Resource allocation in networks via coalitional games
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Resource Allocation in Networks via Coalitional Games

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Resource Allocation in Networks via Coalitional Games

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Abstract

Next-generation wireless and power distribution networks will lead to higher energy consumption and waste. However, the advent of new smart devices opens the possibility of devising cooperative policies for improving their energy efficiency.

The main goal of this dissertation is to manage resource allocation in network engineering problems and to introduce efficient cooperative algorithms to obtain high performance, ensuring fairness and stability. Specifically, this dissertation introduces new approaches for resource allocation in Orthogonal Frequency Division Multiple Access (OFDMA) wireless networks and in smart power grids by casting the problems to the coalitional game framework and by providing a constructive iterative algorithm based on dynamic learning theory.

In OFDMA wireless networks each terminal is assigned to a set of subcarriers. The problem is to find the optimal amount of power transmission over each subcarrier as to achieve the device's demanded data rate exactly. The power distribution is obtained by a dynamic learning algorithm based upon Markov modeling. Simulation results show that the average number of operations of the proposed iterative algorithm are much lower than $K \cdot N$, where K is the number of mobile terminals and N is the number of subcarriers.

In smart power grids, we consider the problem of power trading coordination among micro-grids, e.g., wind turbines and solar panels. To minimize the amount of dissipated power during generation and transfer, we introduce an algorithm which allows the micro-grids to autonomously cooperate and self-organize into a set of coalitions. Our evaluation shows that the new approach enables micro-grids to coordinate for power trading and dissipate only 10% of the power which would be otherwise dissipated by traditional power distribution networks.

Samenvatting

De volgende generatie draadloze netwerken en elektriciteitsnetten zal leiden tot meer consumptie van energie. De opkomst van nieuwe *smart* apparaten maakt het echter mogelijk om coperatief beleid te ontwikkelen dat hun energie-efficintie verbetert.

Het hoofddoel van dit proefschrift is het beheren van de toewijzing van resources in de context van netwerkengineering en het introduceren van efficinte coperatieve algoritmes die hoge prestaties behalen en daardoor eerlijkheid en stabiliteit garanderen. In het bijzonder introduceert dit proefschrift nieuwe methodes voor het toewijzen van resources in draadloze netwerken met Orthogonal Frequency Division Multiple Access (OFDMA) en in *smart* elektriciteitsnetten door het probleem te beschouwen binnen het raamwerk van coalitiespellen en door een constructief, iteratief algoritme te leveren dat is gebaseerd op de theorie van dynamisch leren.

In draadloze netwerken met OFDMA is elke terminal toegewezen aan een verzameling subcarriers. Het probleem is het vinden van de optimale hoeveelheid elektriciteitstransmissie over elke subcarrier teneinde de benodigde datafrequentie exact te behalen. We vinden de elektriciteitsdistributie door middel van een dynamisch leeralgoritme gebaseerd op Markov-modellen. Simulatieresultaten laten zien dat het gemiddelde aantal bewerkingen dat het voorgestelde iteratieve algoritme uitvoert veel lager is dan $K \cdot N$, waar K het aantal mobiele terminals is en N het aantal subcarriers.

In de context van *smart* elektriciteitsnetten beschouwen we de cordinatie van elektriciteitsuitwisseling tussen micronetten, zoals bijvoorbeeld windturbines en zonnepanelen. Om de hoeveelheid dissipatie van energie tijdens generatie en transport te minimaliseren introduceren we een algoritme dat de micronetten toestaat om autonoom samen te werken en zichzelf te organiseren in een verzameling coalities. Onze evaluatie laat zien dat de nieuwe methode micronetten toestaat om hun elektriciteitsuitwisseling te cordineren en een dissipatie te bewerkstelligen die slechts 10% is van die van traditionele elektriciteitsnetten

Contents

List of Figures

List of Tables

List of Acronyms

1/2/3/4/5G	1st/2nd/3rd/4th/5th generation
3GPP	3rd generation partnership project
ADSL	asymmetric digital subscriber lines
AF	amplify and forward
AP	access point
ARQ	automatic repeat request
AWGN	additive white Gaussian noise
BER	bit error rate
BPSK	binary phase-shift keying
BS	base station
CDMA	code division multiple access
CDU	central distribution unit
CF	compress and forward
COFDM	coded orthogonal frequency division multiplexing
CP	cyclic prefix
CSI	channel state information
DAB	digital audio broadcasting
DF	decode and forward
DFT	discrete Fourier transform
DMT	digital multitone
DSP	digital signal processing
DVB-T	digital video broadcasting for terrestrial television
FCC	Federal Communications Commission

FDMA	frequency division multiple access
FEC	forward error correction
FFT	fast Fourier transform
FMT	frequency multitone
GSM	global system for mobile communication
Hz	hertz
ICI	inter carrier interference
ICT	information and communications technology
IFFT	inverse fast Fourier transform
i.i.d.	independent and identically distributed
IP	Internet protocol
ISI	inter symbol interference
LDPC	low-density parity-check
LP	linear programming
LTE	long term evolution
MAC	medium access control
MAI	multiple access interference
MAN	metropolitan area networks
MC	multi-carrier
MC-CDMA	multi-carrier code division multiple access
MG	micro-grid
MIMO	multiple-input multiple-output
NBS	Nash bargaining system
NE	Nash equilibrium
NTU	non transferable utility
OFDM	orthogonal frequency division multiplexing
OFDMA	orthogonal frequency division multiple access
OSI	open systems interconnection
PAPR	peak-to-average power ratio
QAM	quadrature adaptive modulation

QASK	quadrature amplitude shift keying
QoS	quality of service
QPSK	quadrature phase-shift keying
RBS	Raiffa-Kalai-Smorodinsky bargaining
SINR	signal-to-interference-plus-noise ratio
SNR	signal-to-noise ratio
TDMA	time-division multiple access
TU	transferable utility
UMTS	universal mobile telecommunications system
UWB	ultra-wideband
VDSL	very-high-bit-rate digital subcarrier line
WiMAX	worldwide interoperability for microwave access

List of symbols and operators

Throughout this thesis the following notational conventions are used:

absolute value	$\lvert\cdot\rvert$
$\max\{x, 0\}$	$[x]^+$
characteristic vector of coalition $\mathcal{A}$	$\mathbf{1}_{\mathcal{A}}$
pure strategy	a
pure strategy space of player k	$\mathbf{A}_k$
joint pure strategy chosen by the player k	$\mathbf{a}_k \in \times_k \mathbf{A}_k$
set of jointly ϵ-typical	A_ϵ
generic coalition (subset) of players and transmitters	$\mathcal{A}$
complementary set of $\mathcal{A}$	$\mathcal{A}^C$
coalition at time step t	$\mathcal{A}^t$
generic index for carrier subblock	b
number of carrier subblocks	B
power expenditure cost	c
power loss over transmission line l_{ij}	D_{ij}
excess value of the coalition $\mathcal{A}$ with respect to the payoff distribution $\mathbf{u}$	$e\left(\mathcal{A}\,,\,\mathbf{u}\right)$
vector of excess values of all coalitions arranged in non-icreasing order	$\mathbf{E}$
family of coalitions	$\mathcal{F}$
coalitional game	$\mathcal{G}$
real-valued channel response of the channel between terminal k and the base station over subcarrier n	h_{kn}

complex-valued channel response of the channel between terminal k on nth subcarrier	H_{kn}
amplitude of H_{kn}	$\|H_{kn}\|^2$
set of imputations in a game	$\mathcal{I}\left(\mathcal{K}\,,\,\boldsymbol{\nu}\right)$
other feasible imputation	$\mathcal{J}$
generic index for a terminal/player	k
number of terminals/players	K
set of terminals/players	$\mathcal{K}$
transmission line/learner from the sending-end i to the receiving-end j	l_{ij}
generic index for a subcarrier	n
number of carriers	N
set of carriers	$\mathcal{N}$
set of carriers of the bth subblock	$\mathcal{N}^{(b)}$
set of carriers assigned to transmitter k	$\mathcal{N}_k$
(real) Gaussian distribution with mean μ and variance σ^2	$\mathcal{N}(\mu\,,\,\sigma^2)$
transmit power of terminal k over carrier n	p_{kn}
transmit power of terminal k over carrier n at time step t	p^t_{kn}
tentative transmit power of terminal k over carrier n	$\tilde{p}_{kn}$
previous tentative transmit power of terminal k over carrier n	$\hat{p}_{kn}$
maximum transmit power of terminal k over carrier n	$\overline{p}_{kn}$
maximum tentative transmit power of terminal k over carrier n	$\tilde{p}^{\max}_{kn}$
Q value of user k at time step t	Q^t_k
data rate (Shannon capacity) achieved by terminal k	R_k
data rate achieved by terminal k on the carrier n	R_{kn}
achieved data rate between two terminals k and m	$R_{(km)}$
target data rate of terminal k	$R^{\star}_k$
maximum demanded rate of a terminal user	$\overline{R}_k$
minimum demanded rate of a terminal user	$\underline{R}_k$

