

ÉGYPTE GRÉCO-ROMAINE

Fragments of Mathematics on Papyrus


During a recent visit to Vienna, we had our attention drawn by M. Fackelmann to three very small fragmentary papyri, all of which had mathematical texts. In collaboration with our colleague Professor E. M. Bruins (Amsterdam), we were able in three cases (P. Vindob. Gr. Inv. 353 contains 2 problems) to solve the problems dealt with. The third papyrus (P. Vindob. Gr. Inv. 102) preserves too few and too insignificant words for us to decide what problem was treated.

1) P. Vindob. Gr. Inv. 353.

Under this inventory number three fragments written by the same hand are collected. The text on this medium-brown papyrus runs along the fibres. The verso is empty. Fragment a (1.8 x 3.5 cm.) preserves only the drawing of a problem and the symbol for ($\delta\zeta\omicron\nu\varrho\alpha\iota$) followed by the number $\nu\delta$. Fragments b (3.2 x 3.2 cm.) and c (6.5. x 4.8 cm.) belong to one and the same problem. They are separated by a lacuna containing approximately 12 letters. On fragment c parts of the drawing are still visible.

fragm. a (fig. p. 107).

This fragment shows the traces of a straight line drawn at an angle of 45° with the horizontal and the datum ($\delta\zeta\omicron\nu\varrho\alpha\iota$) $\nu\delta$.

$(\delta\zeta\omicron\nu\varrho\alpha\iota)$ $\nu\delta$ 

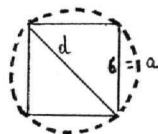
If this result were that of a computation of the area of a circle from its radius, the radius would have an irrational value, $3\sqrt{2}$, as follows from the formula :

$$\begin{aligned}\pi r^2 &= 54 \\ 3r^2 &= 54 \\ r &= 3\sqrt{2}\end{aligned}$$

The approximation for π commonly used in antiquity, 3, is employed here. It is unlikely that an irrational number was the starting point of

the problem, however, and we must therefore look for another solution. A diameter of $6\sqrt{2}$ ($d = 2r$) suggests a circle circumscribed around a square with side $a = 6$; this square would have a diagonal of $6\sqrt{2}$. The traces therefore suggest that the problem, illustrated by the drawing, was: *Given the side of a square $a = 6$, compute the area (S) of the circumscribed circle.* The solution is as follows:

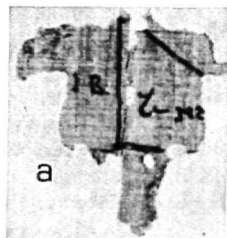
$$\begin{aligned} a &= 6 \\ d^2 &= 2a^2 \\ d^2 &= 72 \\ r^2 &= \frac{d^2}{4} = 18 \\ S &= \pi r^2 = 3 \times 18 = 54 \end{aligned}$$



After having established the text given above we received the message that a small fragment had been found, fitting exactly to the fragment a, bearing the number 12 (cf. the photograph). The circumscribed circle was not drawn! There are many examples in the *Codex Constantinopolitanus Palatii Veteris No. 1* (1) that the scribe did not complete the drawing [which some times was made up by the corrector] or muddled it).

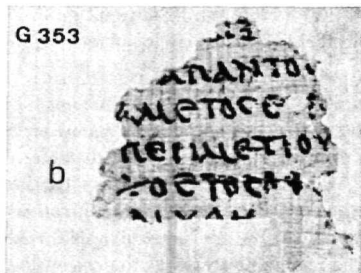
In our analysis the area being expressed — according to the text — in arouras, i.e. squared schoinia, the side of the square measured six schoinia. The frequently used unit of length in field measuring is the plethron, the ht . In our previously published texts (E. M. Bruins — P. J. Sijpesteijn — K. A. Worp, *A Greek Mathematical Papyrus*, Janus LXI, 1974, pp. 297ff.) the unit schoinion was explicitly indicated. Here no unit is given. The schoinion measures 100 ells, but there were in use different ells! In Greece the ell measured normally 6 palms, next to an ell of 7 palms, which happened to be the Samos ell (cf. Herod. II, 168). In Egypt next to the ell of 7 palms a royal ell of 8 palms was in use. This means that the schoinion can correspond to 150, or 175, or 200 feet, whereas the plethron measures 100 feet. For scholarly texts, exercises in computation, the use of a plethron being one half of a schoinion measured in royal ells, is to be preferred for numerical reasons. We do therefore interpret the number 12 at the side of the square as « 12 <plethra> », being the equivalent of 6 schoinia, resulting from our analysis.

