The handle http://hdl.handle.net/1887/32963 holds various files of this Leiden University dissertation

**Author:** Zhai, Jiali Teddy  
**Title:** Adaptive streaming applications : analysis and implementation models  
**Issue Date:** 2015-05-13
Chapter 7

Hard Real-time Scheduling of Adaptive Streaming Applications

The initial scheduling framework in the Daedalus\textsuperscript{RT} design flow considers the CSDF MoC as the analysis model and the PPN MoC as the implementation model. For adaptive streaming applications, we have proposed P\textsuperscript{3}N as the implementation model in Chapter 6. However, an analysis MoC for adaptive streaming applications is still missing in Daedalus\textsuperscript{RT}. More importantly, we need proper operational semantics for such a MoC that potentially allows adaptive execution of the MoC and easy HRT analysis. In this chapter, we propose a new analysis MoC in the Daedalus\textsuperscript{RT} design flow that models adaptive streaming applications.

There already exist some adaptive MoCs in literature [89, 114, 129]. Unfortunately, each of them has certain drawback that does not fulfill our needs. For instance, we would like to explicitly have the notion of mode in an adaptive MoC. A mode of an adaptive MoC is essentially a static MoC, e.g., the CSDF MoC, when the values of all dynamic parameters are fixed. As a result, the existing HRT analysis developed in Daedalus\textsuperscript{RT} for the CSDF MoC can be reused. For the adaptive MoCs shown in Figure 1.6 on page 10, parameterized CSDF and VPDF MoCs are thus excluded from our consideration because they do not have the notion of mode. At the same time, the expressiveness of MCDF is too restricted.

Furthermore, support for the HRT scheduling and the associated analysis is limited in the existing MoCs, especially during mode transitions. In particular, we wish to have a composable analysis for mode transitions. That is, the analysis of any mode transition is independent from the mode transitions occurred in past. This composable analysis will significantly reduce the complexity of the analysis, as the complexity merely depends on the number of allowed transitions. This is crucial for applications with a large number of modes and possible transitions. As a by-product
of this composable analysis, the implementation efficiency of such a HRT system to support adaptive behavior will be much higher. No complex calculation is needed at run-time, as most of parameters (see Section 7.3) can be computed at compile-time.

Based on the discussion above, we develop a new MoC, Mode-Aware Data Flow (MADF), in this chapter that has the advantages of SADF and VPDF. Inspired by SADF, we characterize the adaptive application behavior with individual modes\(^1\) (see Definition 7.2.7) and transitions (see Definition 7.2.11) between them. Similar to VPDF, the length of production/consumption sequences for an actor varies from one mode to another. The length is only fixed when the mode is known. Based on the clear distinction between modes and transitions, we define operational semantics, in particular a novel transition protocol, to avoid timing interference between modes and transitions. As a result, our HRT analysis is simpler than the state-of-the-art timing analysis [47]. To ease discussion, we use additional notations listed in Table 7.1 besides the ones introduced in Chapter 2.

### Scope of Work

We assume that an adaptive streaming application does not have cyclic data dependences. The considered MPSoC platforms in this chapter are homogeneous, i.e., they may contain multiple, but the same type of programmable PEs with distributed memories. Moreover, the platform must be predictable, which means timing guarantees are provided on the response time of hardware components and OS schedulers. The precision-timed (PRET) [79] platform is such an example. On the software side, we assume partitioned scheduling algorithms, i.e., no migration of actors between PEs is allowed. The considered scheduling algorithms on each PE include Fixed-Priority Pre-emptive Scheduling (FPPS) algorithms, such as RM [80], or dynamic

\(^1\)“Scenario” for SADF is equivalent to “mode” in our case.

---

<table>
<thead>
<tr>
<th>Notation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c)</td>
<td>a computation</td>
</tr>
<tr>
<td>(\Delta)</td>
<td>transition delay</td>
</tr>
<tr>
<td>(L)</td>
<td>iteration latency</td>
</tr>
<tr>
<td>(p)</td>
<td>a dynamic parameter</td>
</tr>
<tr>
<td>(\Pi)</td>
<td>a set of parameter vectors defined in Definition 7.2.1</td>
</tr>
<tr>
<td>(\psi)</td>
<td>parameters used for actors defined in Definitions 7.2.4 and 7.2.5</td>
</tr>
<tr>
<td>(x)</td>
<td>Maximum-Overlap Offset (MOO)</td>
</tr>
</tbody>
</table>

Table 7.1: Additional notations used in Chapter 7 besides the ones introduced in Chapter 2.
scheduling algorithms, such as EDF [80].

7.1 Related Work

For FSM-SADF [47], the authors proposed an approach to compute worst-case performance among all mode transitions, assuming the self-timed transition protocol (explained later in Section 7.2.3). Although it is an exact analysis, the approach has inherently exponential time-complexity. Moreover, the approach leads to timing interference between modes upon mode transitions, which makes this approach not applicable for our problem. In contrast, our approach does not introduce interference between modes due to the novel transition protocol proposed in Section 7.2.3. The timing behavior of individual modes and during mode transitions can be analyzed independently. In addition, our approach considers allocation of actors on PEs, which by itself is a harder problem than the one in [47]. In [48], the authors proposed to model scenario transitions in a single FSM. Delays due to scenario reconfiguration are given and explicitly modeled in the FSM. The problem addressed in this chapter is different as we aim at deriving such a delay.

In [45], the author proposes to use a linear model to capture worst-case transition delay and period during scenario transitions of FSM-SADF. Our Maximum-Overlap Offset (MOO, see Section 7.2.3) transition protocol is conceptually very similar to the linear model. However, we obtain the linear model in a different way, specifically simplified for the adopted hard real-time scheduling framework. For instance, finding a reference schedule is not necessary in our case, but being crucial in the tightness of the analysis proposed in [45]. Moreover, our approach solves the problem of changing graph structure during mode transitions, which was not studied in [45].

For VPDF [129], the analysis has been limited to computing buffer sizes under throughput constraints so far. The execution of a VPDF graph on MPSoC platforms under HRT constraints has not been studied. In particular, the allocation of actors and how to switch from one mode to another one are not discussed. Moreover, delay due to mode transitions has not been investigated. Our approach, on the other hand, takes these important factors into account. Therefore, our analysis results are directly reflected in a real implementation.

Mode-controlled data flow (MCDF) [89] is another adaptive MoC whose properties can be partly analyzed at compile-time. The MCDF MoC primarily focuses on SDR applications, where different sub-graphs need to be active in different modes. This is achieved by using switch and select actors. The author implicitly assumes self-timed scheduling during mode transitions. Based on this assumption, a worst-case timing analysis is developed. Similar to the case of SADF, use of the self-timed scheduling introduces timing interference between modes. As a consequence, the
analysis must take into account the sequence of mode transitions of interest. Although the author provides an upper bound of timing behavior for a parameterized sequence of mode transitions, the accuracy is still unknown. In contrast, our approach results in a timing analysis of mode transitions that is independent from already occurred transitions. Moreover, the analysis results are directly reflected in the final implementation. In this sense, our analysis is exact in the timing behavior of mode transitions.

In [93], an analysis is proposed to reason about worst-case response time of a task graph in case of mode change. However, the task graph has very limited expressiveness and is not able to model adaptive application behavior. In this chapter, we define a more expressive MoC that is amenable to adaptive application behavior.

In [104, 108], the authors focus on timing analysis for mode changes of real-time tasks. The starting times of new mode tasks need to be delayed to avoid overloading PEs. The algorithms to compute the starting times were provided. Both works are related to ours because actors allocated on the same PE may also overload the PE after mode transitions. In this case, the starting times of actors in the new mode need to be delayed. In [104, 108], it was assumed that tasks are independent. The proposed algorithms are thus not applicable to the adaptive MoCs, since the starting times of actors in the adaptive MoCs depend on each other due to data dependencies. Moreover, the algorithms in [104, 108] involve high computational complexity because fixed-point equations must be solved at every step in the algorithms.

7.2 Model Definition

7.2.1 Mode-Aware Data Flow (MADF)

Definition 7.2.1 (Mode-Aware Data Flow (MADF)). A Mode-Aware Data Flow (MADF) is a multi-graph defined by a tuple \((\mathcal{A}, \mathcal{A}_c, \mathcal{E}, \Pi)\), where

- \(\mathcal{A} = \{A_1, \ldots, A_{|\mathcal{A}|}\}\) is a set of dataflow actors;
- \(\mathcal{A}_c\) is the control actor to determine modes and their transitions;
- \(\mathcal{E}\) is the set of edges for data/parameter transfer;
- \(\Pi = \{\vec{p}_1, \ldots, \vec{p}_{|\mathcal{A}|}\}\) is the set of parameter vectors, where each \(\vec{p}_i \in \Pi\) is associated with a dataflow actor \(A_i\).