rate vector: $\mathbf{R} = [R_1, \ldots, R_k]$	$\mathbf{R}$
maximum excess of player k against i	s_{ki}
core set of a game	$\mathcal{S}\left(\mathcal{K}\,,\,\boldsymbol{\nu}\right)$
generic time step	t
temperature function	T
payoff (reward) symbol	u
payoff (reward) of player k	u_k
payoff distribution across players	$\mathbf{u}$
OFDM signal bandwidth	W
generic positive constants	α, β
received SNR of terminal k over carrier n	γ_{kn}
carrier spacing	Δf
power step to update the tentative transmit power of terminal k over carrier n	$\Delta\tilde{p}_{kn}$
maximum power step to update the tentative transmit power of terminal k over carrier n	$\overline{\Delta p}_{kn}$
discount factor of the user k	δ_k
tolerance parameter for demanded data rate	ε
normalized average power expenditure of terminal k	ζ_k
stopping criterion of the iterative algorithm	Θ
probability of transmit power update	λ
balanced weight of coalition $\mathcal{A}$	$\mu_{\mathcal{A}}$
uniformly-distributed random variable	ξ_{kn}^t
correlation value between the outputs of source node k and a relay r	ρ_{kr}
correlation value between the outputs of two source nodes	σ_{km}
AWGN power	σ_w^2
correlation value between the outputs of two relays	τ_{rj}
coalition utility function	ν
set of coalition utilities	$\boldsymbol{\nu}$
set of coalition utilities at time step t	$\boldsymbol{\nu}^t$

Shapley value of the player k with respect to coalition payoff ν	$\phi_k(\nu)$
weight coefficient of fairness of a terminal user	φ_k
ergodic set	Φ
set of disjoint coalitions	ψ
set of disjoint coalitions at time step t	ψ^t
set of all possible ψ	$\boldsymbol{\Psi}$
state of the Markov chain	ω
state of the Markov chain at time step t	ω^t
state space of the Markov chain	Ω

Introduction

Motivation

According to the Information and Telecommunication Union, in 2009, the information and communication technology (ICT) sector was responsible for 4% of the global greenhouse gas emissions [1], but its impact is predicted to double by 2020 [2]. Current estimates indicate that ICT is responsible for a fraction of the world energy consumption ranging between 2% to 10% and it is increasing by 16%−20% per year [3]. Within the ICT sector, telecommunication networks take a significant share of the total energy consumption. The number of wireless accesses to the Internet is doubling each year [4]. This ever-increasing traffic demand pushed telecom operators and the industry itself to focus their research on enlarging network capacity, which is usually accompanied by higher energy consumption. The energy costs for mobile network operators can already be as high as 60% of their annual operating budgets. Thus, investments on projects with high energy efficiency goals will have a good investment return not only economically but also environmentally.

Looking back, wireless access technologies have followed different evolutionary paths towards a unified target: performance and efficiency in high mobility environments. In second generation (2G) wireless networks, data services are defined over well-established GSM (global service for mobile). Third generation (3G) wireless networks support very high data rates with larger channel bandwidth and more efficient coding techniques. For even higher data rates and native IP support, the standards 4G and 5G have been released. They use orthogonal frequency division multiple access (OFDMA) [5] as digital modulation and channel access scheme. OFDMA is a multicarrier scheme which can be exploited to increase data rates in a multi-user environment, by dividing a frequency-selective broadband channel into a multitude of orthogonal narrowband subcarriers.

Expanding wireless networks and the advent of new high-definition entertainment services imply that future enhancements will be needed to augment data rates, which will increase energy consumption both at the end-user and at the base station side. Reducing this consumption has been addressed in several studies [4,6] and is considered one of the most important factors to make communication networks energy efficient, reducing the cost of telecommunication services and helping network operators to be environmental friendly. Energy efficiency has become a very important issue in optimizing the current, as well as in designing the future, telecommunication networks. To cope with this, efficient radio resource management systems, in particular *power control* systems, are on demand. Transmission power in wireless cellular networks is a key degree of freedom in the management of interference, energy, and connectivity. The main purpose of power control is to provide each signal in the network with adequate quality without causing unnecessary interference to other users in the system.

"The 3G chipsets that are available to semiconductors work reasonably well except for power. They are real power hogs, so as you know, the handset battery life used to be 5-6 hours for GSM, but when we got to 3G they got cut in half. Most 3G phones have battery lives of 2-3 hours", said Steve Jobs in the early days of universal mobile telecommunications system (UMTS). According to Moore's law, "integrated circuits will double in performance every 18 months", but batteries have not improved much at all; there are deep physical limits. Battery industry foresees that the energy capacity is only doubling every 10 years. This justifies the necessity of an energy efficient resource allocation technique in modern wireless networks. Regardless of involving uplink or downlink communication, resource allocation in OFDMA consists of two sub-problems: subcarrier assignment to each individual wireless terminal, and adjustment of power transmission level over every subcarrier, in order to satisfy the users' QoS requirements and maximize the network sum rate. This generally corresponds to an optimization problem. An intelligent and scalable joint power and bandwidth allocation mechanism is crucial to ensure QoS to the consumer at a reasonable cost [5].

Improving energy efficiency, in a different way, is also a research objective in modern electricity distribution networks such as *smart grids* [7,8]. The term smart grid defines a self-healing network equipped with dynamic optimization techniques that use real-time measurements to maintain voltage levels, increase reliability, and improve asset management. Smart grid is a capillary infrastructure at the medium and low voltage levels that will support local energy trading among producers and consumers. Smart grids are well-known as the modern system of electricity production, distribution, and consumption. Furthermore, smart grids are deemed to improve efficiency, reliability, and sustainability of the production and distribution of electricity. They are able to collect, transmit and use information about the behaviours of electricity producers and consumers in an automated fashion by means of automation and ICT. Smart grids offer the opportunity to purchase energy from renewable and clean resources, e.g., solar and wind.

In traditional power distribution models, consumers acquire power from the central distribution unit. A smart grid is instead a network of smart renewable energy producers-consumers, so-called *micro-grids*, which can also trade power between themselves [8]. A micro-grid is a local renewable electricity generation, e.g., wind turbines and solar panels, which is connected to a collection of individual consumers within an area. A micro-grid can also trade power with the (traditional) central distribution unit if it is necessary. The intelligence of smart grids relies on the real-time exchange of information and control data among micro-grids themselves. Each distributed resource can exchange information with a number of other close-by resources, and subsequently make a local control decision based on this available information. This allows for an energy balance at local levels. Accordingly, a current trend in research goes towards developing technologies, concepts, and strategies to minimize feeding from (traditional) central distribution unit and maximize the local consumption of local energy producer [7]. One key concept in this context is local energy storage. Another idea is the local exchange of energy within a neighborhood of micro-grids [9]. In fact, end-users generate their own energy and the deficit power can be provided by other end-users which have surplus energy generated. In such a context, a local power exchange between end-users is foreseeable without an energy storage equipment. Such a local energy trading among micro-grids can also reduce the amount of power loss over transmission lines. The possibility of local exchange of renewable energy in smart grids allows further efficient utilization, but leads to many challenges as well. The main challenge is coordination, i.e., determining the optimal quantity of power to trade between each pair of micro-grids in order to achieve the best individual and overall performance in terms of minimum price and minimum amount of power loss over transmission lines.

In both problems of radio resource management in OFDMA wireless networks and power trading coordination in smart grids, the main question is how agents, i.e., wireless terminals in OFDMA networks and micro-grids in smart grids, should interact in order to achieve the highest performance, i.e., maximum overall data rate and minimum amount of dissipated power, respectively. A well-devised interaction policy determines the best strategy each player should choose, i.e., the optimal amount of transmission power over each subcarrier for each wireless terminal in OFDMA networks, and the optimal amount of traded power among each couple of micro-grids in smart grids, respectively. Several agents have to coordinate the sharing of a common resource and manage the impacts resulted by decisions of the other agents. The limited resources and the increasing number of users are inevitably accentuating the relevance of good management of the resources, e.g., radio resources in wireless networks and storages in smart grids.

Game theory [10] is a formal, mathematical discipline which studies situations of competition and cooperation between several involved parties. It is a collection of a variety of subfields and techniques, each representing a possibly fundamentally different approach to the description of social interactive decisions. This is a broad definition but consistent with the large number of applications ranging from strategic questions in warfare [11] to issues related to economic competition [12], from social problems of fair distribution [13] to signal processing in wireless engineering [14]; and this list is certainly not exhaustive.

