graders strategy use on multidigit division
28 problems that either did or did not include a context, and found no differences in
29 strategy choice between the two types of problems. Given these inconsistent
30 findings, the effects of contexts on strategy use require further systematic study.

The role of students' language level and gender

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49 Because it is necessary that students accurately understand the usually
50 verbal problem situation of a contextual problem, it is likely that the student's
51 language ability plays an important role. Support for the importance of language in
52 word problem solving comes from the finding that language ability had larger
53 effects on applied problem solving (contextual problem solving) than on
54 computational skills (numerical problems) (Fuchs et al., 2006, 2008; Hickendorff, in
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3 press). Additional support comes from the finding that a common source of errors in
4 word problem solving appears to be misunderstanding of the problem situation
5 (Cummins et al., 1988; Wu & Adams, 2006), and that conceptual rewording of word
6 problems facilitated performance (e.g., Vicente, Orrantia, & Verschaffel, 2007).
7 Therefore, we expect the effect of language ability level to be larger on performance
8 in solving contextual problems than in solving numerical problems.
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14 Ethnic minority students score lower on language ability tests than native
15 students. In addition, international assessments such as TIMSS-2007 (*Trends in*
16 *International Mathematics and Science Study*; Mullis, Martin, & Foy, 2008) and Dutch
17 national assessments (J. Janssen et al., 2005; Hop, 2012) consistently report that
18 ethnic minority pupils lag behind in mathematics achievement too. An obvious
19 question is whether language level plays a role in the performance lag of ethnic
20 minorities on mathematics problems that involve a verbal context. Several research
21 findings with students in secondary education showed that the difficulty of the
22 problem text particularly hampers non-native speakers (Abedi & Hejri, 2004; Abedi
23 & Lord, 2001; Prenger, 2005; Van den Boer, 2003), due to text aspects like the use of
24 unfamiliar vocabulary, passive voice construction, and linguistic complexity.
25 Therefore, we expect differences with respect to the language spoken at home to be
26 larger on the performance in solving contextual problems than in solving numerical
27 problems.
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40 A final student characteristic addressed in the present study is gender.
41 Gender differences in general mathematics performance have been reported
42 frequently. Large-scale international assessments TIMSS-2007 (Mullis et al., 2008)
43 and PISA-2009 (OECD, 2010) showed that boys tend to outperform girls in most of
44 the participating countries, including the Netherlands. This pattern is supported by
45 Dutch national assessments findings: on most mathematical domains boys
46 outperformed girls in third and in sixth grade (J. Janssen et al., 2005; Hop, 2012).
47 However, in grade 6, the multidigit operations were the exception with girls slightly
48 outperforming boys. Moreover, Vermeer et al. (2000) found that in Dutch sixth
49 graders, there were no gender differences in performance on computations, while
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3 boys outperformed girls on applications. The latter difference may possibly be
4 explained by the finding that on application problems, girls had lower levels of
5 subjective competence than boys and attributed bad results to lack of capacity and
6 difficulty of the task. Based on these results, we expect that gender differences in
7 performance to be larger on contextual problems than on numerical problems.
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12 Regarding strategy choice, girls have been found to be more inclined to (quite
13 consistently) rely on rules and procedures and use well-structured strategies,
14 whereas boys have a larger tendency to use more intuitive strategies (Carr & Davis,
15 2001; Carr & Jessup, 1997; Gallagher et al., 2000; Hickendorff et al., 2009, 2010;
16 Hickendorff & Van Putten, 2012; Timmermans, Van Lieshout, & Verhoeven, 2007;
17 Vermeer et al., 2000). There are no empirical findings on whether this pattern is the
18 same for numerical problems as for contextual problems, so further study is needed.
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The current study

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29 The current study's aim was to systematically investigate the effects of
30 presence of a context in mathematics problems on two aspects of problem solving:
31 performance and strategy use (strategy choice and strategy accuracy). To
32 investigate this issue with students and problems typical for the regular school
33 setting, contextual multidigit arithmetic problems on the four basic operations
34 (addition, subtraction, multiplication, and division) were selected from national
35 assessment tests. These problems were administered to a sample of sixth graders in
36 two formats: once with the context, and once without (i.e., in bare numerical
37 format). The total problem set consisted of eight such pairs of problems.
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45 Based on the previous discussion of existing theoretical literature and
46 empirical findings, we had the following expectations. With regard to performance,
47 we expected that two highly related but distinct ability dimensions would be
48 involved in solving the two types of problems, and that contextual problems are
49 more difficult, in particular for students with low language level as well as for girls.
50 With regard to strategy use, we expected that contextual problems would elicit
51 more use of informal, less structured strategies than numerical problems.
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Method*Participants*

Participants were 685 students from grade 6 with mean age 12 years 0 months (SD = 5 months). They originated from 24 different primary schools, with 3 to 82 students participating per school (on average 27.4 students per school; all sixth graders in a class participated). These schools were spread over the entire country of the Netherlands. There were 312 boys, 337 girls, and 36 students with missing gender information. In order to assess language level effects with sufficient power, the schools that were selected had relatively high ethnic minority populations. As a consequence, the current sample of schools and pupils was not entirely representative for the population of Dutch primary schools.

Information on the language spoken at the students' home was gathered (missing data for 42 students). Students were classified into home language *Dutch* (either only Dutch, 517 students, 80%; or Dutch as well as another language, 46 students, 7%) or home language *non-Dutch* (80 students, 12%). The most prevalent non-Dutch language was Arabic (45% of students with home language other than Dutch), followed by Turkish (26%).

Material

Experimental task. The experimental task consisted of 16 multidigit arithmetic problems, consisting of 8 pairs of one contextual and one numerical problem each. There were 2 pairs of problems for each multidigit operation: addition, subtraction, multiplication, and division (with whole number outcomes), see Appendix A.

The contextual problems were selected from the most recent Dutch national assessment (J. Janssen et al., 2005). Two problems were selected for each operation: one at the lower end of the ability scale (i.e., a relatively easy problem with small numbers), and one at the upper end of the scale (i.e., a relatively hard problem with

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3 large numbers). We used problems from the assessments to ensure that they were
4 representative for the type of contextual problems that are used in current
5 educational practices. For the numerical problems, these contextual problems were
6 disposed of their contexts to yield the bare numerical operation required. In this
7 process, decisions had to be made on the order of the numbers in addition and
8 multiplication, and on whether to use a canonical or non-canonical form in
9 subtraction and division. For instance, the first contextual division problem from
10 Appendix A can have different numerical variants, such as $736 : 32 = ?$ (the one
11 chosen in this study) or $32 \times ? = 736$. The choice for a particular numerical problem
12 was made based on the most likely approaches of students to solving typical
13 contextual problems, resulting from earlier studies (Hickendorff et al., 2009;
14 Hickendorff & Van Putten, 2012). Furthermore, in order to avoid testing effects that
15 may have occurred if students had to solve exactly the same numerical operation
16 twice (once with and once without a context), a parallel version of each problem
17 was constructed with numbers and solution steps as similar as possible.
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30 Two different test forms were created, so that the item parallel version was
31 counterbalanced over the test form. That is, in form A item versions *a* were
32 presented as contextual problems and item versions *b* as numerical problems, and
33 in form B this pattern was reversed. For example, the first item pair on Addition in
34 Appendix A presents the problems as presented in form A. In form B, the numbers
35 were switched: i.e., the text of the contextual problem said that 677.50 euro was sold
36 on postcards and 975 euro on stamps, while the numerical problem was $466.50 +$
37 $985 = ?$. Figure 1 presents the specific position of each problem in both task forms.
38 Within each form, paired problems (e.g. A_{1a} and A_{1b}) were presented with 7 other
39 problems in between, to prevent recency effects. The order of the 16 different
40 problems was the same in both task forms, to rule out potentially confounding order
41 effects in combining the data from the two forms.
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52 In the test booklets that students received, a maximum of 3 problems were
53 printed on the left side of a page (A4 size). The right side of each page was left blank,
54 so that students could use that space as scrap paper in solving the problems.
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Standardized tests. The students participating in the current study took part in the 2009 administration of CITO's End of Primary School Test (CITO, 2009) in February. This test is widely used in the Netherlands at the end of primary school, and its purpose is to give advice on the most suitable track of secondary education for each student. To that end, the instrument assesses scholastic achievement level in mathematics, language, and study skills. Over 150,000 Dutch sixth graders participated in the 2009 assessment. The 100-item subtest on language skills consisted of items on writing, spelling, reading comprehension, and vocabulary, and had high internal reliability (KR20 = .89; CITO, 2009). In the current sample, the average number of language items correct was 73.0 (SD = 11.7; missing data for 29 students). This mean score was slightly lower than for the entire population of students participating in the End of Primary School Test, who scored on average 75.2 items correct (SD = 12.0). On the 60-item subtest on mathematics (KR20 = .91), the current sample scored on average 41.7 items correct (SD = 10.6), which was also slightly lower than the average of 42.8 correct (SD = 10.5) for all participants nationwide.