(1) Cf. below p. 111.

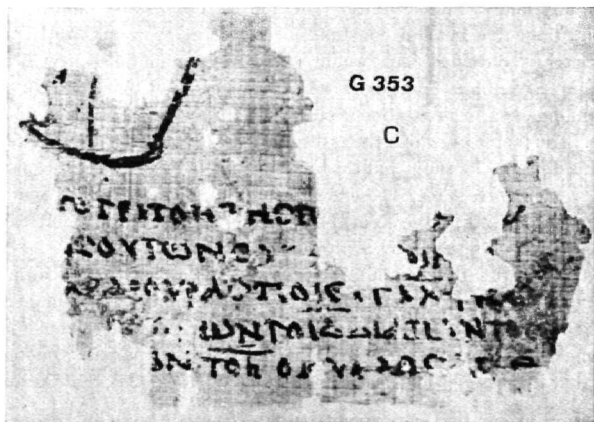


a

G 353



b



G 353

c

fragm. b and c

- 1 [ἀπο]διξ[± 12] τὸ τρίτον τῆς π[ερ]ιμ[έ]τρ[ου]
- 2 [δι]ὰ παντός [μέρισον τ]ρισσοῦτων ὁ ...[...].iv]. δι[-
- 3 ἀμετ<ρ>ος ἔστα[ι (γίνεται) ἰ πός]ας ἀρούσας· ποιεὶ τὰ $\bar{\lambda}$ τῆς
- 4 περιμέτρου [ἐπὶ τὰ $\bar{\lambda}$ (γίνεται) $\bar{\alpha}\nu$ το]ῦτον τὸ $\bar{\omega}$ διὰ παντός·
- 5 (ἀροῦραι) οε. τοσοῦτων τὸ ἔμβαδ[ον] τοῦ. ὁ κύκλος ἀπε-
- 6 δίχθη.

1 ἀποδειξί 5/6 ἀπεδείχθη

These fragments contain the computation, well-known from other sources, of the area of a circle (S) from the known perimeter (p). Mathe-

mathematical literature from antiquity preserves several formulas for computing the area of the circle; among these are:

$$S = \frac{p^2}{4\pi} = \frac{p^2}{12}; S = \frac{\pi d^2}{4} = \frac{3d^2}{4} = d^2 - \frac{d^2}{4}; S = \frac{pd}{4} = \frac{pr}{2}$$

The first line of this problem on our papyrus lies below the remains of a drawing of a circle with a diameter.

The preserved text starts with the general statement: τὸ τρίτον τῆς περιμέτρου διὰ παντός [μέρισον ... το]σοῦτων; after this one expects: ἢ τοῦ κύκλου διάμετρος ἔσται. But the *o* is absolutely certain. The remaining traces at the end of line 2 are too vague to allow us to read anything with certainty. The phrase διὰ παντός indicates the application of a general constant in a general procedure. After the diameter is obtained, one would expect a formulation anew of the question by πόσαι ἀρουραι or πόσας ἀρούρας (scil. ἀποφαίνειν). The beginning of the computation is clear: « Multiply the 30 of the perimeter by ». The factor by which the result must be divided, 14, and the result of this division, 75, show that the total must have been 1,050; the missing factor, therefore, is 35. These are restored in the text above; one may translate: « Multiply the 30 of the perimeter [times the 35, result 1,050; of this] take 1/14 generally: 75 (arouras) ».