Essentially, game theory can be split into two branches: *non-cooperative* and *cooperative* game theory. The distinction between the two is whether or not the players in the game can take joint decisions regarding the strategy to choose.

Non-cooperative games deal with situations in which multiple self-interested entities, or players, simultaneously and independently optimize different objectives and outcomes. Non-cooperative game theory typically results in the study of various equilibria, most notably the Nash equilibrium.

Cooperative game theory examines how strictly rational agents can benefit from voluntary cooperation by reaching bargaining agreements. *Coalitional game theory* is a branch of cooperative game theory in which the central questions are: which coalitions (groups of agents) will actually be formed, and how should payoffs of such a coalition structure be distributed among its members? It is obvious that a cooperative game approach would be efficient if social strategies satisfy both individual interests of the end-users and those of the network service provider.

Besides competition among selfish users, it might be possible that the involved agents, e.g., wireless terminals in cellular network and micro-grids in smart grids, cooperate to access the resources, and through this cooperation effectively achieve a robust allocation strategy which promises significant benefits such as higher throughput and fairness. In this dissertation, we will show that coalitional game theory is an appealing option to formulate the cooperative behaviour of involved agents in network engineering problems. We use coalitional game theory to tackle the problems of resource management in OFDMA-based wireless networks and smart grids with the aim of achieving an energy-efficient network. We will propose resource management policies whose computational burden is as cheap as possible and energy saving is as high as possible.

Main contributions

Due to large number of resources, e.g. subcarriers in OFDMA-based networks and micro-grids in smart grids, there is a great freedom to resource assignment. The main focus of this thesis is to introduce novel cooperative policies among involved agents in wireless networks and smart grids in order to achieve a power efficient resource management with the best performance in terms of energy saving.

Firstly, we will introduce a novel algorithm for resource management in the uplink of OFDMA infrastructure wireless networks. The key requirements of the devised coalitional game-theoretic model are fairness from both end-user and wireless service provider viewpoint, low complexity, and energy efficiency. Our main contributions for OFDMA are the following:

1) A novel approach to resource allocation fairness that improves the common approach of considering only end-users's satisfaction and ignores completely the need of service providers. We propose a more fair approach in which each wireless terminal gets a data rate corresponding exactly to its requirement;

2) Two different approaches to assign subcarriers to different wireless terminals by allowing each subcarrier to be shared by more than one wireless terminal;

3) The definition of power constraints is defined as individual power limitations on

each subcarrier for each user rather than as common constraint on overall energy consumption of each user over all subcarriers [15];

4) The computational burden of the proposed iterative algorithm is the cheapest one in the literature. That is much lower than $K \cdot N$, where K is the number of mobile terminals and N is the number of subcarriers.

We prove the existence of the solution of the cooperative resource allocation game, "the core set solution", by means of the analytical tools of coalitional game theory. A dynamic learning algorithm for reaching one of the core set solutions of the power expenditure scheme is derived, and its convergence is demonstrated based on Markov modeling. The quality of our contributions is tested by simulations which show how the derived framework outperforms the known results [16–19] in terms of complexity, power consumption, and utilization of the spectrum.

The second problem deals with power trading coordination in smart grids. A micro-grid with a surplus amount of power can transfer it to the central distribution unit, and meanwhile can serve micro-grids with deficit power. We investigate the problem with the purpose of minimizing the amount of dissipated power over transmission lines during generation and transfer. The main contribution for smart grids is the following:

- A coalitional game theory based approach which allows micro-grids to autonomously cooperate and self organize into a set of coalitions. In each coalition the micro-grid in excess of power will provide the whole or a fraction of deficit powers of assigned ones in need of power. Micro-grids can also trade with the central distribution unit if it is necessary. Each micro-grid can decide to form a singleton coalition and to trade the whole quantity of power only with the central distribution unit.

We introduce a dynamic learning process which leads micro-grids to a coalition structure guaranteeing that all micro-grids in need of power will be served and all micro-grids in excess of power will be loaded. Another complementary dynamic process leads micro-grid to the best coalition structure wherein the amount of dissipated power is minimized. We model the dynamic processes as Markov chains and show the stability (the convergence to a fixed-point) of both dynamic processes using the Kakutani fixed point theorem. The efficiency of the algorithm is validated with simulation results that show that the overall amount of dissipated power in the proposed cooperative smart grid is only 10% of that in the traditional (non-cooperative) power distribution networks.

Outline of the dissertation

This dissertation consists of three parts. The first part contains two survey-based chapters on basic definitions and notions of coalitional game theory and resource allocation techniques in OFDMA networks, respectively. In the second and third parts, novel cooperative schemes among involved agents in wireless networks and smart grids are introduced to achieve an energy efficient resource management. The remainder of this thesis is organized as follows:

In Chapter 1, we introduce the basic concepts of coalitional game theory and some concepts which turn out to be crucial for applications to communication networks. To this end, we provide motivating examples for the application of coalitional game theory to network engineering problems, and we outline the trends in research into coalitional game theory applications to wireless networks. We will show that this branch of game theory is an appealing tool to tackle different problems in networking and wireless engineering and that the solutions based on coalitional game theory outperform solutions based on non-cooperative game theory. This chapter is a revised and extended version of the following paper:

- F. Shams and M. Luise, "Basics of coalitional games with applications to communications and networking," *EURASIP J. Wireless Communications and Networking*, vol. 2013, no. 1, 2013.

In Chapter 2, after a brief introduction of OFDMA-based technologies, we review the existing techniques concerned with radio resource allocation in OFDM and OFDMA, its multi-user version, where the radio resources can be bandwidth and transmission power. We then discuss the relevant features of each technique and show that existing schemes based on cooperative game theory exhibit good performance in terms of fairness and overall achieved data rate. We provide motivations for the definition of a new fair resource allocation technique in OFDMA. This chapter is a revised and extended version of the following paper:

- F. Shams, G. Bacci, M. Luise, "A survey on resource allocation techniques in OFDM(A) networks," *Computer Networks*, vol. 65, p. 129, 2014.

In Chapter 3, we introduce the resource allocation problem for OFDMA-based wireless networks as a coalitional game. Firstly, we propose two different subcarrier allocation techniques. We then formulate the power control scheme using a novel fairness criterion in which each wireless terminal achieves its data rate demanded exactly. Next, we prove the existence of the core solution(s) of the proposed game, and we describe an iterative algorithm to reach one of such solutions in a centralized fashion. We conclude this chapter by comparing the simulation results with existing ones in the literature. This chapter is a revised and extended form of the following published paper:

- F. Shams, G. Bacci, M. Luise, "An OFDMA resource allocation algorithm based on coalitional games," *EURASIP J. Wireless Communications and Networking*, vol. 2011, no. 1, 2011:46, July 2011.

In Chapter 4, we investigate the problem of power trading coordination among micro-grids (e.g., solar panels, wind turbines, etc.) in smart power grids. With the purpose of minimizing the amount of dissipated power during generation and transfer, we introduce an algorithm based on dynamic learning and coalitional game theory which allows the micro-grids to autonomously cooperate and self-organize into a set of coalitions. The iterative algorithm can be used to reach the best coalition structure with minimum amount of dissipated power in the entire network. This chapter is a revised and extended version of the following paper:

- F. Shams, M. Tribastone, "Power trading coordination in smart grids using dynamic learning and coalitional game theory," in *Int. Conf. on Quantitative Evaluation of SysTems (QEST),* Madrid, Spain, Sep. 2015.

Finally, in Chapter 5 we draw some concluding remarks and touch on a few open issues in the research fields considered in the thesis.

Chapter 1

Coalitional game theory

Game theory is the study of decision making in an interactive environment. *Coalitional games* fulfill the promise of group efficient solutions to problems involving strategic actions. Formulation of optimal player behavior is a fundamental element in this theory. This chapter comprises a self-instructive didactic means indicating how cooperative game theory tools can provide a framework to tackle different network engineering problems. We show that coalitional game approaches achieve an improved performance compare to non-cooperative game theoretical solutions.

This chapter is divided into ten sections. After a brief motivation in the following section, Sect. 1.2 provides an introductory discussion of cooperative game theory. We systematically study fundamental definitions and conditions of cooperative games: superadditivity and convexity. Then, Sect. 1.3 and the sub-section inside discuss the core set solution as the most known solution for payoff distribution. Sect. 1.4 is devoted to a study of a strong payoff distribution, the so-called Shapley value. In Sect. 1.5 we present a systematic study of two others reward division called the *kernel* and *nucleolus*. Then, in Sect. 1.6, we extend the concept of Nash equilibria in cooperative games. Sect. 1.7 is an investigation of the concept of coordinated equilibria where players of game are admitted to pre-communicate among themselves at once. Finally, Sect. 1.8 helps a reader to understand the basic concepts and importance of dynamic learning in cooperative games. Every sections contain some motivation examples that are expedient to understand how different communication networks problems can be modeled as cooperative game. We discuss the features of mentioned approaches in Sect. 1.9 and finally we conclude this chapter in Sect. 1.10.