Procedure

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Experimental task. The experimental task was administered as part of a pretest study for the CITO End of Primary School Test. A test booklet consisted of 6 tasks, divided over the different subjects mathematics, language, and study skills. Students completed each of these tasks on a separate day in January 2009. One of the mathematics tasks included the current experimental task of 16 problems, and an additional 12 problems that were not part of the current study. One of the two experimental task forms (A or B) was assigned to each class.

The task was administered in the classroom, and each student worked individually. Teachers instructed their students that they were free to choose their solution strategy. Students were also told that they could use the blank space next to each problem in the test booklet to make computations, and that they did not need separate scrap paper. Students could take as much time as they needed, so there was no time pressure.

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Standardized tests. The students completed the 2009 End of Primary School Test (CITO, 2009) as part of their final year's standardized assessment in February 2009, which was at most one month after the students participated in the current study.

Solution strategies

The solution strategy used on each trial (student-by-item combination) was categorized based on the notes or solution procedures that students had written in the test booklet. Strategy data were available for 650 students. Three experts (the first author and two trained research assistants) each coded a separate part of the material. Table 1 shows the 9 (addition) or 10 (subtraction, multiplication, and division) different categories of solution strategies that were distinguished, and Appendix B shows examples of categories 1 to 5 for each operation. Below, first the operation-specific categories 1 to 6 are discussed, and after that the operation-general categories 7 to 10 are explained.

Addition. Category 1 (traditional algorithm) coded strategies in which the standard algorithm for adding two or more multidigit numbers was applied. The addends have to be aligned vertically so that digits on the same position represent the same value, and addition proceeds digit-wise from right to left starting with the ones-digits (assuming there are no decimals), next the tens-digits, then the hundreds-digits, and so forth. If the outcome of any particular sub-addition is larger than 10, regrouping has to take place. Category 2 is the RME approach to addition. It contrasts with the traditional algorithm because it proceeds from left to right and it works with numbers instead of single-digits (e.g., $600 + 900 = 1500$ instead of $6 + 9 = 15$). Category 3 and 4 are partitioning strategies, in which either one or more than one of the operands is partitioned or split according to its place value (e.g., 975 is split into 900, 70, and 5). Partitioning of only the second operand is also called the jump or sequential strategy, while partitioning of two or more operands is called the split or decomposition strategy (e.g., Beishuizen, 1993). The final category 6 (note that we left out category number 5 for addition to be consistent with the other

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4 operations) included all kinds of strategies in which some calculations or
5 intermediate solutions were written down from which could be inferred how the
6 answer was obtained, but that did not fit in categories 1 to 4.
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9 *Subtraction.* Category 1 (traditional algorithm) involved application of the
10 standard algorithm for subtraction of two multidigit numbers. Similar to the
11 addition algorithms, the two numbers have to be aligned vertically so that digits on
12 the same position represent the same value, and subtraction proceeds digit-wise
13 from right to left starting with the ones-digits (assuming there are no decimals),
14 next the tens-digits, then the hundreds-digits, and so forth. In case a larger digit has
15 to be subtracted from a smaller one (e.g., 0 - 9), regrouping has to take place.
16 Category 2 is the RME approach to multidigit subtraction. It contrasts with the
17 traditional algorithm because it proceeds from left to right and it works with
18 numbers instead of single-digits. Moreover, there is no need of borrowing: it works
19 with negative numbers instead (e.g., $10 - 80 = -70$). Category 3 and 4 are partitioning
20 strategies, in which either only the subtrahend or both operands are partitioned
21 according to their place value (e.g., 689 is split into 600, 80, and 9). Similar to
22 addition, partitioning of only the subtrahend is also called the jump or sequential
23 strategy, while partitioning of both operands is called the split or decomposition
24 strategy (Beishuizen, 1993). Category 5 involved indirect addition strategies. In
25 these approaches, one starts from the subtrahend and adds on until the minuend is
26 reached (see for example Torbeyns, Ghesquière, & Verschaffel, 2009). The final
27 category 6 (other written strategy) included all kinds of strategies in which some
28 calculations were written down, but that did not fit in categories 1 to 5.
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45 *Multiplication.* The traditional standard algorithm for multiplication
46 (category 1) involves writing the two operands below each other, and multiplying
47 the upper number by each digit of the lower number separately, working from right
48 to left. Then, these partial outcomes are added to obtain the solution. In the RME
49 approach (category 2), both numbers are partitioned and all sub-products are
50 obtained and added. This strategy closely resembles the one in category 4
51 (partitioning of both operands), but the difference lies in the schematic notation that
52 is applied in category 2 which is stressed in the RME approach. Hence, these two
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SOLVING MULTIDIGIT MATHEMATICS PROBLEMS