Throughout this section, we use graph \(G_1\) shown in Figure 7.1 as the running example to illustrate the definition of MADF and the hard real-time scheduling analysis related to MADF. For \(G_1\), \(\mathcal{A} = \{A_1, A_2, A_3, A_4, A_5\}\) is the set of dataflow
actors. $A_c$ is the control actor. $\mathcal{E} = \{E_1, E_2, E_3, E_4, E_5, E_6, E_{11}, E_{22}, E_{44}, E_{55}\}$ is the set of edges. For actor $A_5$, $\bar{p}_5 = [p_5, p_6]$ is the parameter vector. The input port $IP_1$ of actor $A_5$ has a consumption sequence $[1[p_3], 1[0]]$, which can be interpreted as $[p_5, 0]$.

**Definition 7.2.2 (Dataflow Actor).** A dataflow actor $A_i$ is described by a tuple $(\mathcal{I}_i, IC_i, \mathcal{O}_i, C_i, M_i)$, where

- $\mathcal{I}_i = \{IP_1, \ldots, IP_{|\mathcal{I}_i|}\}$ is the set of data input ports of actor $A_i$;

- $IC_i$ is the control input port that reads parameter vector $\bar{p}_i$ for actor $A_i$;

- $\mathcal{O}_i = \{OP_1, \ldots, OP_{|\mathcal{O}_i|}\}$ is the set of data output ports of actor $A_i$;

- $C_i = \{c_1, \ldots, c_{|C_i|}\}$ is the set of computations. When actor $A_i$ fires, it performs a computation $c_k \in C_i$;

- $M_i : \bar{p}_i \rightarrow \{\phi, \bar{C}_i\}$ is a mapping relation, where $\bar{p}_i \in \Pi$, $\phi \in \mathbb{N}^+$, and $\bar{C}_i \subseteq C_i$ is a sequence of computations $[\bar{C}_i(1), \ldots, \bar{C}_i(k), \ldots, \bar{C}_i(\phi)]$ with $\bar{C}_i(k) \in C_i, 1 \leq k \leq \phi$.

Actor $A_2$ in Figure 7.1 has a set of one input port $\mathcal{I}_2 = \{IP_1\}$, a set of one output port $\mathcal{O}_2 = \{OP_1\}$ as well as a control input port $IC_2$. A set of computations $C_2 = \{c_1, c_2, c_3\}$ is associated with $A_2$. The mapping relation $M_2$ is given in Table 7.2. It can be interpreted as follows: If $p_2 = 2$, actor $A_2$ repetitively performs computations according to sequence $\bar{C}_2 = [c_1, c_2]$ every time when firing $A_2$. When $p_2 = 1$, firing $A_2$ performs computation $c_3$. 

Figure 7.1: An example of MADF graph ($G_1$).
Table 7.2: Mapping relation $M_2$ for actor $A_2$ in Figure 7.1.

<table>
<thead>
<tr>
<th>$\vec{p}_2 = [p_2]$</th>
<th>$\phi$</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>$[c_1, c_2]$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>$[c_3]$</td>
</tr>
</tbody>
</table>

Table 7.3: Function $MC_5$ defined for actor $A_5$ in Figure 7.1.

<table>
<thead>
<tr>
<th>$S$</th>
<th>$\mathbb{N}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SI^1$</td>
<td>$[2, 0]$</td>
</tr>
<tr>
<td>$SI^2$</td>
<td>$[1, 1]$</td>
</tr>
</tbody>
</table>

**Definition 7.2.3 (Control Actor).** The control actor $A_c$ is described by a tuple $(IC, O_c, S, M_c)$, where

- $S = \{SI^1, \ldots, SI^{|S|}\}$ is a set of mode identifiers, each of which specifies a unique mode;
- $IC$ is the control input port which is connected to the external environment. Mode identifiers are read through the control input port from the environment;
- $O_c = \{OC_1, \ldots, OC_{|A|}\}$ is a set of control output ports. Parameter vector $\vec{p}_i$ is sent through $OC_i \in O_c$ to actor $A_i$;
- $M_c = \{MC_1, \ldots, MC_{|A|}\}$ is a set of functions defined for each actor $A_i \in A$. For each $MC_i \in M_c$, $MC_i : S \rightarrow \mathbb{N}^{|\vec{p}_i|}$ is a function that takes a mode identifier and outputs a vector of non-negative integer values.

For $G_1$ in Figure 7.1, we have two mode identifiers $S = \{SI^1, SI^2\}$. At run-time, control actor $A_c$ reads these mode identifiers through control port $IC$ (black dot in Figure 7.1). For actor $A_5$, $MC_5 \in M_c$ is given in Table 7.3. As explained previously, the parameter vector for actor $A_5$ is $\vec{p}_5 = [p_5, p_6]$. Therefore, $MC_5$ takes a mode identifier and outputs a 2-dimensional vector as shown in the second column in Table 7.3. For instance, mode $SI^1$ results in a non-negative integer vector $[2, 0]$.

To further define production/consumption sequences with variable length, we use the notation $n[m]$ for a sequence of $n$ elements with integer value $m$, i.e.,

$$n[m] = [\overbrace{m, \ldots, m}^{n \text{ times}}]$$
Definition 7.2.4 (Input Port). An input port $IP$ of an actor is described by a tuple $(CNS, M_{IP})$, where

- $CNS = [\phi_1[cns_1], \ldots, \phi_K[cns_K]]$ is the consumption sequence with $\phi$ phases, where $\phi = \sum_{i=1}^{K} \phi_i$ is determined by the mapping relation $M$ in Definition 7.2.2, and $cns_1, \ldots, cns_K \in \mathbb{N}$;

- $M_{IP} : \vec{p}_i \rightarrow \psi_{IP}$ is a mapping relation, where $\vec{p}_i \in \Pi$ and

$$\psi_{IP} = \{\phi_1, \ldots, \phi_K, cns_1, \ldots, cns_K\}. \quad (7.1)$$

Definition 7.2.5 (Output Port). An output port $OP$ of an actor is described by a tuple $(PRD, M_{OP})$, where

- $PRD = [\phi_1[prd_1], \ldots, \phi_K[prd_K]]$ is the production sequence with $\phi$ phases, where $\phi = \sum_{i=1}^{K} \phi_i$ is determined by the mapping relation $M$ in Definition 7.2.2, and $prd_1, \ldots, prd_K \in \mathbb{N}$.

- $M_{OP} : \vec{p}_i \rightarrow \psi_{OP}$ is a mapping relation, where $\vec{p}_i \in \Pi$ and

$$\psi_{OP} = \{\phi_1, \ldots, \phi_K, prd_1, \ldots, prd_K\}. \quad (7.2)$$

The consumption/production sequence defined here is a generalization of that for the CSDF MoC (see Section 2.2.2 on page 32). We can see that a CSDF actor has a constant $\phi$ phases in its consumption/production sequences, whereas the length of the phase of an MADF actor is parameterized by $\phi = \sum_{i=1}^{K} \phi_i$. In addition, the mapping relation $M_{IP}/M_{OP}$ must be provided by the application designer. Consider the two input ports $IP_1$ and $IP_2$ of actor $A_5$ in Figure 7.1. The mapping relations $M_{IP_1}$ and $M_{IP_2}$ are represented as follows:

$$M_{IP_1} : \vec{p}_5 = [p_5, p_6] \rightarrow \psi_{IP_1} = \{\phi_1, \phi_2, cns_1, cns_2\} = \{1, 1, p_5, 0\}, \quad (7.3)$$

$$M_{IP_2} : \vec{p}_5 = [p_5, p_6] \rightarrow \psi_{IP_2} = \{\phi_1, \phi_2, cns_1, cns_2\} = \{1, 1, 0, p_6\}. \quad (7.4)$$

It can be seen that parameter $p_5$ is mapped to $cns_1$ of $IP_1$, parameter $p_6$ is mapped to $cns_2$ of $IP_2$, and $\phi_1$ and $\phi_2$ both are constant equal to 1. Therefore, the consumption sequence of $IP_1$ is $CNS = [1[p_5], 1[0]]$ and the consumption sequence of $IP_2$ is $CNS = [1[0], 1[p_6]]$. Similarly considering output port $OP_1$ of actor $A_4$, its mapping relation $M_{OP_1}$ is given as:

$$M_{OP_1} : \vec{p}_4 = [p_4] \rightarrow \psi_{OP_1} = \{\phi_1, prd_1\} = \{1, p_4\}. \quad (7.5)$$

In this case, parameter $p_4$ is mapped to $prd_1$, and $\phi_1 = 1$. Therefore, production sequence $PRD = [1[p_4]]$ is obtained for $OP_1$ of $A_4$. 
Definition 7.2.6 (Edge). An edge \( E \in \mathcal{E} \) is defined by a tuple

\[
(A_i, OP, A_j, IP),
\]

where

- actor \( A_i \) produces a parameterized number of tokens to edge \( E \) through output port \( OP \);
- actor \( A_j \) consumes a parameterized number of tokens from \( E \) through input port \( IP \).