1.1 Motivation

The increase of the number of wireless services, combined with demand for high definition multimedia communications, have made the radio resources, and particularly the spectrum and power, a very precious and scarce resource, not because of their unavailability but because they are used inefficiently. For licensed spectrum, the measurements by Shared Spectrum Company [20] shows that the maximal usage of the spectrum is a low percentage of the whole licensed. While the number of users and the spectrum usage steadily increase, the amount of spectrum is still considered a limited resource. Beside, to differentiate between the true signal and background noise is complex for a radio equipment. Generally, this complex process enforces terminals to transmit strong version of signals, that wastes energy of a transmitter.

The modern wireless entities, i.e. wireless terminals and base stations, have considerable capacities to execute dynamic processes. This capability encourages wireless service providers to consider wireless entities as autonomous agents which could cooperate and negotiate with each other to achieve an efficient resource allocation in different situations. Cooperation among wireless terminals is usually intended to achieve a fair radio resource allocation. Cooperation between base stations can be devised to mitigate interference, and promote soft handover where channel gain is varying rapidly which is a challenge in LTE [21].

Game theory is the most prominent tool to analyze interaction issue in social sciences wherein often cooperation amongst autonomous agents is essential for successful task completion. In many settings, groups of competing agents are simultaneously concerned of both individual and overall benefits. In the game theory literature, this branch is known as cooperative game [10, 22]. The players, as the main decision making entities in the game, are considered to negotiate with each other to determine a binding agreement among them. If we assume that all users act rationally and we know what the behavior of the users are, it is possible to determine the overall performance of a system since the actions of one user becomes part of the circumstances for another user. Thus, we are interested in individual performance and overall system performance under a specific set of rules. To fully develop the different possibilities within a game for cooperation among players we have to address which groups the players can achieve collectively. Indeed, if a player assesses that within a certain group it does not receive what it is able to get by itself, then it might decide to abandon the cooperation and pursue an alternative allocation by itself. Cooperative game theory offers the opportunity to extend and expand the treatment of the players in traditional non-cooperative games, especially where selfish players compete over a set of resources. The cooperative game theory is divided into two parts: coalitional game theory and bargaining games [10, 22]. In this contribution we focus on coalitional game theory.

Saad *et al.* in tutorial paper [23] classify coalitional games into three categories: canonical (coalitional) games, coalition formation games and coalitional graph games. In canonical games, no group of players can do worse by joining a coalition than by acting non-cooperatively. In coalition formation games, forming a coalition brings advantage to its members but the gains are limited by a cost for forming the coalition. In coalitional graph games, the coalitional game is in graph form and the interconnection between the players strongly affects the characteristics as well as the outcome of the game.

In the last few years, cooperative game theory has been successfully applied to communications and networking. Hossain *et al.* in [24] provides a guide to state-of-art which unifies the essential information, addressing both theoretical and practical aspects of cooperative communications and networking in the context of cellular design. The current literature is mainly focused on applying cooperative games in various applications such as distributed/centralized radio resource allocation [18,25,26], power control [27, 28], spectrum sharing in cognitive radio [29, 30], cooperative automatic repeat request (ARQ) mechanism [31], cooperative routing [32], and cooperative communications [33, 34]. These problems in wireless networks can be modeled as a cooperative game since it is highly likely that each wireless user can obtain a better utility value by forming groups and controlling resources cooperatively rather than individually. It has been shown that cooperation can result in an enhanced QoS in terms of throughput expansion, bit error rate reduction, or energy saving [24].

Cooperation can be realized at various layers of the network. At the physical layer, different separate antennas can constitute a cluster and then cooperate with each other to exploit multiple-input multiple-output (MIMO) gains. At the MAC sublayer, some wireless terminals can cooperate with each other to share a common wireless medium in an efficient manner and consequently mitigate the interference hazard. There is also the possibility of cooperation of physical and application layers among individual terminals to adapt channel and source codings in multimedia communications.

The altruistic decision of cooperation with others network entities may result in an improvement on overall network performance, and concurrently achieve an egoistic interest of self improvement.

1.2 Preliminaries

Game theory deals with the study, through mathematical models, of conflict situations in which two or more rational players make decisions that will influence each other's welfare. The theory of coalitional games [10,22] also assumes that binding agreements may be established among the players in the course of the conflict situation. In transferable utility (TU) games, agreement may be reached by any subset of the players, and the gain obtained from this agreement is a real number and it is transferable among these players. In non-transferable utility (NTU) games, agreement may be reached by any subset of the players, but the gain may be non-transferable. The main focus of this dissertation will be on the study of TU games.

A TU game is a pair $\mathcal{G} = (\mathcal{K}, \nu)$, where $\mathcal{K} = [1, \ldots, K]$ denotes the set of players and ν the *coalition (characteristic) function* which is interpreted as the maximum outcome (a real number) to each coalition (subset of $\mathcal{K}$) whose players can jointly produce. An NTU game is a pair $\mathcal{G} = (\mathcal{K}, V)$ where V is a mapping which for each coalition $\mathcal{A}$, defines a characteristic set, $V(\mathcal{A})$, satisfying:

1. $V(\mathcal{A})$ is non-empty and closed subset of $\mathbb{R}^{|\mathcal{A}|}$,
2. For each $k \in \mathcal{A}$ there is a $V_k \in \mathbb{R}$ such that $V(\{k\}) = (-\infty\,, V_k]$,
3. $V(\mathcal{A})$ is comprehensive, i.e. for all $\mathbf{u} \in V(\mathcal{A})$ and for all $\mathbf{u}' \in \mathbb{R}^{|\mathcal{A}|}$, if $\mathbf{u}'[k] \leq \mathbf{u}[k]\ \forall\, k \in \mathcal{A}$ then $\mathbf{u}' \in V(\mathcal{A})$,
4. The set $V(\mathcal{A}) \bigcap \left\{\mathbf{u}' \in \mathbb{R}^{|\mathcal{A}|} \mid \mathbf{u}'[k] \geq V_k \ \ \forall\, k \in \mathcal{A}\right\}$ is bounded.

The characteristic set, $V(\mathcal{A})$, is interpreted as the set of achievable outcomes the players in $\mathcal{A}$ can guarantee themselves without cooperating with the players in $\mathcal{K}\backslash\mathcal{A}$. In particular, an NTU-game $\mathcal{G} = (\mathcal{K}, V)$ is called a TU game when the characteristic set for each coalition $\mathcal{A}$, takes the form:

$$V(\mathcal{A}) = \left\{\mathbf{u} \in \mathbb{R}^{|\mathcal{A}|} : \sum_{k \in \mathcal{A}} u_k \leq \nu(\mathcal{A})\right\} \tag{1.1}$$

where $\mathbf{u} = \left[u_1, \ldots, u_{|\mathcal{A}|}\right] \in \mathbb{R}^{|\mathcal{A}|}$ and u_k is the payoff of player k in $\mathcal{A}$ and $\nu : 2^{\mathcal{K}} \longrightarrow \mathbb{R}$. If $\mathcal{A}$ is a coalition (subset) of $\mathcal{K}$ formed in $\mathcal{G}$, then its members get an overall payoff $\nu(\mathcal{A})$, zero for the empty set. Each coalition can be represented as a pure strategy in non-cooperative game theory. There exist only few works on NTU games applications to problems in communications [35]. This is because defining an utility function which meets all conditions of a character set in NTU game is not always feasible.

An important property of interest in characteristic form TU games is *superadditivity*, which, if present, implies that the value of the unite of any two disjoint coalitions is at least as big as the sum of their values.

Definition 1 *A TU game $\mathcal{G}$ is superadditive if*

$$\nu(\mathcal{A}_i \cup \mathcal{A}_j) \geq \nu(\mathcal{A}_i) + \nu(\mathcal{A}_j) \quad \forall\, \mathcal{A}_i, \mathcal{A}_j \subset \mathcal{K} \;\; s.t. \;\; \mathcal{A}_i \cap \mathcal{A}_j = \varnothing \tag{1.2}$$

In a superadditive TU game there are positive synergies and the players prefer to join each other rather than act alone. Under superadditivity condition, the players are willing to form the *grand coalition* (the set $\mathcal{K}$).

Convex, or alternatively *supermodular* coalitional games were introduced by L. Shapley [36]. They model coalitional situations where the marginal contribution of a player to a coalition increases as the coalition becomes larger.