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4 categories are separated because their distinction is educationally relevant more
5 than because it is of interest from a cognitive perspective. In category 3, only one of
6 the operands is partitioned, while the other one is left intact (e.g., $36 \times 27 = 36 \times 20$
7 $+ 36 \times 7$). Category 5 involved repeated addition, in which there is made use of the
8 fact that multiplying by a factor n is equivalent to adding the multiplicand n times.
9 This category included strategies in which either the multiplicand was added n
10 times, or when doubling strategies were used. Again, the final category 6 included all
11 kinds of strategies in which some calculations were written down, but that did not
12 fit in categories 1 to 5.
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20 *Division.* The first category of division was the long division algorithm (note
21 that notation may differ between countries). The algorithm is characterized by
22 starting on the left side with places of highest value of the dividend, and trying to
23 divide the first digit by the divisor (e.g., $7 \div 32$). If that yields a number smaller than
24 1, the first two digits are considered together (73) and the maximum number of
25 times the divisor fits in ($2 \times 32 = 64$) is noted. Then, the difference is determined (73
26 $- 64 = 9$) and the digit from the column to the right is pulled down (making 96). This
27 procedure continues until all digits of the dividend have passed and the remainder
28 is zero, or in case of a nonzero remainder until the required level of precision in the
29 quotient is reached. Categories 2 and 3 involve repeated subtraction strategies (in
30 the Netherlands this is the RME alternative for long division in the mathematics
31 textbooks, Treffers, 1987). Multiples of the divisor are repeatedly subtracted from
32 the dividend. This can be done efficiently with relatively few steps (high-level, HL)
33 or less efficiently with many steps (low-level, LL). In the present study, we defined
34 strategies as high-level when at most 3 steps (the minimum number of steps + 1)
35 were taken. It is worth noting that the most efficient repeated subtraction strategy
36 resembles the traditional algorithm, with the main difference that in the algorithm
37 one ignores the place value of the digits, while one works with whole numbers in
38 repeated subtraction (e.g., with 640 instead of 64). Categories 4 and 5 resemble
39 categories 2 and 3, respectively, but they differ in the approach: repeatedly adding
40 multiples of the divisor until the dividend is reached, as opposed to repeatedly
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3 subtracting from the dividend until zero is reached. The same distinction between
4 high-level (maximum 3 steps) and low-level approaches was made. As with the
5 other operations, category 6 (other written strategy) involved strategies in which
6 some calculations were written down, but that could not be classified in categories 1
7 to 5.
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12 *Remainder categories.* The remaining strategy categories 7 to 10 were the
13 same for the four operations. Category 7 (no written working) includes all trials in
14 which an answer was written down, but nothing else (i.e., no calculations or
15 intermediate solutions), so it is very likely that the answer was computed mentally
16 (supported by findings of Hickendorff et al., 2010). Category 8 (wrong procedure)
17 includes trials in which the wrong procedure was applied, such as adding the two
18 numbers in a division problem. In trials classified in category 9 (unclear strategy) it
19 was unclear how the student arrived at the answer (s)he had given, in some cases
20 because the written solution steps were erased. The final category 10 (skipped
21 problem) included trials in which the problem was skipped entirely, i.e., no answer
22 was given and no solution steps were written down.
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33 *Reliability.* The solution strategies of 45 students (720 trials; 180 trials per
34 operation) were double-coded by two independent raters to assess the agreement
35 in categorization. Cohen's kappa (Cohen, 1960) on the cross-tabulation of the
36 categorization of the two raters was computed as a measure of inter-rater reliability.
37 Kappa values were sufficiently high with .82, .79, .89, and .92 for addition,
38 subtraction, multiplication, and division, respectively, indicating substantial and
39 satisfactory agreement.
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Data Analysis and Results

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49 In all data-analyses, the data were collapsed over the two test forms A and B,
50 thereby counterbalancing potential differences between parallel item versions
51 within problem pairs. The results are presented in two parts: performance and
52 strategy use.
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SOLVING MULTIDIGIT MATHEMATICS PROBLEMS

Performance

The proportion correct on contextual problems and numerical problems were very close on addition (.70 vs. .72), subtraction (.72 vs. .73), multiplication (.69 vs. .68), and division (.75 vs. .73). Recall that the main question was: What is the effect of presence of a context on performance?, and that this question was addressed in two ways. First, we established whether separate dimensions of individual differences were involved in solving items with and without a context (the multidimensionality hypothesis). Second, the effect of problem format on the difficulty level of an item was tested (the sources-of-difficulty hypothesis).

Because the accuracy data concern repeated dichotomous observations (i.e., each student answers 16 items), analysis techniques that can take this multivariate character into account are needed. Aggregating over items by computing total proportion correct, as is characteristic of the Classical Test Theory (CTT) approach, has several disadvantages. One salient disadvantage is that the resulting proportion correct scores are potentially hampered by ceiling and floor effects, biasing subsequent statistical analyses on these scores. Modern test theory, specifically item response theory (IRT) modeling, overcomes these limitations by accounting for the influence of item characteristics on the responses (for a detailed discussion of the advantages of IRT over CTT, see Embretson & Reise, 2000). In IRT models, one or more continuous latent variables are introduced that model individual differences in ability level, affecting the probability to solve a particular item correctly. Specifically, in the most simple unidimensional IRT model – the Rasch model – the probability P_{is} that person s solves item i correctly is modeled as a logistic function of the difference between the person’s latent ability level θ_s and the item’s difficulty level

$$\beta_i: P_{is} = \frac{\exp(\theta_s - \beta_i)}{1 + \exp(\theta_s - \beta_i)}.$$

Both hypotheses were addressed with item response theory (IRT) models. We used the `lmer` function from the `lme4` package (Bates & Maechler, 2010) available in the statistical computing program R (R Development Core Team, 2009) to estimate the model parameters. For further details on how to fit IRT models with

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lmer, see De Boeck et al. (2011). The main assumptions of IRT models are that the model involves the correct number of dimensions of individual differences and that the item response curves P_{is} follow the specified mathematical function (Embretson & Reise, 2000). The first assumption about dimensionality is at the core of the current investigation. The second assumption will be tested by checking for item fit statistics that quantify how well the observed item responses are predicted by the estimated IRT model parameters (Embretson & Reise, 2000).

Multidimensionality. To explore whether different latent abilities were involved in solving items with and without a context, we used multidimensional IRT (MIRT) modeling (see Reckase, 2009). Specifically, a confirmatory two-dimensional IRT model was used, in which each item was assigned a priori to one of the two dimensions *solving numerical problems* and *solving contextual problems*. Figure 2 shows a graphical display of this model. Such a model belongs to the class of between-item or simple structure Rasch models, in which it is assumed that multiple related subscales or ability dimensions underlie test performance, and that each item in the test is only related to one of these subscales (Adams, Wilson, & Wang, 1997).

Our main interest was in the estimate of the latent correlations between the two ability dimensions θ_{num} and θ_{con} . A latent correlation estimate in a MIRT model is not attenuated by measurement error: it is an unbiased estimate of the true correlation between the latent variables (Adams & Wu, 2000; Wu & Adams, 2006). Therefore, it is a better alternative than computing the correlation on ability estimates of consecutive unidimensional models, or on classical test theory approaches that are based on the proportion of items solved correctly (as was done in the studies by Fuchs et al., 2006, 2008).

The results of fitting a 2-dimensional between-item Rasch model showed that the latent correlation between the ability to solve numerical problems and the ability to solve contextual problems was estimated at 1.000.² Therefore, we conclude that solving numerical problems and solving mathematics problems in a

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context involves one (latent) ability factor. Furthermore, we computed item fit statistics for the unidimensional Rasch model with the `ltm` package (Rizopoulos, 2006). There were no items with significant misfit (lowest p -value was .059), indicating that the unidimensional Rasch model adequately predicted the observed item response patterns.

Sources of item difficulty. The next step was to assess the effect of item format on the difficulty level of an item. For that end, we used an extended Linear Logistic Test Model (LLTM; Fischer, 1987). The LLTM is an example of a broad class of explanatory IRT models, in which predictors on the item level, the person level, and on the item-by-person level can be incorporated in the model (De Boeck & Wilson, 2004). The LLTM allows for decomposition of item difficulty β_i into the effects of K different item features, in a multiple regression-like manner: $\beta_i = \sum_{k=1}^K \tau_k q_{ik} + \tau_0$. The q_{ik} entries of the so-called Q-matrix specify the involvement of item feature k in item i , and have to be assigned a priori. The LLTM has the drawback that it assumes that the K item features predict item difficulty without error. To relax this assumption, the LLTM can be extended by incorporating error that is randomly distributed over items in the model, yielding the LLTM + e model (De Boeck, 2008; R. Janssen, Schepers, & Peres, 2004).