Considering edge \( E_5 \) in Figure 7.1, it connects output port \( OP_1 \) of actor \( A_4 \) to input port \( IP_2 \) of actor \( A_5 \).

Definition 7.2.7 (Mode of MADF). A mode \( SI^i \) of MADF is a live CSDF [30] graph, denoted as \( G^i \), obtained by setting values of \( \Pi \) in Definition 7.2.1 as follows:

\[
\forall k \in \Pi : \bar{p}_k = MC_k(SI^i), \tag{7.6}
\]

where function \( MC_k \) is given in Definition 7.2.3.

Definition 7.2.8 (Mode of MADF Actor). An actor \( A_k \) in mode \( SI^i \), denoted by \( A^i_k \), is a CSDF [30] actor obtained from \( A_k \) as follows:

\[
\bar{p}_k = MC_k(SI^i). \tag{7.7}
\]

Figure 7.2(a) shows the CSDF graph in mode \( SI^1 \) and Figure 7.2(b) shows the CSDF graph in mode \( SI^2 \). Consider function \( MC_5 \) for actor \( A_5 \) in Table 7.3 with parameter vector \( \bar{p}_5 = [p_5, p_6] \). For instance, mode \( SI^1 \) results in \( \bar{p}_5 = [p_5, p_6] = [2, 0] \), where parameter values \( p_5 = 2 \) and \( p_6 = 0 \). Consequently, according to mapping relations \( M_{IP_1} \) and \( M_{IP_2} \) given in Equations 7.3 and 7.4, \( cns_1 = p_5 = 2 \) can be obtained for input port \( IP_1 \) and \( cns_2 = p_6 = 0 \) for \( IP_2 \). This determines actor \( A_5^1 \) shown in Figure 7.2(a) for mode \( SI^1 \).

Definition 7.2.9 (Inactive Actor). An MADF actor \( A^k_i \) is inactive in mode \( SI^k \) if the following conditions hold:

1. \( \forall IP \in \mathcal{I}_i : CNS = [0, \ldots, 0] \);
2. \( \forall OP \in \mathcal{O}_i : PRD = [0, \ldots, 0] \).

Otherwise, \( A^k_i \) is called active in mode \( SI^k \).

For actor \( A_4^1 \) shown in Figure 7.2(a), it has consumption and production sequence \([0]\). Therefore, actor \( A_4 \) is said to be inactive in mode \( SI^1 \).
7.2. Model Definition

During execution of a MADF graph, it can be either in a steady-state or mode transition.

Definition 7.2.10 (Steady-state). A MADF graph is in a steady-state of a mode $SI_i$, if it satisfies Equation (7.6) with the same $SI_i$ for all its actors.

Definition 7.2.11 (Mode Transition). A MADF graph is in a mode transition from mode $SI_o$ to $SI_l$, where $o \neq l$, if some actors have $SI_o$ for Equation (7.7) and the remaining active actors have $SI_l$ for Equation (7.7).

In the steady-state of a MADF graph, all active actors execute in the same mode. As defined previously in Definition 7.2.7 and shown in Figure 7.2(a) and Figure 7.2(b), the steady-state of the MADF graph has the same operational semantics as a CSDF [30] graph. We use $A_{ik}^x$ to denote the $x$th firing of actor $A_i$ in mode $SI_k$. At $A_{ik}^x$, it executes computation

$$\tilde{C}_i((x - 1) \mod \phi + 1),$$
where \( \bar{C}_i \) is given in Definition 7.2.2. The number of tokens consumed and produced are specified according to Definitions 7.2.4 and 7.2.5, respectively. For instance, the \( x \)th firing of \( A^k_i \) produces the following number of tokens through an output port \( OP \):

\[
PRD((x - 1) \mod \phi + 1).
\]

In each mode \( SI^k \), the MADF graph is a live CSDF graph and thus has the notion of graph iterations with a non-trivial repetition vector \( \vec{q}_k^s \in \mathbb{N}_{|A|} \) resulting from Equation (2.14) on page 32. Next, we further define mode iterations.

**Definition 7.2.12 (Mode Iteration).** One iteration \( It^k \) of a MADF graph in mode \( SI^k \) consists of one firing of control actor \( A_c \) and \( q^k_i \in \vec{q}_k^s \) firings of each MADF actor \( A^k_i \).

Consider the two modes shown in Figure 7.2(a) and Figure 7.2(b). Repetition vectors \( \vec{q}^1 \) and \( \vec{q}^2 \) are:

\[
\vec{q}^1 = [4, 2, 2, 0, 2], \\
\vec{q}^2 = [2, 1, 1, 1, 2].
\] (7.8)

For any mode of a MADF graph, i.e., a live CSDF graph, under any valid schedule, it has (eventually) periodic execution in time. This holds for CSDF graphs under self-timed schedule [110], K-periodic schedule [31], and SPS [22]. The length of the periodic execution, called iteration period, determines the minimum time interval to complete one graph iteration (cf. Definition 7.2.12). The iteration period, denoted by \( H^k \), is equal for any actor in the same mode \( SI^k \). During a periodic execution, the starting time of each actor \( A^k_i \), denoted by \( S^k_i \), indicates the time distance between the start of source actor \( A^k_{src} \) and the start of actor \( A^k_i \) in the same iteration period. Based on the notion of starting times, we define iteration latency \( L^k \) of a MADF graph in mode \( SI^k \) as follows:

\[
L^k = S^k_{snk} - S^k_{src},
\] (7.9)

where \( S^k_{snk} \) and \( S^k_{src} \) are the earliest starting times of the sink and source actors, respectively. Figure 7.3 illustrates the execution of both modes \( SI^1 \) and \( SI^2 \) given in Figure 7.2 under the self-timed schedule. A rectangle denotes WCET of an actor firing. The WCETs of all actors in both modes are given in the third row of Table 7.4 on page 132. Now, it can be seen in Figure 7.3 that iteration period \( H^1 = H^2 = 8 \). Based on the starting time of each actor, we obtain iteration latencies \( L^1 = S^1_{snk} - S^1_{src} = 10 - 0 = 10 \) and \( L^2 = S^2_{snk} - S^2_{src} = 10 - 0 = 10 \) as shown in Figure 7.3.
## 7.2. Model Definition

### 7.2.3 Mode Transition

While the operational semantics of a MADF graph in steady-state are the same as that of a CSDF [30] graph, the transition of MADF graph from one mode to another is the crucial part that makes it fundamentally different from CSDF. The protocol for mode transitions has strong impact on the compile-time analyzability and implementation efficiency. In this section, we propose a novel and efficient protocol of mode transitions for MADF graphs.

During execution of a MADF graph, mode transitions may be triggered at runtime by receiving a Mode Change Request (MCR) from the external environment. We first assume that a MCR can be only accepted in the steady-state of a MADF graph, not in an ongoing mode transition. This means that any MCR occurred during an ongoing mode transition will be ignored. Consider a mode transition from $S_I^o$ to $S_I^l$. The transition is accomplished by the control actor reading mode identifier $S_I^l$ from its control input port (see the black dot in Figure 7.1) and writing parameter values of $\vec{p}_i$ to the control output port connected to each dataflow actor $A_i^l$ according to function $MC_i$ given in Definition 7.2.3. Then, $A_i^l$ reads new parameter values $\vec{p}_i$ from its control input port and sets the sequence of computations according to mapping relation $M_i$ in Definition 7.2.2. The production and consumption sequences are obtained in accordance with $M_{IP}$ and $M_{OP}$ in Definition 7.2.4 and Definition 7.2.5, respectively. Similar to the P$^3$N MoC, we further define that mode transitions are only allowed at quiescent points [94].

**Definition 7.2.13** (Quiescent Point of MADF). For a transition from mode $S_I^o$ to $S_I^l$, a quiescent point of a MADF actor $A_i$ is a firing $(A_i^l, x)$ in a mode iteration $It^l$
that satisfies
\[ \neg \exists (A^i, y) \in \text{It}^i : y < x. \]  

(7.10)

Figure 7.4 shows an execution of \( G_1 \) in Figure 7.1 with two mode transitions. For instance, the MCR at time \( t_{MCR1} = 1 \) denotes a transition request from mode \( S^2 \) to \( S^1 \). The mode transition of actor \( A_1 \) is only allowed at the quiescent point (time 2 in Figure 7.4) right before the first firing in mode iteration \( \text{It}^1 \) of mode \( S^1 \).