Definition 2 *A TU game $\mathcal{G}$ is convex or supermodular if for all $k \in \mathcal{K}$:*

$$\nu(\mathcal{A}_i \cup \{k\}) - \nu(\mathcal{A}_i) \;\leq\; \nu(\mathcal{A}_j \cup \{k\}) - \nu(\mathcal{A}_j) \quad \forall\, \mathcal{A}_i \subseteq \mathcal{A}_j \subset \mathcal{K} \backslash \{k\} \tag{1.3}$$

Equivalently:

Definition 3 *A TU game $\mathcal{G}$ is convex or supermodular if:*

$$\nu(\mathcal{A}_i) + \nu(\mathcal{A}_j) \;\leq\; \nu(\mathcal{A}_i \cap \mathcal{A}_j) + \nu(\mathcal{A}_i \cup \mathcal{A}_j) \quad \forall\, \mathcal{A}_i, \mathcal{A}_j \subseteq \mathcal{K} \tag{1.4}$$

Convexity means that there are increasing returns to scale. Note that a convex game is superadditive. To better understand the importance of convexity approach in network probems, we verify the convexity condition in a K-user channel access game. The payoff of each coalition of players (transmitters) is defined as the outer MAC capacity region. [37, Lemma 1] shows that in a multiple access channel scenario, the inequality (1.4) is not met. This means the game is not convex, and thus adding a new player does not give benefit to others transmitters.

1.3 The core solution

A central question in a coalitional game is how to divide the extra earnings (or cost savings) among the members of the formed coalition. In a TU game, an allocation is a function $\mathbf{u}$ from $\mathcal{K}$ to $\mathbb{R}$ that specifies for each player $k \in \mathcal{K}$ the payoff $u_k \in \mathbb{R}$ that this player can expect when it cooperates with the other players. The payoff of each player can show the cost borne by the player, the power of influence, and so on depending on the problem setting.

Definition 4 *Let $\mathcal{K}$ be the set of K players of the superadditive TU game $\mathcal{G}$, and let ν be the payoff of the game. The set of all "imputations" of $\mathcal{G}$ is the set:*

$$\mathcal{I}(\mathcal{K}, \boldsymbol{\nu}) = \left\{ \mathbf{u} \in \mathbb{R}^K : \left| \begin{array}{ll} i) & \sum_{k \in \mathcal{K}} u_k = \nu(\mathcal{K}) \\ ii) & u_k \geq \nu(\{k\}) \quad \forall\, k \in \mathcal{K} \end{array} \right. \right\} \tag{1.5}$$

where $\mathbf{u} = [u_1, \ldots, u_k, \ldots, u_K] \in \mathbb{R}^K$ is the imputation vector of the players. The former condition is called the feasibility*, and the latter* individually rational *condition.*

The *core* concept was introduced in [38] and is the most attractive and natural way to define a payoff distribution: if a payoff distribution is in the core, no agent has any incentive to be in a different coalition. The core of a TU game is the subset of all imputations $\mathbf{u} \in \mathcal{I}(\mathcal{K}, \boldsymbol{\nu})$ that no other imputation *directly dominates*, that is $\nexists\, \mathbf{u}' \in \mathcal{I}(\mathcal{K}, \boldsymbol{\nu})$ s.t. $u'_k > u_k \;\; \forall\, k \in \mathcal{K}$. As can be seen, for coalitional games as well as non-cooperative games, the notion of dominance is essentially equivalent; the payoffs under the various situations are compared and one situation dominates the others if these payoffs are higher. The core actually presents a condition stronger than Nash equilibrium in non-cooperative game: no group of agents should be able to profitably deviate from a configuration in the core. Equivalently, no set of players can benefit from forming a new coalition, which corresponds to the group rationality assumption.

In an NTU game $\mathcal{G} = (\mathcal{K}, V)$, the core apportionment is defined as [22, Ch. 12]:

Definition 5 *Let $\mathcal{K}$ be the set of K players of the superadditive NTU-game $\mathcal{G}$, and let V be the payoff of the game. The core of $\mathcal{G}$ is the set*

$$\mathcal{S}(\mathcal{K}, \mathbf{V}) = \{\, \mathbf{u} \in V(\mathcal{K}) : \;\; \forall\, \mathbf{u}' \in V(\mathcal{A}) \;\; \exists\, k \in \mathcal{A} \;\; s.t. \;\; u_k \geq u'_k \,\} \tag{1.6}$$

where $\mathbf{u}$ is the payoff distribution across players, and $u_k \in \mathbf{u}$ if and only if no coalition can improve upon u_k.

In a TU game $\mathcal{G} = (\mathcal{K}, \nu)$, the core apportionment is defined as follows:

Definition 6 *Let $\mathcal{K}$ be the set of K players of the superadditive TU game $\mathcal{G}$, and let ν be the payoff of the game. The core of $\mathcal{G}$ is the set*

$$\mathcal{S}(\mathcal{K}, \boldsymbol{\nu}) = \left\{ \mathbf{u} \in \mathbb{R}^K : \left| \begin{array}{ll} i) & \sum_{k\in\mathcal{K}} u_k = \nu(\mathcal{K}) \\ ii) & \sum_{k\in\mathcal{A}} u_k \geq \nu(\mathcal{A}) \quad \forall\, \mathcal{A} \subset \mathcal{K} \end{array} \right. \right\} \tag{1.7}$$

where $\mathbf{u} = [u_1, \ldots, u_k, \ldots, u_K] \in \mathbb{R}^K$ is the payoff distribution across players, and $u_k \in \mathbf{u}$ if and only if no coalition can improve upon u_k. The second condition is called non-blocking *condition.*

The core consists of the set of allocations that can be blocked by any coalition of agents. If for some set of agents $\mathcal{A}$, the non-blocking condition does not hold, then the agents in $\mathcal{A}$ have an incentive to collectively deviate from the coalition structure and to divide $\nu(\mathcal{A})$ among themselves. In general, the core of a given TU game $(\mathcal{K}, \nu)$ is found by linear programming (LP) as:

$$\min_{\mathbf{u}\in\mathbb{R}^K} \sum_{k\in\mathcal{K}} u_k\,; \qquad \text{s.t.} \sum_{k\in\mathcal{A}} u_k \geq \nu(\mathcal{A}) \quad \forall\, \mathcal{A} \subseteq \mathcal{K} \tag{1.8}$$

Madiman in [39] introduces some intuitive applications of core solution to information theory contexts e.g. source coding and multiple-access channel, and summarize some of its limitations in multi user scenarios. Li *et al.* in [40] show that the cooperation among wireless nodes and core apportionment can increase spectrum efficiency in a TDMA cooperative communication. In [41], Niyato *et al.* applies the core solution in a coalition among different wireless access networks to offer a stable and efficient bandwidth allocation.

Indeed, there is a number of realistic application scenarios, in which the emergence of the grand coalition is either not guaranteed, or might be perceivably harmful, or is plainly impossible [42]. For a non-superadditive coalitional game, the coalition formation process does not lead the players to form the grand coalition. In this case, Def. 6 does not apply. Let us redefine the core set in a general (not necessarily superadditive) coalitional formation TU game [26]. Let $\psi = [\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_m]$ denote a partition of the set $\mathcal{K}$ wherein $\mathcal{A}_i \cap \mathcal{A}_j = \varnothing$ for $i \neq j$, $\bigcup_{i=1}^m \mathcal{A}_i = \mathcal{K}$ and $\mathcal{A}_i \neq \varnothing$ for $i = 1, \ldots, m$, and let $\boldsymbol{\Psi}$ denote the set of all possible partitions ψ. Let us also define $\mathcal{F} = [\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_n]$, such that $\bigcup_{i=1}^{n} \mathcal{A}_i = \mathcal{K}$ and $\mathcal{A}_i \neq \varnothing$ for $i = 1, \dots, n$, as a family of (not necessarily disjoint) coalitions.

Definition 7 *A "core apportionment"* $\mathbf{u} \in \mathbb{R}^K$ *is a payoff distribution with the following property:*

$$\mathcal{S}(\mathcal{K}, \boldsymbol{\nu}) = \left\{ \mathbf{u} \in \mathbb{R}^K : \left| \begin{array}{ll} i) & \sum_{k \in \mathcal{K}} u_k = \max_{\psi \in \boldsymbol{\Psi}} \sum_{\mathcal{A} \in \psi} \nu(\mathcal{A}) \\ ii) & \sum_{k \in \mathcal{A}} u_k \geq \nu(\mathcal{A}) \quad \forall \mathcal{A} \subset \mathcal{K} \end{array} \right. \right\} \tag{1.9}$$

Note that, if $\mathcal{G}$ is superadditive, then $\max_{\psi \in \boldsymbol{\Psi}} \sum_{\mathcal{A} \in \psi} \nu(\mathcal{A}) = \nu(\mathcal{K})$.