There were three item features: operation required (nominal variable with 4 categories: addition, subtraction, multiplication, and division; recoded into 3 dummy variables), number size of the problem (dichotomous variable with 2 categories: small or large), and item format (dichotomous variable with 2 categories: numerical or contextual). Our main interest was in the effect of item format, statistically correcting for the covariates operation and number size. In addition, we corrected for possible group differences in ability level between students who were administered test form A or test form B, by including 'test form' in the model as well. Because this was a variable on the student level, the final IRT models that we used could be characterized as *latent regression LLTM + e* models.

Results showed that the main effect of item format was not significant ($\tau_{\text{context}} = .05, p = .30$). Testing for interactions between item format and the other two item

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3 features, it was found that the interaction between item format and number size
4 was not significant (Likelihood Ratio $\chi^2(1) = .0, p = 1.00$), while the interaction
5 between item format and operation was significant ($\chi^2(3) = 16.4, p < .001$). Further
6 inspection of this latter interaction showed that the effect of context format was
7 significant only on division problems, $\tau_{\text{context, division}} = .35, p < .001$, while for the other
8 three operations the effect of context format was not significant ($\tau_{\text{context, addition}} = -.15,$
9 $p = .10$; $\tau_{\text{context, subtraction}} = -.08, p = .36$, and $\tau_{\text{context, multiplication}} = .08, p = .35$). So, only for
10 division problems the context made the item more difficult compared to the bare
11 numerical problem. On the other three operations the contextual problems and the
12 numerical problems were just as difficult.
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21 Final analyses were carried out to assess whether the effect of item format
22 (numerical vs. contextual) on item difficulty depended on gender, language
23 achievement level, or home language. First, the main effects of the student
24 characteristics on performance are reported. Gender had a significant effect on
25 performance: girls had significantly more correct answers than boys ($\beta_{\text{girl}} = .23, p =$
26 $.03$). In contrast, the effect of home language on performance was not significant
27 ($\beta_{\text{other than Dutch}} = -.04, p = .82$). Furthermore, the effect of language achievement level
28 was highly significant and positive: $\beta_{\text{language}} = .58, p < .001$. Finally, we focus on the
29 interactions between student characteristics and item format. The interaction
30 between item format and gender ($\chi^2(4) = 4.4, p = .35$), home language ($\chi^2(4) = 1.1,$
31 $p = .89$), and language achievement level ($\chi^2(4) = 4.1, p = .39$) all turned out to be
32 nonsignificant. So, the effects of presenting an item in a context did not depend on
33 either students' gender, home language or language achievement level.
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Strategy use

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48 *Strategy choice.* Table 2 shows the distribution of the strategy categories for
49 addition, subtraction, multiplication, and division, split by problem format
50 (numerical versus contextual). There were only small differences with respect to
51 problem format in the distribution of strategies: The percentages of trials solved by
52 each strategy were very similar for the numerical problems as for the contextual
53 problems, with the largest difference being 3 percent points.
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4 In contrast, operation required seemed to have large effects on strategy
5 choice distribution. First, differences in the use of the traditional algorithm
6 (category 1) emerged. It was the dominant strategy for addition and subtraction
7 (used on around 70% of the trials). Also for multiplication it was the most prevalent
8 strategy although its dominance was less pronounced, being used on over 50% of
9 the multiplication trials. For division, however, the traditional algorithm was used
10 on only 15% of all division trials. Second, the frequency of answering without
11 written working (category 7) was about the same for each operation, occurring on
12 between 14% and 17% of all trials. Moreover, wrong procedures (category 8),
13 unclear strategies (category 9), and skipping the entire problem (category 10) did
14 not occur very often. These occurred with about the same frequency on each
15 operation, although slightly more often on division. Third, some operation-specific
16 patterns emerged. Most notably, on division the high-level repeated subtraction
17 strategy was the most prevalent strategy, used on over 50% of all division trials. On
18 multiplication, partitioning of 1 operand (category 3) was used quite often (17% of
19 all trials). Finally, on addition, the RME approach was used relatively often (9% of all
20 trials) compared to the other operations.
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34 In order to statistically test the effects of operation and problem format on
35 strategy choice distribution, we first collapsed some of the categories to make
36 strategy categories comparable across operations, and also to obtain categories
37 filled with a substantial number of observations. We recoded the 9 or 10 operation-
38 specific strategies into 4 operation-general categories: the traditional algorithm
39 (former category 1), non-traditional strategies (former categories 2 to 6), no written
40 working (former category 7), and other trials (former categories 8 to 10). Figure 3
41 presents the proportion of choice of each of these four strategy types on numerical
42 and contextual problems, per operation.
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50 Next, we estimated a multinomial logistic model for correlated responses
51 using a random effects model (Hartzel, Agresti, & Caffo, 2001) in the SAS procedure
52 NLMIXED, as described by Kuss & McLerran (2007). The main assumption of this
53 model is that the random effects follow a multivariate normal distribution, which is
54 a common assumption in mixed effect models (such as multilevel analysis) that is
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3 hard to test. As predictor variables in the model, we first included only variables on
4 the item level: operation required (nominal variable with 4 categories: addition,
5 subtraction, multiplication, or division; recoded into 3 dummy variables) and
6 problem format (numerical or contextual), and their interaction.
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10 Operation had a highly significant effect (Likelihood Ratio $\chi^2(9) = 3995.4, p <$
11 $.001$) on the distribution of strategies, as is also obvious from Figure 3. However, the
12 main effect of problem format was not significant ($\chi^2(3) = 4.9, p = .18$), and neither
13 was the interaction between problem format and operation ($\chi^2(9) = 15.8, p = .07$).
14 Therefore, we may conclude that the presence of a context did not influence the
15 distribution of solution strategies on any of the four operations.
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22 Final analyses were carried out to assess whether the effect of item format
23 (numerical vs. contextual) depended on gender, language achievement level, or
24 home language. First, the main effects of the student characteristics on strategy
25 choice distribution are reported. The effect of home language was not significant (χ^2
26 $(3) = .4, p = .95$), so students who spoke Dutch at home had the same strategy
27 distribution as students who spoke another language at home. In contrast, gender
28 ($\chi^2(3) = 36.4, p < .001$) as well as language achievement level ($\chi^2(3) = 187.7, p <$
29 $.001$) had a significant effect on strategy distribution. Table 3 shows that girls were
30 more likely than boys to choose the traditional algorithm, and less likely to answer
31 without written working. To visualize the effect of language achievement level, it
32 was recoded into three categories based on the population percentile rank (based
33 on all participants of the End of Primary School Test 2009): low (up to percentile
34 33), medium (percentiles 34 to 66), and high (percentile 67 and higher). Table 3
35 shows that the probability to choose the traditional algorithm increased with higher
36 language level, while the probability of answering without written working as well
37 as choosing one of the other strategies decreased with higher language level.
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51 Finally, the interaction effects between problem format (numerical vs.
52 contextual) on the one hand, and the student characteristics on the other hand, were
53 not significant regarding gender ($\chi^2(3) = .4, p = .95$), home language ($\chi^2(3) = .9, p =$
54 $.82$), or language achievement level ($\chi^2(3) = 2.7, p = .44$). Thus, we may conclude
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3 that the finding that the presence of a context did not affect strategy choice
4 distribution holds for all subgroups of students.
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8 *Strategy accuracy.* Finally, we investigated to what extent the problem format
9 (numerical vs. contextual) affected the accuracy of each of the strategies, i.e., the
10 proportion correct per strategy. To that end, we again used explanatory IRT models
11 (De Boeck & Wilson, 2004). This time, not only predictors on the item level were
12 included, but also the strategy used was included as a person-by-item predictor (see
13 also Hickendorff et al., 2009, 2010). All trials in which the solution strategy was
14 classified in the Other category were excluded from the analyses, because this was a
15 small heterogeneous group of trials with many skipped problems. As a result, we
16 analyzed the accuracy differences between the traditional algorithm, non-traditional
17 strategies, and no written working.
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26 Results showed that these three strategies differed significantly in accuracy
27 ($\chi^2 (2) = 184.6, p < .001$), and that the accuracy differences depended on the
28 operation required ($\chi^2 (6) = 55.0, p < .001$). However, the operation-specific
29 accuracy differences between the strategies did not depend on the item format
30 (numerical vs. contextual), $\chi^2 (8) = 9.0, p = .34$. Figure 4 shows the estimated
31 proportion correct of each strategy for students at the mean of the latent ability
32 scale, by operation.
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39 The following pattern emerged: the traditional algorithm had higher
40 accuracy than the non-traditional strategies, which in turn were more accurate than
41 no written working. Statistical testing of these differences showed the following: the
42 regression parameters of the accuracy difference between the traditional algorithm
43 and non-traditional strategies were $\beta = .32 (p = .02)$, $\beta = .96 (p < .001)$, $\beta = .55 (p <$
44 $.001)$, and $\beta = .10 (p = .60)$, for addition, subtraction, multiplication, and division,
45 respectively. So, on addition, subtraction, and multiplication the traditional
46 algorithm was significantly more accurate than the non-traditional strategies, while
47 the difference was not significant on division. Next, the regression parameters for
48 the accuracy difference between non-traditional strategies and no written working
49 were $\beta = .58 (p < .001)$, $\beta = .09 (p = .67)$, $\beta = .74 (p < .001)$, and $\beta = 1.72 (p < .001)$, for
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3 addition, subtraction, multiplication, and division, respectively. Thus, on addition,
4 multiplication, and division, non-traditional strategies were significantly more
5 accurate than no written working, while the difference was not significant on
6 subtraction. Moreover, it is worth noticing that the estimated accuracy of no written
7 working on division problems was much lower than on the other three operations.
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Discussion