Definition 7.2.13 defines mode transitions of MADF graphs as a partially ordered actor firings. However, it does not specify at which time instance a mode transition actually starts. Therefore, below, we focus on the transition protocol that defines the points in time for occurrences of mode transitions. To quantify the transition protocol, we introduce a metric, called transition delay, to measure the responsiveness of a protocol to a MCR.

**Definition 7.2.14 (Transition Delay).** For a MCR at time \( t_{MCR} \) calling for a mode transition from mode \( S^o \) to \( S^l \), the transition delay \( \Delta^{o \rightarrow l} \) of a MADF graph is defined as

\[ \Delta^{o \rightarrow l} = \sigma_{\text{snk}}^{o \rightarrow l} - t_{MCR}, \]  

(7.11)

where \( \sigma_{\text{snk}}^{o \rightarrow l} \) is the earliest starting time of the sink actor in the new mode \( S^l \).

In Figure 7.4, we can compute the transition delay for \( MCR1 \) occurred at time \( t_{MCR1} = 1 \) as \( \Delta^{2 \rightarrow 1} = 18 - 1 = 17 \).

**Self-timed (ST) Transition Protocol**

In the existing adaptive MoCs like FSM-SADF [47], a protocol, referred here as Self-Timed (ST) transition protocol, is adopted. The ST protocol specifies that actors are scheduled in the self-timed manner not only in the steady-state, but also during a mode transition. For FSM-SADF upon a MCR, a firing of a FSM-SADF actor in the new mode can start immediately after the firing of the actor completes the old mode iteration. The only possible delay is introduced due to availability of input data. One reason behind the ST protocol is that the ST schedule for a (C)SDF graph (steady-state of FSM-SADF) leads to its highest achievable throughput. However, the ST protocol generally introduces interference of one mode execution with another one. The time needed to complete mode transitions also fluctuate as the transition delay of an ongoing transition depends on the transitions occurred in the past. We consider this as an undesired effect because mode transitions using the ST protocol become potentially slow and unpredictable. Another consequence of the

---

2The steady-state of SADF is defined similarly to that of MADF. The only difference is that a scenario of FSM-SADF is a SDF graph, whereas a mode of MADF is a CSDF graph.
incurred interference between modes using the ST transition protocol is the high time complexity of analyzing transition delays, because transition delays cannot be analyzed independently for each mode transition. The analysis proposed in [47] uses an approach based on state-space exploration, which has the exponential time complexity.

Consider $G_1$ in Figure 7.1 and an execution of $G_1$ with the two mode transitions illustrated in Figure 7.4. The execution is assumed under the ST schedule for both steady-state and mode transitions of $G_1$. After $MCR1$ at time $t_{MCR1}$, the transition from mode $SI^2$ to $SI^1$ introduces interference to execution of the new mode $SI^1$ from execution of the old mode $SI^2$. The interference increases the iteration latency of the new mode $SI^1$ to $L^1 = S_5^1 - S_1^1 = 18 - 2 = 16$ from initially 10 as shown in Figure 7.3(a) when $G_1$ is only executed in the steady-state of mode $SI^1$. Even worse, the interference is further propagated to the second mode transition after $MCR2$ at time $t_{MCR2}$. In this case, the iteration latency $L^2 = S_5^2 - S_1^2 = 42 - 23 = 19$ is increased from initially 10 as shown in Figure 7.3(b) when $G_1$ is only executed in the steady-state of mode $SI^2$. This example thus clearly shows the problem of the ST protocol. That is, it introduces interference between the old and new modes due to mode transitions, thereby increasing the iteration latency of the new mode in the steady-state after the transition. Furthermore, the increase of iteration latency also potentially increases transition delays as it will be shown in the next section.

Maximum-Overlap Offset (MOO) Transition Protocol

To address the problem of the ST transition protocol explained above, we introduce in this chapter our new transition protocol, called Maximum-Overlap Offset (MOO).
Figure 7.5: An illustration of the Maximum-Overlap Offset (MOO) calculation.

**Definition 7.2.15 (Maximum-Overlap Offset (MOO)).** For a MADF graph and a transition from mode $S^o$ to $S^l$, Maximum-Overlap Offset (MOO), denoted by $x$, is defined as

$$x = \begin{cases} 
\max_{A_i \in A^o \cap A^l} (S^o_i - S^l_i) & \text{if } \max_{A_i \in A^o \cap A^l} (S^o_i - S^l_i) > 0 \\
0 & \text{otherwise,}
\end{cases} \quad (7.12)$$

where $A^o \cap A^l$ is set of actors active in both modes $S^o$ and $S^l$.

Basically, we first assume that the new mode $S^l$ starts immediately after the source actor $A^o_{src}$ of the old mode $S^o$ completes its last iteration $H^o$. All actors $A^l_i$ of the new mode execute according to the earliest starting times $S^l_i$ and iteration period $H^l$ in the steady-state. Under this assumption, if the execution of the new mode overlaps with the execution of the old mode in terms of iteration periods $H^o$ and $H^l$, we then need to offset the starting time of the new mode by the maximum overlap among all actors. In this way, the execution of the new mode will have the same iteration latency as that of the new mode in the steady-state, i.e., no interference between the execution of both old and new modes.

Consider MCR1 at time $t_{MCR1}$ shown in Figure 7.4. Obtaining MOO $x$ is illustrated in Figure 7.5. We first assume that the new mode $S^l$ starts at the time when the source actor $A^2$ completes the last iteration at time 8 (see bold, dashed line in Figure 7.5). Actors $A^l_i$ in the new mode start as if they executed in the steady-state of mode $S^l$. Then, we can see that, for actor $A_3$, the execution of $A^l_3$ in
the new mode $S_{1}^{1}$ according to $S_{3}^{1}$ in Figure 7.3(a) overlaps 4 time units (solid bar in Figure 7.5) with the execution of $A_{3}^{2}$ in the old mode $S_{2}^{2}$ in terms of iteration periods $H_{2}$ and $H_{1}$. This is also the maximum overlap between the execution of actors in modes $S_{2}^{2}$ and $S_{1}^{1}$. According to Definition 7.2.15, $x$ can be obtained through the following equations:

\[
\begin{align*}
S_{1}^{2} - S_{1}^{1} &= 0 - 0, \\
S_{2}^{2} - S_{2}^{1} &= 1 - 1 = 0, \\
S_{3}^{2} - S_{3}^{1} &= 9 - 5 = 4, \\
S_{5}^{2} - S_{5}^{1} &= 10 - 10 = 0.
\end{align*}
\]

Therefore, it results in an offset $x = \max(0, 0, 4, 0) = 4$ to the start of mode $S_{1}^{1}$ and is shown in Figure 7.6. The starting time of the new mode $S_{1}^{1}$, namely the source actor $A_{1}^{1}$, must be first delayed to the time when $A_{1}^{2}$ completes the iteration period $H_{2}$ in the last iteration, namely time 8 as shown as the first bold line in Figure 7.6. In addition, the MOO $x = 4$ must be further added to the starting time of $A_{1}^{1}$ (the second bold line in Figure 7.6). Figure 7.6 also shows another transition from mode $S_{1}^{1}$ to $S_{2}^{2}$ with a MCR occurred at time $t_{MCR2} = 23$. The starting time of the source actor $A_{1}^{1}$ in the new mode $S_{2}^{2}$ must be first delayed to the time 28 (the third bold line in Figure 7.6), namely the time when $A_{1}^{1}$ completes the last iteration in the old mode.
\[ S^1 \]. To calculate the MOO \( x \) for this transition, the following equations hold:

\[
\begin{align*}
S_1^1 - S_1^2 &= 0 - 0, \\
S_2^1 - S_2^2 &= 1 - 1 = 0, \\
S_3^1 - S_3^2 &= 5 - 9 = -4, \\
S_5^1 - S_5^2 &= 10 - 10 = 0.
\end{align*}
\]

Thus, the equations above result in \( x = \max(0, 0, -4, 0) = 0 \). For this transition, the new mode \( S^2 \) starts at time 28 as shown in Figure 7.6.

The MOO protocol offers several advantages over the ST protocol. Essentially, the MOO protocol retains the iteration latency of the MADF graph in the new mode the same as the initial value, thereby avoiding the interference between the old and new modes. For instance, after \( MCR1 \) and \( MCR2 \) in Figure 7.6, mode \( S^1 \) and \( S^2 \) still have the initial iteration latency \( L^1 = 10 \) and \( L^2 = 10 \) as shown in Figure 7.3. Therefore, efficiently computing the starting time of MADF actors in the new mode becomes feasible and it plays an important role in deriving a hard-real time schedule for the MADF actors. As a result, analysis of the worst-case transition delay is much simpler (see Theorem 7.3.1) than that of the ST protocol, because the transition delay does not depend on the order of the transitions occurred previously.