The core allocation set can be found through linear programming and its existence, in general, depends upon the feasibility of (1.8). Unfortunately, the core is a strong notion, and there exist many games where it is empty. We can study the non-emptiness of the core without explicitly solving the core equation. The following notation helps to simplify the dual of (1.8).

Definition 8 *A superadditive TU game $\mathcal{G}$ for a family $\mathcal{F}$ of coalitions is* totally balanced *if, for any $\mathcal{A} \in \mathcal{F}$, the inequality*

$$\sum_{\mathcal{A} \in \mathcal{F}} \mu_{\mathcal{A}} \cdot \nu(\mathcal{A}) \leq \nu(\mathcal{K}) \tag{1.10}$$

holds, where $\mu_{\mathcal{A}}$ is a collection of numbers in $[0, 1]$ *(balanced collection of weights) such that*

$$\sum_{\mathcal{A} \in \mathcal{F}} \mu_{\mathcal{A}} \cdot \mathbf{1}_{\mathcal{A}} = \mathbf{1}_{\mathcal{K}} \tag{1.11}$$

with $\mathbf{1}_{\mathcal{A}} \in \mathbb{R}^K$ denoting the characteristic vector whose elements are

$$(\mathbf{1}_{\mathcal{A}})[i] = \begin{cases} 1, & i \in \mathcal{A} \\ 0, & \textit{otherwise} \end{cases} \tag{1.12}$$

The following pathbreaking result in the theory of TU games was independently gave by Bondareva [43] and L. Shapley [44].

Lemma 1 ([10]) *A totally balanced TU game has a non-empty core set.*

Where forming the grand coalition is not guaranteed, the following notation is applied.

Definition 9 *A (not necessarily superadditive) TU game $\mathcal{G}$ for a family $\mathcal{F}$ of coalitions is totally balanced if, for every balanced collection of weights $\mu_{\mathcal{A}}$, and for any $\mathcal{A} \in \mathcal{F}$,*

$$\sum_{\mathcal{A}\in\mathcal{F}} \mu_{\mathcal{A}} \cdot \nu\left(\mathcal{A}\right) \leq \max_{\psi\in\mathbf{\Psi}} \sum_{\mathcal{A}\in\psi} \nu\left(\mathcal{A}\right) \tag{1.13}$$

So, if a TU game is totally balanced, then the core is non empty and therefore it is a convenient solution concept on the class of totally balanced TU games. There is an interesting relation between convex and balanced games.

Lemma 2 ([22]) *A convex game is totally balanced, but the converse is not necessarily true.*

The other key feature of coalitional convex games is

Lemma 3 (L. Shapley [36]) *The core set of a convex game is unique.*

Now, we illustrate an intuitive example of *power distribution* based on core set solution. This example is an extended form of the example established by [45, Ch. 12]. The network sketched in Fig. 1.1 wishes to allocate power among three players $\mathcal{K} = \{k_1, k_2, k_3\}$, according to their will to cooperate with each other. A power of 1 mW is provided to the network if three players decide to cooperate, or equivalently if the grand coalition will form. If only one player refuses to cooperate, a power of 0.8 mW will be assigned to the pair of cooperating nodes. The coalition game of Fig. 1.1 is defined by:

$$\nu\left(\mathcal{A}\right) = \begin{cases} 0 & \text{if } \left|\mathcal{A}\right| = 1; \\ 0.8 & \text{if } \left|\mathcal{A}\right| = 2; \\ 1 & \text{if } \left|\mathcal{A}\right| = 3. \end{cases} \tag{1.14}$$

The players of each coalition will cooperate with each other. The player of a singleton coalition will be isolated.

Each player receives a positive payoff if it decides to cooperate, whereas all players receive zero if no agreement is bound. To divide the total payoff (power) in some appropriate way, we rest on the core set definition. It is straightforward to show

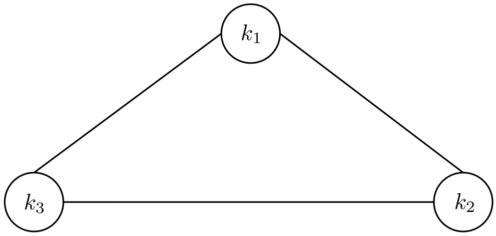


Fig. 1.1: *The network allocates power among three players according to their will to cooperate with each other. A selfish player receives zero, a pair of cooperative players receive* $0.8\,\mathrm{mW}$*, and the network supply* $1\,\mathrm{mW}$ *to the grand coalition.*

that the coalitional TU game defined by (1.14) is superadditive. From Eqs. 1.3 and 1.4, it is easy to show that TU game (1.14) is not convex (supermodular). To check whether the core set of TU game (1.14) is empty or not, we resort to the balanced solution. TU game (1.14) is not balanced even though assigning the balanced weights as $\mu_{\mathcal{A}} = 1$ for singleton coalitions, and $\mu_{\mathcal{A}} = 0$ otherwise, inequality (1.10) holds. By using the fact that there exists other balanced collection of weights in which $\mu_{\mathcal{A}} = \frac{1}{2}$ for $|\mathcal{A}| = 2$, and $\mu_{\mathcal{A}} = 0$ otherwise, the game is not balanced, and its core set may be empty. Note that, this result *does not* mean that the core set of the game is *surely* empty.

Now, we heuristically find a core apportionment studying various possible networks. When there is no cooperation among players, the players are not provided with any power. That is, $\mathcal{F} = [\,\{\,k_1\,\}\,,\,\{\,k_2\,\}\,,\,\{\,k_3\,\}\,]$ with payoff distribution:

$$u_{k_1} = u_{k_2} = u_{k_3} = 0$$

If only one player decides to stay alone, the payoff 0.8 is equally divided between the two cooperative players and the isolated player gets zero. That is, for instance, $\mathcal{F} = [\,\{\,k_1\,,\,k_2\,\}\,,\,\{\,k_3\,\}\,]$ with payoff distribution:

$$\begin{cases} u_{k_3} = 0 \\ u_{k_1} = u_{k_2} = 0.4 \end{cases}$$

Now, we suppose a player, for example, k_2 decides to cooperate with both k_1 and k_3, but the two players k_1 and k_3 do not bind an agreement to mutually cooperate. It is reasonable to suppose that the player k_2 can act as a relay between k_1 and k_3

and it must be provided more power. That is $\mathcal{F} = [\{k_1, k_2\}, \{k_2, k_3\}]$ with payoff distribution:

$$\begin{cases} u_{k_1} = u_{k_3} = 0.2 \\ u_{k_2} = 0.6 \end{cases}$$

Finally, in the complete network each player receives the same payoff. That is $\mathcal{F} = [\{k_1, k_2, k_3\}]$ with payoff distribution:

$$u_{k_1} = u_{k_2} = u_{k_3} = 1/3$$

As can be easily seen, the above argument satisfies feasibility and non-blocking conditions of the core set apportionment in Def. 6. It is worthwhile to note that the core set definition does not imply an even division of the whole payoff across players. Thus, it is clear that this game consists of multiple core sets. The power distribution problem can also be solved by cooperative game-theoretic bargaining solutions; e.g. Nash bargaining game and Auction [10].

1.3.1 On core stability

The goal of the network Fig. 1.1 is to allocate power among players in order to stimulate all of them to cooperate. Obviously, each player tries to get the highest possible payoff. Let us predict the behavior of the players after having known the definition of the game. Suppose that the players k_1 and k_2 find an opportunity to meet each other. Obviously, they quickly take advantage to cooperate and achieve payoff distribution $\mathbf{u} = [0.4, 0.4, 0]$. Then, it is profitable for player k_1 to invite player k_3 to join and therefore, improving its own payoff from 0.4 to 0.6 and that of player k_3 from zero to 0.2. On the other hand, this new agreement causes a decreasing payoff of player k_2 from 0.4 to 0.2, and now the players k_2 and k_3 have an incentive to cooperate and increase their proper payoff from 0.2 to $1/3$. Note that this agreement makes the player k_1's payoff dcrease from 0.6 to $1/3$. The unfavorable decision of player k_2 would tempt player k_1 to retaliate. A negotiation between k_1 and k_3 to release cooperation with k_2 results increasing their payoffs and boiling down k_2's payoff to zero. The result of above argument concerns that the network is sustained by only one pair cooperation under the threat of: "If you cooperate with the third player, then I will do the same".[1] It is fairly clear that the players would seek to

[1] *Two is cooperation, three is a crowd.*

cooperate only as pairs for the purpose of negotiation, and not cooperate in the grand coalition framework, even though the game is superadditive. This is due to fact of being superadditive but not balanced. The pairs can be changed as time goes on. In fact, the core apportionment suffers the lack of "farsighted" (i.e., long-term) stability.