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16 The current study aimed to assess the effects of presenting multidigit
17 arithmetic problems in a realistic context typical for school mathematics, on two
18 aspects of sixth graders' problem solving: performance and solution strategy use.
19 First, regarding performance, multidimensional IRT models showed that the same
20 latent ability dimension was involved in solving numerical and contextual problems.
21 Moreover, explanatory IRT models showed that presenting an arithmetic problem in
22 a context increased the difficulty level of the division problems, but did not affect the
23 difficulty level of addition, subtraction, and multiplication problems. Importantly,
24 these performance effects were independent of students' gender, home language,
25 and language achievement level. Second, the presence of a context did not affect the
26 strategy choice distribution, nor the strategy accuracy, irrespective of students'
27 gender, home language, and language achievement level. In summary, we conclude
28 that, contrary to our expectations, the effects of presenting arithmetic problems in a
29 realistic context typical for school mathematics tests were nonexistent in addition,
30 subtraction, and multiplication, and that only a small difference in problem difficulty
31 was found for division.
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45 Regarding performance, based on earlier research findings (Fuchs et al.,
46 2006, 2008; Hickendorff, in press) we expected that related but separate ability
47 dimensions would be involved in solving the two types of problems. A possible
48 explanation for the difference between the current results and previous findings
49 may lie in the differences between the age groups (first to third graders versus sixth
50 graders). It has been argued that children in higher grades, who have had more
51 years of formal schooling, have more developed cognitive schemata to solve word
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3 problems (De Corte et al., 1985; Vicente et al., 2007). Possibly, these cognitive
4 schemata are so well-developed at grade 6 that students no longer perceive
5 differences between such typical contextual problems and numerical problems. This
6 would result both in indistinguishability of the latent ability dimensions involved, as
7 well as in absence of an effect on problem difficulty (as was also found in another
8 study in sixth grade by Vermeer et al., 2000). Importantly, this pattern held for girls,
9 for students with low language ability level, and for students from non-native origin.
10 It thus seems that, at the end of primary school, these students are not hampered by
11 the verbal nature of the contextual problems in mathematics, which is a very
12 relevant finding for assessment practices.
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22 Furthermore, contextual problems did not elicit different solution strategies,
23 contrary to expectations based on research on word problems with young children
24 (Verschaffel et al., 2000) and on algebra problem solving with older students
25 (Koedinger & Nathan, 2004; Koedinger et al., 2008), as well as to expectations based
26 on RME theory (e.g., Van den Heuvel-Panhuizen et al., 2009). However, the absence
27 of effects on strategy choice is more congruent with the findings from Van Putten et
28 al. (2005), who also found no difference in solution strategy choice on multidigit
29 division problems in grade 4.
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36 In the following, we will address the limitations of this study and the
37 implications of its findings, and we will pay special attention to two interesting
38 patterns found in the present study: one regarding gender differences and the other
39 regarding solution strategy use across the four arithmetic operations.
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Limitations

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46 The major limitations of the current study lie in the problem set. First and
47 foremost, a rather diverse set of problems and contexts was used in order to obtain
48 a set of problems representative of everyday-life school mathematics. This was done
49 because the ecological validity of the study's findings was deemed more important
50 than a systematic investigation of the effects of different problem characteristics,
51 such as number features, semantic structure, and types of contexts. As a
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consequence, however, the conclusions may not generalize beyond the types of typical contextual arithmetic problems used in this study, i.e., one-step problems without redundant information or misleading key words. Furthermore, it was not possible to address the effects of specific characteristics of the context, such as linguistic complexity of the problem text (Abedi & Hejri, 2004; Abedi & Lord, 2001) and the effect of an illustration in the problem (Berends & Van Lieshout, 2009).

In addition, due to practical constraints, the number of problems is rather small, in particular when the results per operation are reported. Therefore, these results must be interpreted with caution and should be replicated in future research. However, given the lack of effects of contexts across operations, we believe that it is a rather robust pattern. Furthermore, we found that the (absence of) context effects persisted across different subgroups of students, further supporting the robustness of the pattern.

Furthermore, two types of problem matching were used. The first type was between contextual problems and their corresponding numerical format. However, this correspondence is not univocal, and the possibility that a different choice of matching numerical problems would result in different performance and/or strategy use could not be ruled out. We tentatively argue, however, that for the age group in the current study it may play a minor role given the students' ample experience with switching the order of the numbers and changing the form of a problem.

The second type of problem matching was the construction of parallel problems, in order to prevent testing effects if students had to solve a problem with the exact same numbers twice (once with, and once without a context). Arguably, the parallel problems were probably not exactly parallel in practice because of their unique numerical features. Fortunately, the counterbalancing of parallel problem version and problem format (contextual versus numerical) statistically controls for these possible differences.

Practical implications

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The results of the current study showed only minor effects of presenting multidigit arithmetic problems in a realistic context at the end of primary school. Because the problems used were taken from the Dutch national assessments at the end of primary school, we tentatively conclude that at least for multidigit addition, subtraction, multiplication, and division problems, the outcomes would have been the same if more or only numerical problems would have been included. This is an important observation, because the mathematics assessments have been criticized for their large demands on language abilities as there are so many verbal problems. In the domain of multidigit arithmetic at the end of primary school, the results of the current study do not seem to support this criticism. Whether these conclusions may be extended to other domains of mathematics (e.g., fractions) and/or other assessments such as TIMSS (grade 4 and grade 8) and PISA (15-year-olds) has to be studied in further research.