Concerning the transition delay, it may be the case that the MOO protocol results in initially longer transition delay than the ST protocol does due to the offset given in Definition 7.2.15. For \( MCR1 \) occurred at time \( t_{MCR1} \), the transition delay of the MOO protocol is \( \Delta^{2 \rightarrow 1} = 22 - 1 = 21 \) as shown in Figure 7.6, whereas the transition delay of the ST protocol is equal to \( \Delta^{2 \rightarrow 1} = 18 - 1 = 17 \) as shown in Figure 7.4. On the other hand, let us consider the same transition request \( MCR2 \) occurred at time \( t_{MCR2} = 23 \) shown in Figures 7.4 and Figure 7.6. For \( MCR2 \), the ST protocol results in transition delay \( \Delta^{1 \rightarrow 2} = 42 - 23 = 19 \) as shown in Figure 7.4. In contrast, the transition delay for the MOO protocol is \( \Delta^{1 \rightarrow 2} = 38 - 23 = 16 \) as shown in Figure 7.6. The MOO protocol could provide shorter transition delay than the ST protocol, thereby faster responsiveness to a mode transition.

### 7.3 Hard real-time Scheduling of MADF

Based on the proposed MOO protocol for mode transitions, in this section, we propose a HRT scheduling framework for MADF. We further show an analysis technique for mode transitions in MADF to reason about transition delays, such that timing constraints can be guaranteed. The HRT scheduling framework for acyclic MADF graphs is an extension of the SPS [22] framework initially developed for acyclic CSDF graphs.
As explained in Section 2.3, the key concept of the SPS framework is to derive a periodic taskset representation for a CSDF graph. Since the steady-state of a mode can be considered as a CSDF graph according to Definitions 7.2.7 and 7.2.10, it is thus straightforward to represent the steady-state of a MADF graph as a periodic taskset and schedule the resulting taskset using any well-known HRT scheduling algorithm. Using the SPS framework, we can derive the two main parameters for each MADF actor in mode $SI^k$, namely the period ($T^k_i$ in Equation (2.16) on page 34) and the earliest starting time ($S^k_i$ in Equation (2.17) on page 35). Under SPS, the iteration period in mode $SI^k$ is obtained as $H^k = q^k_i T^k_i$, $\exists A^k_i \in A$. Below, we focus on determining the earliest starting time of each actor in the new mode upon a transition. From the earliest starting time, we can reason about the transition delay to quantify the responsiveness of a transition.

Upon a MCR, a MADF graph can safely switch to the new mode if all of its actors have completed their last iteration in the old mode. In this case, the firings of MADF actors in the new mode do not overlap with the firings of actors in the old mode. This is called synchronous protocol $[104]$ in real-time systems with mode change. One of its advantages is the simplicity, i.e., the synchronous protocol does not require any schedulability test at both compile-time and run-time. However, other protocols lead to earlier starting times than the synchronous protocol. Therefore, the synchronous protocol sets an upper bound on the earliest starting time for each MADF actor in the new mode.

**Lemma 7.3.1.** For a MADF graph $G$ under SPS and a MCR from mode $SI^o$ to $SI^l$ at time $t_{MCR}$, the earliest starting time of actor $A^l_i$, $\hat{\sigma}^{o \rightarrow l}_i$, is upper bounded by

$$\hat{\sigma}^{o \rightarrow l}_i = F^o_{src} + S^o_{snk} + S^l_i,$$  

(7.13)

where $F^o_{src}$ indicates the time when the source actor $A^o_{src}$ completes its last iteration $It^o$ of the old mode $SI^o$ and is given by

$$F^o_{src} = t^o_s + \left\lfloor \frac{t_{MCR} - t^o_s}{H^o} \right\rfloor H^o.$$  

(7.14)

$t^o_s$ is the starting time of mode $SI^o$ and $H^o$ is the iteration period of mode $SI^o$.

**Proof.** As explained previously for a transition from mode $SI^o$ to $SI^l$, the upper bound of the earliest starting time for each actor $A^l_i$ is computed in such a way that no firings of actors $A^o_i$ and $A^l_i$ occur simultaneously. This means, the start of an actor $A^l_i$ must be later than all actors $A^o_i$ have completed the last iteration $It^o$ of the old
mode $SP^o$. Given that mode $SP^o$ starts at time $t_s^o$, the completion time of all actors $A_i^o$ in the last iteration $It^o$ can be thus computed as

$$F^o_{snk} = t_s^o + \left[ \frac{t_{MCR} - t_s^o}{H^o} \right] H^o + S^o_{snk} + H^o. \quad (7.15)$$

where $F^o_{snk}$ is the time when the old mode $SP^o$ completes the last iteration $It^o$. It is assumed that the sink actor $A^o_{snk}$ is the last actor to complete the iteration, i.e., \( \forall A_i^o \in A, S_i^o \leq S^o_{snk} \). Given Equation (7.14), Equation (7.15) can be rewritten as

$$F^o_{snk} = t_s^o + \left[ \frac{t_{MCR} - t_s^o}{H^o} \right] H^o + S^o_{snk} = F^o_{src} + S^o_{snk}.$$ 

Now, starting the source actor $A^l_{src}$ at any time later than $F^o_{snk}$ is valid without introducing simultaneous execution of actors $A_i^o$ and $A_i^l$. Therefore, the earliest starting time of source actor $A^l_{src}$ is $\hat{\sigma}^o_{src} = F^o_{snk}$. For any actor $A^l_i \in A \setminus A^l_{src}$, its earliest starting times must satisfy Equation (2.17) on page 35 imposed by the SPS framework. That is, the earliest starting starting time $\hat{\sigma}^l_{i \rightarrow o}$ of actor $A^l_i$ can be obtained by adding $S^l_i$ to $\hat{\sigma}^o_{src}$.

Let us consider the actor parameters given in Table 7.4 for $G_1$ in Figure 7.1. The third row shows the WCET for each actor in modes $S^1$ and $S^2$. Based on WCETs, the period (fourth row in Table 7.4) and the earliest starting time (fifth row in Table 7.4) for each actor in the steady-state of both modes are obtained according to Equations 2.16 and 2.17, respectively. Given $q^2$ in Equation (7.8), we can also compute iteration period $H^2 = q^2_{I^2} = 2 \times 4 = 8$. Now consider the mode transition from mode $S^2$ to $S^1$ shown in Figure 7.7. Assume that the MCR occurs at time $t_{MCR} = 13$ and mode $S^2$ starts at time $t_s^2 = 8$. The completion time of the last iteration $It^2$ is equal to the completion time of the sink actor $A^2_s$ computed as

$$F^2_{snk} = t_s^2 + \left[ \frac{t_{MCR} - t_s^2}{H^2} \right] H^2 + S^2_s = 8 + \left[ \frac{13 - 8}{8} \right] 8 + 20 = 36.$$
Figure 7.7: Upper bounds of earliest starting times for transition from mode $S^{P_2}$ to $S^{P_1}$.

In Figure 7.7, $F_{\text{snk}}^2$ corresponds to the earliest starting time of the source actor $A^1_1$ (bold dashed line). Finally, we can compute the earliest starting time for each actor in the new mode $S^{P_1}$ by adding $S^1_1$. Considering for instance the sink actor $A^1_5$ in the new mode with $S^1_5 = 14$, the upper bound of its earliest starting time can be obtained as

$$\sigma^{2\rightarrow1}_5 = F_{\text{src}}^2 + S^2_5 + S^1_5 = F_{\text{snk}}^2 + S^1_5 = 36 + 14 = 50.$$ 

We can thus compute the transition delay (cf. Definition 7.2.14) as

$$\Delta^{2\rightarrow1} = \sigma^{2\rightarrow1}_5 - t_{\text{MCR}} = 50 - 13 = 37.$$

Although the upper bound of the earliest starting times is easy to obtain for MADF actors in the new mode, it does not provide a responsive mode transition. Therefore, here we aim at deriving a lower bound of the earliest starting times with the proposed MOO protocol.

**Lemma 7.3.2.** For a MADF graph under SPS and a MCR from mode $S^{P_o}$ to $S^{P_l}$ at time $t_{\text{MCR}}$, the earliest starting time of actor $A^l_i$ using the MOO protocol is lower bounded by $\sigma^{o\rightarrow l}_i$ given as

$$\sigma^{o\rightarrow l}_i = F^o_{\text{src}} + x + S^l_i,$$

(7.16)

where $F^o_{\text{src}}$ is given in Equation (7.14) and $x$ is given in Equation (7.12).