A coalition structure based on core set, is not adequately farsighted to avoid the elusiveness of negotiation structure. At first sight, the core appears to be an extremely myopic notion, requiring the stability of a proposed allocation to deviations or blocks by coalitions, but not examining the stability of the deviations themselves. In general, the stability requirement is that the outcome be immune to deviations of a certain sort by coalitions. To provide the formal definition of farsighted stability, we need some additional notation.

Definition 10 ([46]) *For* $\mathbf{u}, \mathbf{u}' \in \mathcal{I}(\mathcal{K}, \boldsymbol{\nu})$, $\mathbf{u}$ indirectly dominates $\mathbf{u}'$, *which is denoted by* $\mathbf{u}' \ll \mathbf{u}$, *if there exist a finite sequence of imputations* $\mathbf{u}' = \mathbf{u}_1, \mathbf{u}_2, \ldots,$ $\mathbf{u}_m = \mathbf{u}$ *and a finite sequence of nonempty coalitions* $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_m$ *such that for each* $j = 1, 2, \ldots, m-1$*: i) by the deviation of* $\mathcal{A}_j$*, the imputation of* $\mathbf{u}_j$ *is replaced to* $\mathbf{u}_{j+1}$*, and ii)* $\mathbf{u}_j[k] < \mathbf{u}[k]$ *for all* $k \in \mathcal{A}_j$*.*

Condition (i) says that each coalition $\mathcal{A}_j$ has the power to replace imputation $\mathbf{u}_j$ by imputation $\mathbf{u}_{j+1}$, and the condition (ii) says that each player in $\mathcal{A}_j$ strictly prefers imputation $\mathbf{u}$ to imputation $\mathbf{u}_j$. It is clear that the indirect dominance relation contains the direct dominance relation.

Definition 11 ([46, 47]) *Let* $\mathcal{G} = (\mathcal{K}, \nu)$ *be a TU game. A subset* $\mathcal{J}$ *of* $\mathcal{I}(\mathcal{K}, \boldsymbol{\nu})$ *is a* farsighted stable *set if: i) for all* $\mathbf{u}, \mathbf{u}' \in \mathcal{J}$*, neither* $\mathbf{u} \ll \mathbf{u}'$ *nor* $\mathbf{u}' \ll \mathbf{u}$*, and ii) for all* $\mathbf{u}' \in \mathcal{I}(\mathcal{K}, \boldsymbol{\nu}) \backslash \mathcal{J}$ *there exists* $\mathbf{u} \in \mathcal{J}$ *such that* $\mathbf{u}' \ll \mathbf{u}$*. Conditions i) and ii) are called* internal stability *and* external stability*, respectively.*

By internal stability, there is no imputation in $\mathcal{J}$ that is dominated by another imputation in $\mathcal{J}$. By external stability, an imputation outside a stable set $\mathcal{J}$ is unlikely to be attained. Let us introduce three other different payoff distribution concepts which capture foresight of the players.

1.4 Shapley value

The Shapley value is an alternative solution for the payoff distribution in TU games. The Shapley value has long been a central solution concept in coalitional game theory. It was introduced by L. S. Shapley in the seminal paper [48] and it was seen as a reasonable way of distributing the gains of cooperation, in a fair and unique way, among the players in the game. In the Shapley solution, those who contribute more to the groups that include them are paid more. Let us denote $\phi_k(\nu)$ as the Shapley value of player k in the TU game defined by ν. The surprising result due to Shapley is the following theorem.

Theorem 1 *There is a unique single-valued solution to TU games satisfying efficiency, symmetry, additivity and dummy. It is the well-known Shapley value, the function that assigns to each player k the payoff:*

$$\phi_k(\nu) = \sum_{\substack{\forall \mathcal{A} \subseteq \mathcal{K} \\ s.t.\ k \in \mathcal{A}}} \frac{(|\mathcal{A}|-1)! \cdot (K-|\mathcal{A}|)!}{K!} \left(\nu(\mathcal{A}) - \nu(\mathcal{A} \backslash \{k\}) \right) \tag{1.15}$$

The expression $\nu(\mathcal{A}) - \nu(\mathcal{A}\backslash\{k\})$ is the marginal payoff of player k to the coalition $\mathcal{A}$. The Shapley value can be interpreted as the expected marginal contribution made by a player to the value of a coalition, where the distribution of coalitions is such that any ordering of the players is equally likely. The Shapley value can be interpreted as follows [48]. Suppose the grand coalition of all players is being formed in a sequence by including one player at a time. In general, the marginal contribution of a particular player depends on its position in the sequence, which is determined randomly. There are $K!$ possible orderings of the players in total. Consider any coalition $\mathcal{A}$ containing k and observe that the probability that player k enters the coalition to find precisely the players in $\mathcal{A}\backslash\{k\}$ already there is $\frac{(|\mathcal{A}|-1)! \cdot (K-|\mathcal{A}|)!}{K!}$. That is, out of $K!$ permutations of $\mathcal{K}$, there are $(|\mathcal{A}|-1)!$ different orders of in which the players in $\mathcal{A}\backslash\{k\}$ can preceed k and $(K-|\mathcal{A}|)!$ orders in which the remaining $\mathcal{K}\backslash\mathcal{A}$ can follow. When leaving a coalition $\mathcal{A}$, a player k receives the amount by which his exclusion changes (increase or decreases) the profits of the group, $\nu(\mathcal{A}) - \nu(\mathcal{A}\backslash\{k\})$. That makes the Shapley value exponentially hard to compute. Shapley characterized such value as the unique solution that satisfies the following four axioms:

1. *Efficiency*: The payoffs must add up to $\nu(\mathcal{K})$, which means that all the grand

coalition surplus is allocated. That is:

$$\sum_{k \in \mathcal{K}} \phi_k(\nu) = \nu(\mathcal{K})$$

In the absence of superadditivity, instead we use: $\max_{\psi \in \mathbf{\Psi}} \sum_{\mathcal{A} \in \psi} \nu(\mathcal{A})$.

2. *Symmetry*: This axiom requires that the names of the players play no role in determining the value. If two players are substitutes because they contribute the same to each coalition, the solution should treat them equally. That is:

$$\nu(\mathcal{A} \cup \{k\}) = \nu(\mathcal{A} \cup \{i\}) \implies \phi_k(\nu) = \phi_i(\nu).$$

3. *Additivity*: The solution to the sum of two TU games must be the sum of what it awards to each of the two games. That is:

$$\phi_k(\nu + \omega) = \phi_k(\nu) + \phi_k(\omega) \qquad \forall\, k \in \mathcal{K}.$$

4. *Dummy player*: The player k is dummy (null) if $\nu(\mathcal{A} \cup \{k\}) = \nu(\mathcal{A})$ for all $\mathcal{A}$ not containing k. If a player k is dummy, the solution should pay it nothing; i.e. $\phi_k(\nu) = 0$.

The Shapley value is a feasible allocation, but need not be individually rational. Whenever the TU game is superadditive, the Shapley value is feasible and individually rational, but need not be in the core and hence can be directly dominated by another imputation. Reference [36] shows that the Shapley value of a supermodular TU-game is a core imputation, that is, the Shapley value is not dominated. For a superadditive TU game The Shapley value is an internal and external stable imputation, and for NTU games, it is formulated in [49, 50]. To make an example, let us calculate the Shapley value of the players in the power distribution game of Fig. 1.1:

$$\nu = \begin{cases} 0 & \{k_1\}, \{k_2\}, \{k_3\}; \\ 0.8 & \{k_1, k_2\}, \{k_1, k_3\}, \{k_2, k_3\}; \\ 1 & \{k_1, k_2, k_3\}. \end{cases} \implies$$

$$\phi_{k_1}(\nu) = \phi_{k_2}(\nu) = \phi_{k_3}(\nu) =$$
$$0 + \frac{1!\,.\,1!}{3!}(0.8 - 0) + \frac{1!\,.\,1!}{3!}(0.8 - 0) + \frac{2!\,.\,0!}{3!}(1 - 0.8) = 1/3.$$

Young in [51] defines an equivalent definition for Shapley value. He withdraws the additivity axiom, and instead, adds an axiom of marginality.

1. *Marginality*: If the marginal contribution to coalitions of a player in two games is the same, then the the award of the player must be the same. That is, if:

$$\nu\left(\mathcal{A}_i\right)-\nu\left(\mathcal{A}_i \backslash\{k\}\right)=\omega\left(\mathcal{A}_j\right)-\omega\left(\mathcal{A}_j \backslash\{k\}\right) \quad \forall\, \mathcal{A}_i \in \nu \text{ and } \forall\, \mathcal{A}_j \in \omega\,,$$

then $\phi_k(\nu)=\phi_k(\omega)$.