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Although the presence of a context appeared not to affect arithmetic problem solving at the end of primary school, this does not mean that the shift towards dominance of contextual problems in mathematics tests and mathematics education is without consequences. Research findings in earlier grades (Fuchs et al., 2006, 2008; Hickendorff, in press) did show differences between mathematics problems with and without a context, and also showed that language ability had a larger effect on solving contextual problems. The present results implied that these differences have diminished and disappeared at the end of primary school. However, this does not preclude that students with low verbal abilities had more difficulties to obtain the same performance level on contextual problem solving as on numerical problem solving.

Gender differences

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Interestingly, girls outperformed boys on the multidigit arithmetic problems, in contrast to the findings of international (PISA, TIMSS) educational assessments. Dutch national assessments, however, reported the trend that multidigit operations were the only domain of mathematics on which boys did not outperform girls, and in the 2004 assessment, girls even showed a small advantage in this domain (J.

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Janssen et al., 2005). A possible explanatory mechanism is differences in strategy use: girls were found to use the more accurate traditional algorithm more often than boys, who in turn were more inclined to answer without written work. This pattern has been reported consistently (Hickendorff et al., 2009, 2010; Hickendorff & Van Putten, 2012) and is also in line with more general research findings concerning gender differences in strategy choice (Carr & Davis, 2001; Carr & Jessup, 1997; Gallagher et al., 2000; Timmermans et al., 2007; Vermeer et al., 2000).

Furthermore, we expected differential effects of the presence of a context for boys and girls, based on findings of Vermeer et al. (2000). This expectation was not confirmed in the current study. A tentative explanatory mechanism may be that grade 6 students (boys and girls) have such well-developed cognitive schemata for solving typical school word problems that they no longer perceive differences in contextual problems and numerical problems. This reasoning is supported by the absence of an effect of the presence of a context for both boys and girls.

Solution strategies for multidigit arithmetic

Although there were no marked effects of the presence of a context on arithmetic problem solving, the present study yields unique empirical data on strategic competence of sixth graders from a reform-based educational environment. That is, strategy use (choice and accuracy) was studied across the four basic operations with multidigit numbers simultaneously, in one common framework. In particular, strategic competence in addition/subtraction on the one hand, and multiplication/division on the other hand, have not previously been studied simultaneously in one study to our knowledge, and several interesting patterns emerged.

First, the dominance of the traditional algorithm decreased from addition (69%) and subtraction (74%) to multiplication (51%) and then again to division (15%). The Dutch national assessments showed a very similar trend in instructional practices: 69%, 72%, 57%, and 17% of the 118 participating grade 6 teachers reported instructing only the traditional algorithm for addition, subtraction,

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3 multiplication, and division, respectively (J. Janssen et al., 2005, p. 44). Therefore, it
4 may be that students' choice for the traditional algorithm is determined to a large
5 extent by the teacher's instruction. The traditional algorithm turned out to yield the
6 highest probability of a correct answer on addition, subtraction, and multiplication,
7 but on division it did not differ in accuracy from the non-traditional strategies
8 (similar to findings of Hickendorff et al., 2009). However, because students were
9 free to choose their solution strategy, strategy accuracy data may be biased by
10 selection effects (cf. Siegler & Lemaire, 1997). That is, different students choose
11 different strategies on different items, thereby affecting the accuracy rates. A
12 possible way to assess strategy accuracy without bias would be to implement the
13 choice/no choice method (Siegler & Lemaire, 1997).
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24 Second, division stood out compared to the other operations in the frequency
25 of use of non-traditional strategies: 56% of all division trials were solved by
26 repeated subtraction. Again, this is in line with the instructional approach in
27 mathematics textbooks and teacher reports, in which repeated subtraction is the
28 dominant strategy. This may also have been reflected in the strategy accuracy: on
29 each operation except division, non-traditional strategies were less accurate than
30 the traditional algorithm.
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37 Third, for each operation, on a substantial percentage of trials (14-17%)
38 students answered without writing down any working. This frequency fell in the
39 range of frequencies reported in previous studies on multidigit multiplication and
40 division (Hickendorff et al., 2009, 2010; Hickendorff & Van Putten, 2012; Van Putten
41 et al., 2005). A previous study showed that it predominantly involved mental
42 computation (i.e., students calculating in their head; Hickendorff et al., 2010).
43 Importantly, it was the least accurate strategy (although the difference with non-
44 traditional strategies was not significant on subtraction), and this is a consistent
45 research finding. Therefore, we plead for more systematic research into this
46 phenomenon, that may yield educational recommendations for instructing students
47 when and when not to use a written strategy.
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SOLVING MULTIDIGIT MATHEMATICS PROBLEMS

Appendix A

The 8 problem pairs in test form A (texts translated from Dutch).

Note. In the Netherlands, a comma is used as decimal separator.

addition problems

A class of students sells postcards and stamps for charity. They sold for € 466,50 on postcards and for € 985 on stamps. How much did they earn?

$$677,50 + 975 =$$

scarf: €

coat: € 298

shirt: €

hat: € 9,95

Here, you see the price tags of the clothes that Francien has bought.
How much did she pay in total?

$$19,95 + 198,50 + 129 + 8,80 =$$

subtraction problems

Mrs. De Vries has € 3010 on her bank account. She withdraws € 689 to buy a new bike.
How much is left on her account?

$$4020 - 787 =$$

DE BANK		BANK STATEMENT	
1 - 1 - 2003		SAVINGS ACCOUNT	BALANCE
			1862,85

DE BANK		BANK STATEMENT	
1 - 1 - 2004		SAVINGS ACCOUNT	BALANCE
			2348,77

de heer/
mevrouw
D.E. SALDO
CENTENLAAN 3
1111 EUROPARK

How much more was there on the account on 1-1-2004 compared to 1-1-2003?

$$3618,88 - 2923,95 =$$

SOLVING MULTIDIGIT MATHEMATICS PROBLEMS

multiplication problems

Charles has to photocopy 36 pages. He needs 27 copies of each page.
How many copies are that in total?

$$37 \times 24 =$$



Mother buys 17 meters of this curtain material.
How much does she have to pay?

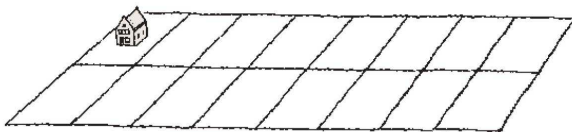
$$26 \times 20,1 =$$

division problems

<i>Invoice for "The Sunflower"</i>			
	number	price per book	total price
history textbooks	32		€ 736

The Sunflower has bought 32 new history textbooks.
How much is the price per book?

$$864 : 36 =$$



From a 5120 m² piece of land, 16 building parcels are made. Each building parcel will have the same area.
What area will each building parcel have?

$$5880 : 14 =$$

THE EFFECTS OF CONTEXTS ON MATHEMATICS PROBLEM SOLVING

Appendix B

Examples of solution strategies; numbers referring to Table 1.