**Proof.** Under the MOO protocol, the start of actor $A^l_i$ must be later than the time when $A^o_i$, if any, completes its last iteration in the old mode $S^{P_o}$. We assume that the source actor $A^l_{\text{src}}$ is the first actor to start in the new mode $S^{P_l}$, i.e., $\forall A^l_i \in A, S^l_i \geq S^l_{\text{src}}$. Thus, the starting time of the source actor $A^l_{\text{src}}$ is at least equal to the completion
time of the last iteration of $A^o_{src}$, denoted by $F^o_{src}$. Given $F^o_{src}$ in Equation (7.14), it thus holds $\sigma^{o\rightarrow l}_{src} \geq F^o_{src}$. Then, the offset $x$ because of the MOO protocol given in Equation (7.12) must be taken into account. Consequently, the earliest starting time of $A^l_{src}$ is lower bounded by $\sigma^{o\rightarrow l}_{src} = F^o_{src} + x$. For any actor $A^l_i \in A \setminus A^l_{src}$, its earliest starting times must satisfy Equation (2.17) on page 35 imposed by the SPS framework. Hence, the earliest starting time $\sigma^{o\rightarrow l}_i$ of actor $A^l_i$ can be obtained by adding $S^l_i$ to $\sigma^{o\rightarrow l}_i$.

Let us consider again the transition from mode $SI^2$ to $SI^l$. With the MOO protocol, the mode transition is illustrated in Figure 7.8. Upon the MCR at time $t_{MCR} = 13$ and $t_{S^2} = 8$, source actor $A^2_1$ completes its last iteration $It^2$ in the old mode $SI^2$ at the time (cf. Equation (7.14)) given as

$$
F^2_{src} = F^2_1 = t^2_S + \left\lfloor \frac{t_{MCR} - t^2_S}{H^2} \right\rfloor H^2 = 8 + \left\lfloor \frac{13 - 8}{8} \right\rfloor 8 = 16
$$

This is the earliest possible time at which mode transition is allowed. For MOO, $x$ can be computed according to Equation (7.12). Therefore, the following equations hold:

$$
\begin{align*}
S^2_1 - S^1_1 &= 0 - 0, \\
S^2_2 - S^1_2 &= 4 - 2 = 2, \\
S^2_3 - S^1_3 &= 12 - 6 = 6, \\
S^2_5 - S^1_5 &= 20 - 14 = 6.
\end{align*}
$$
It thus yields \( x = \max(0, 2, 6, 6) = 6 \), i.e., an offset \( x = 6 \) is added to \( F^2_{\text{src}} \). It can be seen in Figure 7.8 that the source actor \( A^1_1 \) starts at time \( F^2_{\text{src}} + x = 16 + 6 = 22 \). Finally, the earliest starting times of actors in mode \( SI^1 \) can be determined by adding \( S^1_i \). Considering for instance \( A^1_5 \) in the new mode, the lower bound of its earliest starting time can be obtained as:

\[
\hat{\sigma}^{2 \rightarrow 1}_5 = F^2_{\text{src}} + x + S^1_5 = 16 + 6 + 14 = 36.
\]

Now, the transition delay (cf. Definition 7.2.14) can be obtained as

\[
\hat{\Delta}^{2 \rightarrow 1} = \hat{\sigma}^{2 \rightarrow 1}_5 - t_{\text{MCR}} = 36 - 13 = 23.
\]

### Scheduling Analysis under a Fixed Allocation of Actors

During a mode transition of a MADF graph according to the MOO protocol, actors execute simultaneously in the old and new modes. The derived starting time in Lemma 7.3.2 for each actor is only the lower bound because the allocation of actors on PEs is not taken into account yet. That means, the derived starting times according to Lemma 7.3.2 can be only achieved during mode transitions when each actor is allocated to a separate PE. In a practical system where multiple actors are allocated to the same PE, the PE may be potentially overloaded during mode transitions. To avoid overloading PEs, the earliest starting times of actors may be further delayed.

**Lemma 7.3.3.** For a MADF graph under SPS, a MCR from mode \( SI^o \) to \( SI^l \), and an \( m \)-partition of all actors \( \Psi = \{\Psi_1, \ldots, \Psi_m\} \), where \( m \) is the number of PEs, the earliest starting time of an actor \( A^l_i \) without overloading the underlying PE is given by

\[
s^{o \rightarrow l}_i = F^o_{\text{src}} + \delta^{o \rightarrow l} + S^l_i, \quad (7.17)
\]

where \( F^o_{\text{src}} \) is computed by Equation (7.14) and \( \delta^{o \rightarrow l} \) is obtained as

\[
\delta^{o \rightarrow l} = \min_{t \in [x, S^o_{\text{snk}}]} \{ t : U^o_j(k) \leq UB, \forall k \in [t, S^o_{\text{snk}}] \land \forall \Psi_j \in \Psi \}. \quad (7.18)
\]

UB denotes the utilization bound of the scheduling algorithm used to schedule actors on each PE. \( \Psi_j \) contains the set of actors allocated to PE\(_j\). \( U^o_j(k) \) is the total utilization of PE\(_j\) at time \( k \) demanded by both mode \( SI^o \) and \( SI^l \) actors, and is given by

\[
U^o_j(k) = \sum_{A^o_d \in \Psi_j} \left( u^o_d - b(k - S^o_d) \cdot u^o_d \right) + \sum_{A^l_d \in \Psi_j} \left( h(k - S^l_d - t) \cdot u^l_d \right), \quad (7.19)
\]
A_1 \in \Psi_i \text{ is an actor active in the old mode } SI_{o} \text{ and allocated to PE}_i. A_1 \in \Psi_i \text{ is an actor active in the new mode } SI_{l} \text{ and allocated to PE}_i. h(t) \text{ is the Heaviside step function.}

**Proof.** Lemma 7.3.2 shows the lower bound of the earliest starting time for actor A_i in the new mode SI_{l}. However, starting A_i at time \( \delta_{i}^{o \rightarrow l} \) may overload PE_j, i.e., the resulting total utilization of PE_j, denoted by \( U_j(\delta_{i}^{o \rightarrow l}) \), exceeds UB. Therefore, in this case, the earliest starting time \( \sigma_{i}^{o \rightarrow l} \) must be delayed by \( \delta_{i}^{o \rightarrow l} \) such that \( U_j(\sigma_{i}^{o \rightarrow l}) \leq UB \) holds. From Equation (7.17) and Equation (7.16), we can see that \( \delta_{i}^{o \rightarrow l} \) is lower bounded by \( x \) which corresponds to the MOO protocol. In addition, \( \delta_{i}^{o \rightarrow l} \) is upper bounded by \( S_{snk}^{o} \) if we consider Equation (7.17) and Equation (7.13) on page 131.

\( \delta_{i}^{o \rightarrow l} \) of interest is the minimum time \( t \) in the bounded interval \([x, S_{snk}^{o}]\) that satisfies two conditions.

**Condition 1:** for each PE \( j \), the total utilization cannot exceed UB at time \( t \), i.e., \( U_j(t) \leq UB \). The total utilization \( U_j(t) \) in Equation (7.19) consists of two parts, namely \( U_j^{o}(t) \) and \( U_j^{l}(t) \). \( U_j^{o}(t) \) denotes the PE capacity occupied by the actors in mode \( SI_{o} \) that are not completed yet. Additional PE capacity \( U_j^{l}(t) \) is demanded by the already released actors in the new mode \( SI_{l} \).

**Condition 2:** We need to check all time instants \( k > t \) in the interval \([t, S_{snk}^{o}]\), such that \( U_j(k) \leq UB \), to guarantee that each PE \( j \) is not overloaded during the mode transition.

Figure 7.9 shows all actors of \( G_1 \) in Figure 7.1 allocated to 3 PEs and let us assume that the actors allocated to each PE are scheduled using the EDF algorithm. The utilization bound of EDF is given as \( UB = 1 \) [80]. Given this allocation and the transition from mode \( SI_{2} \) to \( SI_{1} \) shown in Figure 7.8, the lower bound of the earliest
starting time $\delta^{2 \rightarrow 1} = 22$ for actor $A_1^1$ cannot be achieved. At time 22, only actor $A_2^1$ has completed the last iteration $I_2^1$ on PE $1$. Starting the new mode $S_1^1$ at time 22 corresponds to $\delta^{2 \rightarrow 1} = x = 6$. The total utilization of PE $1$ demanded by the actors in the old mode $S_2^2$ at time 22, i.e., $U_1^2(6)$, can be computed as follows:

$$U_1^2(6) = \sum_{A_d^2 \in \Psi_1} u_d^2 - h(6 - S_d^2) \cdot u_d^2, \ d \in \{1, 3, 4, 5\}$$

$$= u_1^2 - h(6) \cdot u_1^2 + u_3^2 - h(-6) \cdot u_3^2 + u_4^2 - h(-2) \cdot u_4^2 + u_5^2 - h(-14) \cdot u_5^2$$

$$= 0 + u_3^2 + u_4^2 + u_5^2 = \frac{1}{8} + \frac{3}{8} + \frac{1}{4} = \frac{3}{4}$$