At first sight, this computation does not follow any of the formulas given above, and no explanation is given of the source of the numbers 35 and 14 which are used as multiplier and divisor. The perimeter *p* must be one of the factors in the numerator, but it is not squared. One observes that if the radius had been computed to be 5 (i.e. 1/2*d*), that the multiplication in our text corresponds to $S = \frac{pr}{2}$, with both numerator and denominator multiplied by 7. Thus $S = \frac{30 \times (5 \times 7)}{2 \times 7}$. But why, we ask, should the writer have introduced this wholly gratuitous and clumsy procedure of multiplying by $\frac{7}{7}$? The answer lies in the common use of the better approximation for π , $\frac{22}{7}$, when one of the formulas which involves π (as the one used does not), is employed. For example, if one computes $S = \frac{\pi d^2}{4}$, using $\frac{22}{7}$ for π , one gets $S = \frac{22d^2}{28}$, or $\frac{11d^2}{14}$. By introducing 7 into the numerator, the writer could compute in such a way that until the last step all numbers arising are integers. Then, after all other computations are carried out, the result is divided

by 7 in order to eliminate the factor, and to make the final effect that of multiplying by $\frac{7}{7}$, i.e. 1. But this procedure has meaning *only* if π appears in the equation. In our case, the writer had already used a figure of $\pi = 3$ in computing the diameter, and the use of 7's therefore has no real purpose. For a figure of $\pi = 3$, the computation is in fact correct, even if clumsy.

When we come to the final phrase, however, we find the following: « So much the area : 378. The circle has been determined ». No computation is stated which would produce 378. The solution here lies in the $\frac{22}{7}$ approximation for π , once again. The way in which the procedure of 7's was conceptualized was actually that a measuring unit 7 times smaller than the real one was hypothesized, making all measurements 7 times as large. In the case of areas, thus, one would introduce the 7 twice in the numerator (once in p , once in r). One would then divide twice by 7 at the conclusion. But our writer had multiplied only r by 7, as we have seen. At this point, then, following mechanically the wrong procedure (for this formula), he multiplied again by 7. But he did not multiply his answer 75 times 7; rather, he took through an error the answer above to the previous problem, 54; for $7 \times 54 = 378!$ To make matters worse, he did not then divide by 7 again, as would be necessary, but let the answer stand, neither erasing it nor putting $\sigma\phi\acute{\alpha}\lambda\mu\alpha$ by its side, but confidently asserting « the circle has been determined ». This ingenious solution supposes, rightly as we think, that the three fragments all belong to the same papyrus.

Notes :

2-3 : For omission of ρ after a stop cf. F. T. Gignac, *A Grammar of the Greek Papyri of the Roman and Byzantine Periods*, Milano, 1976, p. 107f.

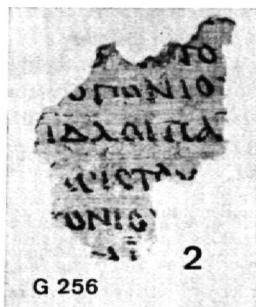
4 : The scribe should have written $\tau\delta^7$.

5 : $\cdot\prime$: We take this symbol as $\acute{\alpha}\rho\omicron\upsilon\epsilon\alpha\iota$ (cf. H. C. Youtie, *Scriptiunculae*, II, p. 915, 5 n.) though we do not totally exclude that it stands for $\gamma\acute{\iota}\nu\epsilon\tau\alpha\iota$ / $\gamma\acute{\iota}\nu\omicron\nu\tau\alpha\iota$. A similar symbol was taken by H. C. Youtie, *Ostraca from Karanis IV*, 1140, ZPE 18, 1975, p. 28, to stand for $\acute{\alpha}\rho\tau\acute{\alpha}\beta\alpha\iota$ (cf. O. Mich. I, 415, 7). The symbol is also known to stand for $\delta\beta\omicron\lambda\omicron\iota$ (cf. A. Blanchard, *Sigles et abréviations dans les papyrus documentaires grecs : recherches de paléographie*, BICS 30, 1974, p. 37f. Cf also E. M. Thompson, *An Introduction to Greek and Latin Palaeography*, Oxford, 1912, p. 84).