addition	subtraction	multiplication	division	
<p>①</p> $\begin{array}{r} ^1 ^1 \\ 677,50 \\ + 975 \\ \hline 1652,50 \end{array}$	<p>①</p> $\begin{array}{r} ^9 ^{10} \\ 30\cancel{10} \\ - 689- \\ \hline 2321 \end{array}$	<p>①</p> $\begin{array}{r} 36 \\ \underline{27x} \\ 252 \\ 720+ \\ \hline 972 \end{array}$	<p>①</p> $\begin{array}{r} 32/736 \setminus 23 \\ \underline{64} \\ 96 \\ \underline{96} \\ 0 \end{array}$	
<p>②</p> $\begin{array}{r} 677,50 \\ + 975 \\ \hline 1500 \\ 140 \\ 12 \\ \hline 0,50 \\ \hline 1652,50 \end{array}$	<p>②</p> $\begin{array}{r} 3010 \\ - 689- \\ \hline 3000 \\ -600 \\ -70 \\ -9 \\ \hline 2321 \end{array}$	<p>②</p> $\begin{array}{r} 36 \\ \underline{27x} \\ 42 \\ 210 \\ 120 \\ \hline 600+ \\ 972 \end{array}$	<p>②</p> $\begin{array}{r} 736 \\ \underline{640-} \quad 20x \\ 96 \\ \underline{96-} \quad 3x \\ 0 \quad 23x \end{array}$	<p>③</p> $\begin{array}{r} 736 \\ \underline{320-} \quad 10x \\ 416 \\ \underline{320-} \quad 10x \\ 96 \\ \underline{64-} \quad 2x \\ 32 \\ \underline{32-} \quad 1x \\ 0 \quad 23x \end{array}$
<p>③</p> $\begin{aligned} 677,50 + 900 &= 1577,50 \\ 1577,50 + 70 &= 1647,50 \\ 1647,50 + 5 &= 1652,50 \end{aligned}$	<p>③</p> $\begin{aligned} 3010 - 600 &= 2410 \\ 2410 - 80 &= 2330 \\ 2330 - 9 &= 2321 \end{aligned}$	<p>③</p> $\begin{aligned} 36 \times 20 &= 720 \\ 36 \times 7 &= 252 \\ 720 + 252 &= 972 \end{aligned}$		
<p>④</p> $\begin{aligned} 600 + 900 &= 1500 \\ 70 + 70 &= 140 \\ 7,50 + 5 &= 12,50 \\ 1500 + 140 + 12,50 &= \\ &1652,50 \end{aligned}$	<p>④</p> $\begin{aligned} 3000 - 600 &= 2400 \\ 2400 - 80 &= 2320 \\ 2320 + 10 - 9 &= 2321 \end{aligned}$	<p>④</p> $\begin{aligned} 30 \times 20 &= 600 \\ 30 \times 7 &= 210 \\ 6 \times 20 &= 120 \\ 6 \times 7 &= \underline{42+} \\ &972 \end{aligned}$	<p>④</p> $\begin{aligned} 10 \times 32 &= 320 \\ 20 \times 32 &= 640 \\ 3 \times 32 &= 96 \\ 23 \times 32 &= 736 \end{aligned}$	
	<p>⑤</p> $\begin{aligned} 689 + 11 &= 700 \\ 700 + 300 &= 1000 \\ 1000 + 2010 &= 3010 \\ 11 + 300 + 2010 &= 2321 \end{aligned}$	<p>⑤</p> $\begin{array}{r} 2 \times 27 = 54 \\ 27 \quad \quad 4 \times 27 = 108 \\ 27 \quad \quad 8 \times 27 = 216 \\ \dots \quad \quad 16 \times 27 = 432 \\ \underline{27+} \quad \quad 32 \times 27 = 864 \\ 972 \quad \quad 864 + 108 = 972 \end{array}$	<p>⑤</p> $\begin{aligned} 2 \times 32 &= 64 \\ 4 \times 32 &= 128 \\ 8 \times 32 &= 256 \\ 16 \times 32 &= 512 \\ 20 \times 32 &= 640 \\ 22 \times 32 &= 704 \\ 23 \times 32 &= 736 \end{aligned}$	

THE EFFECTS OF CONTEXTS ON MATHEMATICS PROBLEM SOLVING

Footnotes

¹ We defined *word problems* as problems containing only text, while the category *contextual* problems encompasses word problems, but also contains problems that include an illustration that may hold essential information for problem solving. In this study, we focus on the more general contextual problems.

² We cross-checked this latent correlation estimate by using other estimation methods and software. In a first approach, item parameter were estimated for each dimension separately using conditional maximum likelihood (Verhelst & Glas, 1995) and the latent correlation between the dimensions was estimated in a separate step. Second, we used a Bayesian framework: the MIRT models were formulated as normal-ogive instead of logistic models, and parameters were estimated using an MCMC-procedure (see also Albert, 1992 for unidimensional IRT models and Béguin & Glas, 2001 for MIRT models), that was programmed into R (R Development Core Team, 2009)). Finally, we used the NLMIXED-procedure from SAS (see De Boeck & Wilson, 2004 and Hickendorff, in press) to fit the 2PL extension of the MIRT model, allowing for the nonzero discrimination parameters to be different from each other. In all three alternative programs, the latent correlation was estimated at 1.000, so we conclude that it is a robust result.

THE EFFECTS OF CONTEXTS ON MATHEMATICS PROBLEM SOLVING

Table 1

Solution strategies categories.

	addition	subtraction	multiplication	division
1	traditional algorithm	traditional algorithm	traditional algorithm	traditional algorithm
2	RME approach	RME approach	RME approach	repeated subtraction (HL)
3	partitioning 1 operand	partitioning 1 operand	partitioning 1 operand	repeated subtraction (LL)
4	partitioning ≥ 2 operands	partitioning both operands	partitioning both operands	repeated addition (HL)
5	-	indirect addition	repeated addition	repeated addition (LL)
6	other written strategy	other written strategy	other written strategy	other written strategy
7	no written working	no written working	no written working	no written working
8	wrong procedure	wrong procedure	wrong procedure	wrong procedure
9	unclear strategy	unclear strategy	unclear strategy	unclear strategy
10	skipped problem	skipped problem	skipped problem	skipped problem

THE EFFECTS OF CONTEXTS ON MATHEMATICS PROBLEM SOLVING

Table 2

Distribution of solution strategy categories of numerical (num) problems and contextual (context) problems, in proportions per operation. Strategy categories refer to Table 1.

strategy	addition		subtraction		multiplication		division	
	num	context	num	context	num	context	num	context
1 (traditional)	.68	.70	.74	.73	.52	.51	.15	.14
2 *	.09	.09	.01	.01	.03	.02	.53	.50
3 *	.01	.01	.01	.00	.18	.17	.05	.04
4 *	.04	.03	.01	.01	.07	.07	.03	.03
5 *	n.a.	n.a.	.02	.04	.01	.02	.01	.01
6 *	.02	.01	.01	.01	.01	.01	.02	.03
7 (no written)	.14	.14	.17	.17	.16	.16	.14	.17
8 (wrong)	.00	.00	.00	.00	.00	.01	.01	.02
9 (unclear)	.01	.01	.01	.01	.01	.01	.02	.02
10 (skipped)	.01	.00	.01	.01	.01	.02	.04	.04
<i>N</i> observations	1300	1300	1300	1300	1300	1300	1300	1300

* operation-specific strategy categories, see Table 1.

THE EFFECTS OF CONTEXTS ON MATHEMATICS PROBLEM SOLVING

Table 3

Strategy choice distribution (in proportions), by gender and language achievement level.

strategy	gender		language achievement		
	boy	girl	low	medium	high
traditional	.45	.59	.47	.55	.57
non-traditional	.29	.29	.29	.29	.29
no written working	.22	.09	.19	.14	.12
other	.04	.03	.06	.02	.02
<i>N</i> observations	4768	5216	3776	3712	2608

*THE EFFECTS OF CONTEXTS ON MATHEMATICS PROBLEM SOLVING***Figure Captions**

Figure 1. Design of experimental task forms. A = Addition, S = Subtraction, M = Multiplication, and D = Division. Problem indices 1 (small numbers) and 2 (large numbers) denote the specific pair within each operation, indices *a* and *b* denote the two parallel versions within each problem pair. Problems in unshaded cells represent numerical problems, problems in cells shaded grey are the contextual problems.

Figure 2. Graphical representation of between-item two-dimensional IRT model.