Enabling $A_1^1$ in the new mode $S_1^1$ at time 22 would yield

$$U_1(6) = U_1^2(6) + u_1^1 = \frac{3}{4} + \frac{1}{2} > UB = 1,$$

thereby leading to being unschedulable on PE $1$. In this case, the earliest times of all actors in mode $S_1^1$ must be delayed by $\delta^{2 \rightarrow 1} = 8$ to time 24 as shown in Figure 7.10. At time 24, the total utilization demanded by mode $S_1^2$ actors is

$$U_1^2(8) = \sum_{A_d^2 \in \Psi_1} u_d^2 - h(8 - S_d^2) \cdot u_d^2, \ d \in \{1, 3, 4, 5\}$$

$$= u_1^2 - h(8) \cdot u_1^2 + u_3^2 - h(-4) \cdot u_3^2 + u_4^2 - h(0) \cdot u_4^2 + u_5^2 - h(-12) \cdot u_5^2$$

$$= 0 + u_3^2 + u_5^2 = \frac{1}{8} + \frac{3}{8} = \frac{1}{2},$$

Figure 7.10: Earliest starting times for transition $S_1^2$ to $S_1^1$ on 2 PEs shown in Figure 7.9.
Now, enabling $A_1^1$ in the new mode at time 24 results in the total utilization of PE$_1$ as

$$U_1(8) = U_1^2(8) + u_1^1 = \frac{3}{8} + \frac{1}{2} < 1.$$  

Next, assuming that the new mode $SI^l$ starts at time 24, we need to check that the remaining actors in the new mode $SI^l$, namely $A_3^1$ and $A_5^1$, can start with $S_3^1$ and $S_5^1$ respectively without overloading PE$_1$. For instance, enabling $A_3^1$ at time 24 results in

$$\sigma_2 \rightarrow 1^3 = 24 + S_{1^3} = 24 + 6 = 30.$$  

At time 30, the total utilization of PE$_1$ can be obtained according to Equation (7.19) as follows:

$$U_1^2(8 + 6) = \sum_{A_d^1 \in \Psi_1} u_d^2 - b(14 - S_d^2) \cdot u_d^2, \ d \in \{1, 3, 4, 5\}$$

$$= u_1^2 + u_3^2 - b(2) \cdot u_3^2 + u_4^2 - b(6) \cdot u_4^2 + u_5^2 - b(-6) \cdot u_5^2$$

$$= 0 + 0 + u_5^2 = \frac{1}{4},$$

$$U_1^1(8 + 6) = \sum_{A_d^1 \in \Psi_1} \left( b(14 - S_d^1 - 8) \cdot u_d^1 \right), \ d \in \{1, 3, 5\}$$

$$= b(6)u_1^1 + b(6)u_3^1 + b(-8)u_5^1$$

$$= \frac{1}{2} + \frac{1}{4} = \frac{3}{4},$$

$$U_1(8 + 6) = U_1^2(8 + 6) + U_1^1(8 + 6) = 1 = UB.$$  

Hence, actors $A_3^1$, $A_1^1$, and $A_5^1$ are schedulable on PE$_1$ using EDF. Similarly, starting $A_5^1$ at time $\sigma_3^{2\rightarrow 1} = 24 + S_3^1 = 38$ still keeps the resulting set of actors schedulable on PE$_1$.

Using Lemma 7.3.3, we can quantify the maximum and minimum transition delays for any transition from mode $SI^o$ to $SI^l$.

**Theorem 7.3.1.** For a MADF graph under SPS, a fixed allocation of all MADF actors $\Psi = \{\Psi_1, \ldots, \Psi_m\}$ to $m$ PEs, and a MCR from mode $SI^o$ to $SI^l$, the minimum transition delay is given by

$$\Delta_{\min}^{o \rightarrow l} = \delta^{o \rightarrow l} + S_{snk}^l,$$  

and the maximum transition delay is given by

$$\Delta_{\max}^{o \rightarrow l} = \delta^{o \rightarrow l} + S_{snk}^l + H^o,$$  

where $\delta^{o \rightarrow l}$ is computed by Lemma 7.3.3, $S_{snk}^l$ is the starting time of the sink actor in the new mode $SI^l$, and $H^o$ is the iteration period of the old mode $SI^o$. 

Proof. For a MCR from mode $S^o$ to $S^l$, the transition delay $\Delta^{o\rightarrow l}$ of a MADF graph is given in Definition 7.2.14 as $\Delta^{o\rightarrow l} = \sigma^{o\rightarrow l}_{\text{snk}} - t_{\text{MCR}}$, where the earliest starting time of the sink actor is calculated as $\sigma^{o\rightarrow l}_{\text{snk}} = F_{\text{src}}^o + \delta^{o\rightarrow l} + S^l_{\text{snk}}$ according to Lemma 7.3.3. Therefore, $\Delta^{o\rightarrow l}$ can be rewritten as $\Delta^{o\rightarrow l} = F_{\text{src}}^o + \delta^{o\rightarrow l} + S^l_{\text{snk}} - t_{\text{MCR}}$. Essentially, $\Delta^{o\rightarrow l}$ is composed of three parts. In the first part, the MOO transition protocol together with a fixed allocation of the MADF actors determine $\delta^{o\rightarrow l}$. The second part $S^l_{\text{snk}}$ results from the SPS framework. These two parts thus can be determined at compile-time. The third part $F_{\text{src}}^o - t_{\text{MCR}}$ depends on when the MCR occurs, namely at $t_{\text{MCR}}$, which can only be determined at run-time. In the following, we distinguish two cases for $t_{\text{MCR}}$:

Case 1: Assume that the MCR occurs at the end of an iteration of the source actor in the old mode $S^o$, i.e., $t_{\text{MCR}} = F_{\text{src}}^o$. Then, the source actor shall be only delayed by $\delta^{o\rightarrow l}$ to start in the new mode $S^l$ according to Lemma 7.3.3, thereby guaranteeing the fastest possible start of the new mode $S^l$. As a consequence, it results in the minimum possible transition delay. Therefore, substituting $t_{\text{MCR}} = F_{\text{src}}^o$, we obtain

$$\Delta_{\min}^{o\rightarrow l} = F_{\text{src}}^o + \delta^{o\rightarrow l} + S^l_{\text{snk}} - F_{\text{src}}^o = \delta^{o\rightarrow l} + S^l_{\text{snk}}.$$  

Case 2: Assume that the MCR occurs at the beginning of an iteration of the source actor in the old mode $S^o$, i.e., $t_{\text{MCR}} = F_{\text{src}}^o - H^o$. Then, the source actor cannot start in the new mode before it completes the whole iteration in the old mode $S^o$ followed by the delay $\delta^{o\rightarrow l}$ according to Lemma 7.3.3. Therefore, the maximum transition delay is computed as follows:

$$\Delta_{\max}^{o\rightarrow l} = F_{\text{src}}^o + \delta^{o\rightarrow l} + S^l_{\text{snk}} - (F_{\text{src}}^o - H^o) = \delta^{o\rightarrow l} + S^l_{\text{snk}} + H^o.$$  

It can be seen from Theorem 7.3.1 that the maximum and minimum transition delays solely depend on the allocation of MADF actors and the old and new modes in question, irrespective of the previously occurred transitions. The old and new modes determine $H^o$ and $S^l_{\text{snk}}$, respectively, while the allocation of MADF actors determines the value of $\delta^{o\rightarrow l}$. Here, the offset $x$ due to our MOO protocol is captured in $\delta^{o\rightarrow l}$ and can be considered as performance overhead if $x \neq 0$. The other parts, namely $H^o$ and $S^l_{\text{snk}}$, in the maximum and minimum transition delays cannot be avoided as they will be present in any transition protocol.

### 7.4 Case Study

In this section, we present a case study of using the proposed MADF MoC and the developed HRT scheduling explained in Section 7.3. With the case study, we
show that the MADF MoC is able to capture different application modes and the transitions between them. Then, the main focus of the case study is to analyze the transition delays and to demonstrate the effectiveness of the proposed MOO transition protocol.