2) P. Vindob. Gr. Inv. 256.

A medium-brown papyrus. The text runs along the fibres. The verso is empty. 2 x 3 cm.

- 1 traces
 2]...το[
 3 ἰσο]γωνίων [
 4]. ἰδ̄ λοιπά̄ .[
 5 β]άσις τοῦ[των
 6 ἰσο]γωνίων [
 7 λοι]πά̄ ἰ[



This very small fragment contains only some words of the statement of a problem and its solution. Due to the stereotypic and strict terminology we can be almost sure about the problem and its solution which has been treated. The polygons are divided into two groups; the *ἰσόπλευρα* and the *ἰσογώνια*. An isosceles triangle cannot be *ἰσογώνιον*. A triangle which is *ἰσογώνιον* is automatically *ἰσόπλευρον*; the two concepts are identical for the triangle for which the normal terminology leads to call it *ἰσόπλευρον*. The «equalangled» has therefore a sense *only for polygons with more than three sides*. Then the concepts are different. For the quadrilaterals e.g. an «isogonial» quadrilateral is a rectangle, an «isopleuron» quadrilateral is a rhomb, a quadrilateral having both properties is regular, a square. The word *βάσις*, which has been preserved, suggests that *only one* length is playing a role; it points to the side of a regular polygon. The numerical factor 14 suggests circles to play a role and the word *λοιπά*, following the number 14 indicates the subtracting of a circular area. The problem to compute the remaining area, between 4 touching circles, of the same diameter, having the centres in the vertices of a regular polygon, is well known. The basis of the polygon is the diameter of the circles. The solution is very simple for a square, as from the total area of the square just four quarters of the circle, i.e. the circle must be subtracted. In modern formulas:

$$d^2 - \frac{\pi d^2}{4} = d^2 - \frac{11d^2}{14} = \frac{3d^2}{14}.$$

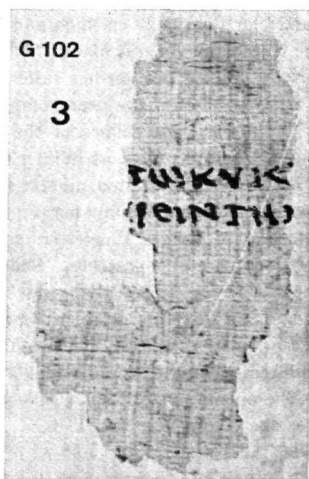
The computations of the area of the pen-

tagon, hexagon,....., dodekagon have been preserved in a codex from Constantinopel (cf. E. M. Bruins, *Codex Constantinopolitanus Palatii Veteris No. I*, Leiden, 1964), e.g., but more than 4 sides leads to tedious calculations. One can either compute the square and the circle separately and determine the remainder, the difference, or, as is done in the quoted codex, fol. 18^v, just square the diameter, the basis, multiply into 3 and divide by 14. For $d = 7$ this leads to $d^2 = 49$, $3d^2 = 147$, divided by 14, result $10\frac{1}{2}$. As the traces in line 7 correspond to $\lambda\sigma\iota\pi\alpha\delta$ $\overline{\iota\lambda}$ the problem and the solution are uniquely determined, even from these scanty remains.

3) P. Vindob. Gr. Inv. 102.

A medium - brown papyrus. The text runs along the fibres. The verso is empty. At the top 1.8 and at the bottom approx. 3 cm. have been left free. 5.3 x 3.2 cm.

- 1 $\epsilon\sigma\iota\tau\omega\ \kappa\acute{o}\kappa\lambda\iota\omicron\varsigma$
 2 $\]\rho\epsilon\iota\upsilon\ \tau\acute{\eta}\nu\ [$



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