Figure 3. Strategy choice proportion of recoded solution strategies on numerical (num) and contextual (context) problems, per operation.

Figure 4. Estimated mean accuracy of the three strategies, by operation.

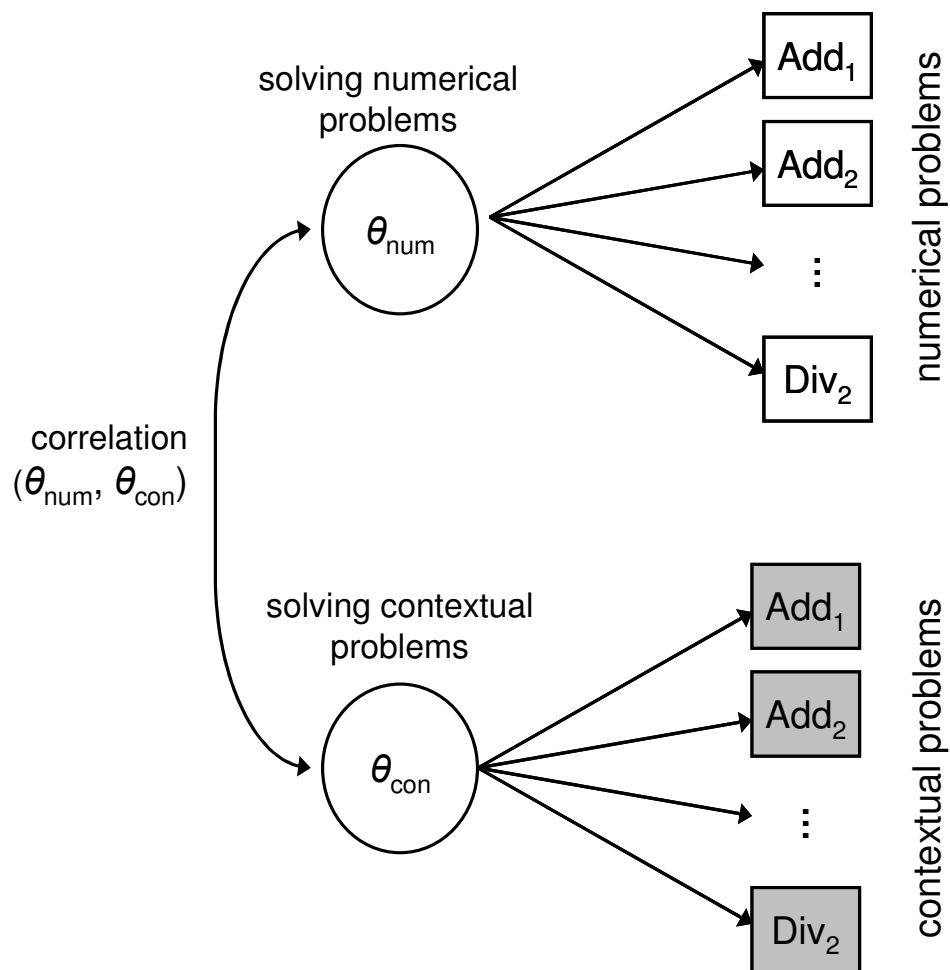
THE EFFECTS OF CONTEXTS ON MATHEMATICS PROBLEM SOLVING

Figure 1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
form A	A _{1b}	S _{2a}	A _{2a}	S _{1b}	M _{1b}	D _{2a}	M _{2a}	D _{1b}	A _{1a}	S _{2b}	A _{2b}	S _{1a}	M _{1a}	D _{2b}	M _{2a}	D _{1b}
form B	A _{1b}	S _{2a}	A _{2a}	S _{1b}	M _{1b}	D _{2a}	M _{2a}	D _{1b}	A _{1a}	S _{2b}	A _{2b}	S _{1a}	M _{1a}	D _{2b}	M _{2a}	D _{1b}

THE EFFECTS OF CONTEXTS ON MATHEMATICS PROBLEM SOLVING

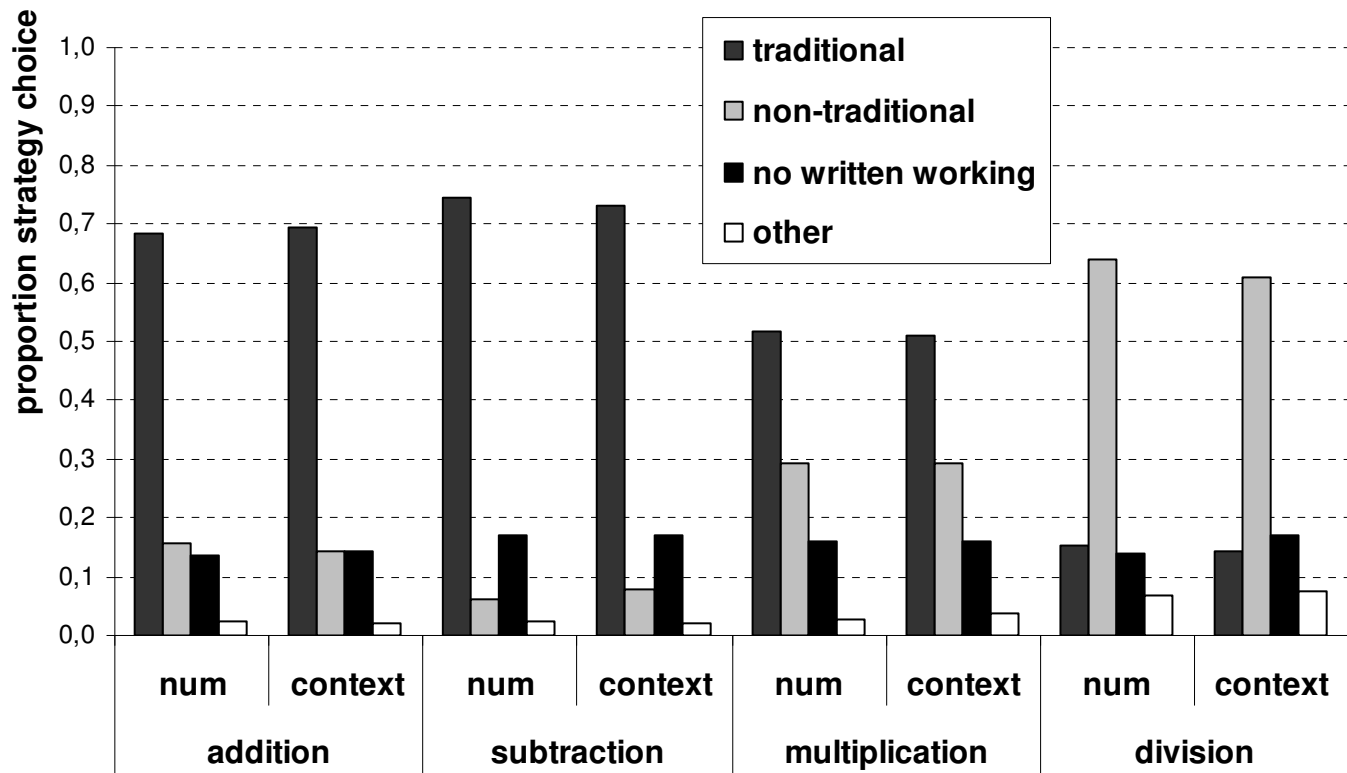
Figure 2



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THE EFFECTS OF CONTEXTS ON MATHEMATICS PROBLEM SOLVING

Figure 3



THE EFFECTS OF CONTEXTS ON MATHEMATICS PROBLEM SOLVING

Figure 4