We consider a real-life adaptive application from the StreamIT benchmark suit [54], called Vocoder, which implements a phase voice encoder and performs pitch transposition of recorded sounds from male to female. We modeled Vocoder with a MADF graph with 4 modes, which capture different workloads. The MADF graph of Vocoder is shown in Figure 7.11. Depending on the desired quality of audio encoding and various performance requirements, users may switch between four different modes of Vocoder at run-time. The four modes $S = \{SI^8, SI^{16}, SI^{32}, SI^{64}\}$ specify different lengths of the Discrete Fourier Transform (DFT), denoted by $dl \in \{8, 16, 32, 64\}$. Mode $SI^8$ ($dl = 8$) requires the least amount of computation at the cost of the worst voice encoding quality among all DFT lengths. Mode $SI^{64}$ ($dl = 64$) produces the best quality of voice encoding among all modes, but is computationally intensive. The other two modes $SI^{16}$ and $SI^{32}$ explore the trade-off between the quality of the encoding and computational workload. A transition from one mode to any other one is possible, thereby resulting in totally 12 possible transitions. At run-time, reconfiguration of the parameter $dl$ is triggered by the environment, e.g., the user in this case. Subsequently, control actor $A_c$ propagates $dl$ to the dataflow actors shown in Figure 7.11 through the dashed-lined edges.

We measured the WCETs of all dataflow actors in Figure 7.11 in the four modes on an ARM Cortex-A9 [1] PE. All dataflow actors were compiled using the compiler arm-xilinx-eabi-gcc 4.7.2 with the vectorization option. The WCETs of all actors in all four modes are given in Table 7.5. It is worth to note that in mode $SI^8$, actors Spec2Env and male2female exhibit exceptionally high WCETs. It is because parameter $dl$ represents the size of the inner-most loop in the computation of actors Spec2Env and male2female. Small $dl$ (in this case $dl = 8$) leads to the fact that the inner-most loop cannot be vectorized by the compiler. In the other modes from $SI^{16}$ to $SI^{64}$, larger sizes of the inner-most loop ($dl$ equal to 16, 32, and 64, respectively) lead to full vectorization of the computation of actors Spec2Env and male2female. Therefore, in these three modes, the WCETs of actors Spec2Env and male2female are even smaller than the ones in mode $SI^8$. The dataflow actors of Vocoder are allocated to 4 PEs as shown in Figure 7.12. This allocation guarantees that the shortest periods (maximum throughput) in the steady-states of all modes can be achieved.

Table 7.6 shows the performance results for the four modes in their steady-state under SPS. For instance, the second column at the first row in Table 7.6 indicates that it is guaranteed for sink actor WriteWave to produce 256 samples per 917,451
Figure 7.11: MADF graph of Vocoder.
Table 7.5: WCETs of all actors in Vocoder (in clk.).

<table>
<thead>
<tr>
<th>Mode</th>
<th>ReadWave</th>
<th>DFT</th>
<th>AddCosWin</th>
<th>Rec2Polar</th>
<th>Unwrap</th>
<th>Spec2Env</th>
<th>male2female</th>
<th>Polar2Rec</th>
<th>InvDFT</th>
<th>WriteWave</th>
</tr>
</thead>
<tbody>
<tr>
<td>SI</td>
<td>8</td>
<td>3704</td>
<td>3704</td>
<td>35,121</td>
<td>7,168</td>
<td>1,095</td>
<td>1,095</td>
<td>71,337</td>
<td>1,392</td>
<td>1,392</td>
</tr>
<tr>
<td>SI</td>
<td>16</td>
<td>3704</td>
<td>3704</td>
<td>35,121</td>
<td>7,168</td>
<td>1,095</td>
<td>1,095</td>
<td>71,337</td>
<td>1,392</td>
<td>1,392</td>
</tr>
<tr>
<td>SI</td>
<td>32</td>
<td>3704</td>
<td>3704</td>
<td>35,121</td>
<td>7,168</td>
<td>1,095</td>
<td>1,095</td>
<td>71,337</td>
<td>1,392</td>
<td>1,392</td>
</tr>
<tr>
<td>SI</td>
<td>64</td>
<td>3704</td>
<td>3704</td>
<td>35,121</td>
<td>7,168</td>
<td>1,095</td>
<td>1,095</td>
<td>71,337</td>
<td>1,392</td>
<td>1,392</td>
</tr>
</tbody>
</table>

Clock cycles in mode $S^8$. This is the “worst-case” performance among all four modes because the Spec2Env actor exhibits exceptionally high workload (cf. WCETs in Table 7.5 and Definition 2.3.2 on page 34) in mode $S^8$. Consequently, actor Spec2Env becomes the “bottleneck” actor, so that mode $S^8$ cannot be scheduled with
higher throughput (shorter period). Nevertheless, all mode $S^{8}$ actors as a whole require a total utilization of only 1.24 (see the third column in Table 7.6) which is the least among all modes. From Table 7.6, we can see that MADF together with the SPS framework brings another advantage of efficiently utilizing PE resources. For example, in case that Vocoder is switched to a mode with lower utilization, idle capacity of PEs can be efficiently utilized by admitting other applications at run-time without introducing interference to the currently running Vocoder.

Now, we focus on the performance results of the MOO protocol, namely transition delays, for all possible transitions between the four modes of Vocoder. Table 7.7 shows both the minimum and maximum transition delays in accordance with Theorem 7.3.1 for all transitions. We can see in the second column of Table 7.7 that, in the best case, the transition delays for 6 out of 12 transitions remain the same as the iteration latencies of the new modes. This can be seen as $\chi = 0$ shown in the fourth column. In these 6 transitions, the proposed MOO protocol does not introduce any extra delay. In the 6 remaining transitions, as expected, the MOO protocol introduces offset $\chi > 0$ to the transitions from an old mode with a longer iteration latency to a new mode with a shorter iteration latency. For instance, the largest $\chi$ (in bold shown in Table 7.7) happens in case of a transition from mode $S^{8}$ with the longest iteration latency (see the fourth column in Table 7.6) to mode $S^{16}$.
Table 7.7: Performance results for all mode transitions of Vocoder.

<table>
<thead>
<tr>
<th>Transition (Sp to Sl)</th>
<th>$\Delta_{\text{min}}^{o\rightarrow l}$ (in clk.)</th>
<th>$\Delta_{\text{max}}^{o\rightarrow l}$ (in clk.)</th>
<th>$x$ (in clk.)</th>
<th>$\delta^{o\rightarrow l}$ (in clk.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Sp^8 \rightarrow Sp^{16}$</td>
<td>3,636,815</td>
<td>4,554,266</td>
<td>1,234,264</td>
<td>1,234,264</td>
</tr>
<tr>
<td>$Sp^8 \rightarrow Sp^{12}$</td>
<td>2,903,988</td>
<td>3,821,439</td>
<td>1,478,540</td>
<td>1,478,540</td>
</tr>
<tr>
<td>$Sp^8 \rightarrow Sp^8$</td>
<td>2,728,479</td>
<td>3,645,930</td>
<td>1,537,043</td>
<td>1,537,043</td>
</tr>
<tr>
<td>$Sp^{16} \rightarrow Sp^8$</td>
<td>2,402,550</td>
<td>2,551,480</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Sp^{16} \rightarrow Sp^8$</td>
<td>7,339,608</td>
<td>7,488,538</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Sp^{12} \rightarrow Sp^{16}$</td>
<td>2,402,550</td>
<td>2,580,731</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Sp^{12} \rightarrow Sp^8$</td>
<td>1425,448</td>
<td>1,603,629</td>
<td>234,012</td>
<td>234,012</td>
</tr>
<tr>
<td>$Sp^{16} \rightarrow Sp^8$</td>
<td>7,339,608</td>
<td>7,517,789</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Sp^{12} \rightarrow Sp^{16}$</td>
<td>2,402,550</td>
<td>2,702,869</td>
<td>977,102</td>
<td>977,102</td>
</tr>
<tr>
<td>$Sp^{12} \rightarrow Sp^8$</td>
<td>2,402,550</td>
<td>2,702,869</td>
<td>1,211,114</td>
<td>1,211,114</td>
</tr>
<tr>
<td>$Sp^8 \rightarrow Sp^8$</td>
<td>7,339,608</td>
<td>7,639,927</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

with the shortest iteration latency. To quantify $x$, we compute the percentage of $x$ compared to both minimum and maximum transition delays as

$$\Omega_{\text{min}} = \frac{x}{\Delta_{\text{min}}^{o\rightarrow l}} \times 100\%, \quad \Omega_{\text{max}} = \frac{x}{\Delta_{\text{max}}^{o\rightarrow l}} \times 100\%.$$ 

$\Omega_{\text{min}}$ varies from the worst-case 56% to the best case 16% with an average of 41%, whereas $\Omega_{\text{max}}$ varies from the worst-case 44% to the best case 14% with an average of 33%. Therefore, the increase of the transition delays due to the MOO protocol is reasonable for this real-life application.

Next, we consider the effect of the actor allocation shown in Figure 7.12 on the earliest starting times of actors in the new mode upon a transition (cf. Lemma 7.3.3). In this particular example, we find out that no extra delay is incurred to any actor in all transitions due to the fixed actor allocation. This can be seen from the fourth and fifth columns in Table 7.7, where $\delta^{o\rightarrow l} = x$. 