Marginality is an idea with a strong tradition in economic theory. In Young's definition, marginality is assumed and additivity is dropped. Young in [51] shows that the Shapley value is unique.

Theorem 2 (Young [51]) *There exists a unique single-valued solution to TU games satisfying efficiency, symmetry and marginality, and this solution is the Shapley value.*

In the network engineering literature, S. Kim in [52] proposes an energy efficient routing protocol based on the Shapley value. The concept of Shapley value is used by Khouzani *et al.* [53] to achieve a fair aggregate cost of link sharing, among primary and secondray users in a cognitive network. Using the Shapley value, a suitable network resource sharing among multimedia users is fairly achievable, as Park *et al.* propose in [54].

1.5 The kernel and nucleolus

Let $\mathcal{G}=(\mathcal{K}, \nu)$ be a coalitional game with transferable payoff. The *excess* of the coalition $\mathcal{A}$ with respect to the payoff vector $\mathbf{u} \in \mathbb{R}^K$ is defined as

$$e\left(\mathcal{A}\,, \mathbf{u}\right)=\nu\left(\mathcal{A}\right)-\sum_{k \in \mathcal{A}} u_k \tag{1.16}$$

A positive excess can be interpreted as an incentive for a coalition to generate more utility. Using the excess notion, the core apportionment in a TU game can be redefined as:

$$\left\{\, \mathbf{u} \in \mathbb{R}^K: \ e\left(\mathcal{K}\,, \mathbf{u}\right)=0 \quad \text{, and} \quad e\left(\mathcal{A}\,, \mathbf{u}\right) \leq 0 \quad \forall\, \mathcal{A} \subset \mathcal{K} \,\right\} \tag{1.17}$$

The *maximum excess* of player k against i is defined as

$$s_{ki}\left(\mathbf{u}\right)=\max\left\{\, e\left(\mathcal{A}\,, \mathbf{u}\right) \mid \mathcal{A} \subset \mathcal{K},\ \ k \in \mathcal{A},\ \ i \in \mathcal{K} \backslash \mathcal{A} \,\right\} \tag{1.18}$$

If player k departs from $\mathbf{u}$, the most it can hope to gain (the least to lose) without the consent of player i is the amount of maximum excess. Extensions of the excess for NTU games is formalized in [55].

As defined by Osborne and Rubinstein [10, Ch. 14], a coalition $\mathcal{A}_i$ is an *objection* of k against i to $\mathbf{u}$, if $\mathcal{A}_i$ includes k but not i and $u_i > \nu(\{i\})$. Equivalently, $\mathcal{A}_i$ is a coalition that contains k, excludes i and which gains too little. A coalition $\mathcal{A}_j$ is a *counter-objection* to the objection $\mathcal{A}_i$ of k against i, if $\mathcal{A}_j$ includes i but not k and $e(\mathcal{A}_j\,,\mathbf{u}) \geq e(\mathcal{A}_i\,,\mathbf{u})$. Equivalently, $\mathcal{A}_j$ is a coalition that contains i and excludes k and that gains even less. Objections and counter-objections are exchanged between members of the same coalition in $\mathcal{A}_i$.

The idea captured by the *kernel* is that if at a non empty imputation $\mathbf{u}$ the maximum excess of player k against any other player i is less than the maximum excess of player i against the player k, then player k should get less. Of course, the players cannot get less than their individual worths if $\mathbf{u}$ is an imputation. The definition of the kernel follows:

Definition 12 *The kernel is the set of all imputations* $\mathbf{u}$ *with the property that for every objection* $\mathcal{A}_i$ *of any player* k *against any other player* i *to* $\mathbf{u}$ *there is a counter-objection of* i *to* $\mathcal{A}_i$ *such that:*

a) $s_{ki}(\mathbf{u}) = s_{ik}(\mathbf{u})\,;$ *or*

b) $s_{ki}(\mathbf{u}) < s_{ik}(\mathbf{u})$ *and* $u_k = \nu(\{k\})\,;$ *or*

c) $s_{ki}(\mathbf{u}) > s_{ik}(\mathbf{u})$ *and* $u_i = \nu(\{i\})$.

The kernel is the set of imputations $\mathbf{u}$ such that for any coalition $\mathcal{A}_i$, for each objection $\mathcal{A}_j$ of a user $k \in \mathcal{A}_i$ over any other member $i \in \mathcal{A}_i$, there is a counter-objection of i to $\mathcal{A}_j$. The kernel is contained in the (nonempty) core in any assignment game ν [56, Th. 1]. In Fig. 1.1, the unique kernel element is the equal split $\mathbf{u} = [\,1/3\,,\,1/3\,,\,1/3\,]$, otherwise for the single player coalition objection of the player with the minimum payoff, there is no any counter-objection.

The last type of a stable imputation we will study is the *nucleolus*. With the nucleolus no confusion regarding the player set can arise. The basic motivation behind the nucleolus is that one can provide an allocation that minimizes the excess of the coalitions in a given coalitional game $\mathcal{G} = (\mathcal{K}, \nu)$. For a TU game $\mathcal{G} = (\mathcal{K}, \nu)$ and the payoff vector $\mathbf{u} \in \mathbb{R}^K$, let us denote $\mathbf{E}(\mathbf{u}) = [\cdots \geq e(\mathcal{A}, \mathbf{u}) \geq \cdots : \emptyset \neq \mathcal{A} \neq \mathcal{K}]$ as a $2^K - 2$ dimensional vector whose components are the values of the excess function for all $\mathcal{A} \subset \mathcal{K}$, arranged in a non-increasing order. The nucleolus of a game is the imputation which minimizes the excess with respect to the lexicographic order [2] over the set of imputations. The nucleolus of $\mathcal{G}$ with respect to $\mathcal{I}(\mathcal{K}, \boldsymbol{\nu})$ is given by:

$$\{\mathbf{u} \in \mathcal{I}(\mathcal{K}, \boldsymbol{\nu}) \mid \mathbf{E}(\mathbf{u}) \preceq_{lex} \mathbf{E}(\mathbf{u}') \quad \forall\, \mathbf{u}' \in \mathcal{I}(\mathcal{K}, \boldsymbol{\nu})\} \tag{1.19}$$

The definition of the nucleolus of a coalitional game in characteristic function form entails comparisons between vectors of exponential length. Thus, if one attempts to compute the nucleolus by simply following its definition, it would take an exponential time. In the network engineering literature, Han and Poor in [57] apply the Shapley value, excess and nucleolus solutions to study a possible cooperative transmission among intermediate nodes to help relay the information of wireless users.

This defining property makes the nucleolus appealing as a fair single-valued solution. It is easy to see that, whenever the core of a game is nonempty, the nucleolus lies in it [22]. Moreover, the nucleolus always belongs to the kernel and satisfies the symmetry and dummy axioms of Shapley: dummy players receive zero payoffs. If a null player is removed from the game, the payoff allocation of the remaining players is uninfluenced by its departure. Because of these desirable properties, the nucleolus solution has found a lot of applications in cost sharing and resource allocation as Maschler in [12] reports. However, the nucleolus possesses certain features that makes it less agreeable. The original definition treats the excesses of any two coalitions as equally important, regardless of coalition sizes and coalition composition. Some unappealing features of utility distribution, derived with the nucleolus are listed in [51]. For instance, the nucleolus lacks many monotonicity properties. That is, if a game changes so that some player's contribution to all coalitions increases then the player's allocation should not decrease. Monotonicity states that as the underlying data of game change, the utility must change in a parallel fashion.

[2] The lexicographic order between two vectors $\mathbf{u}$ and $\mathbf{u}'$ is defined by $\mathbf{u} \preceq_{lex} \mathbf{u}'$, if there exists an index k such that $\mathbf{u}[l] = \mathbf{u}'[l]$ for all $l < k$, and $\mathbf{u}[k] < \mathbf{u}'[k]$.

1.6 Cooperative Nash equilibria

Coalitional games aim at identifying the best coalitions of the agents and a fair distribution of the payoff among the agents. The classic core solution is an extension of the Nash equilibrium, since the coalitions bind agreements of agents with each other and earns a vector value rather than a real number. In [58, Sec. 7.6] it is shown that the core set of an underlying coalitional game, if it exists, asymptotically coincides with the set of Nash equilibria of the repeated game, in the long run. The result of the Nash equilibrium is not always a satisfactory outcome for an external observer (e.g., prisoner's dilemma game). R. Aumann[3] in [59] and Bernheim *et al.* in [60] introduce a stronger notion of Nash equilibria based on coalitional game theory. First, let us review the definition of the Nash equilibrium where each pure strategy in a static game is presented as a coalition in a coalitional game. Thus, each player belongs to only one coalition.

Definition 13 *A pure strategy (coalition) combination* $\psi = [\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_m]$ *wherein* $\mathcal{A}_i \bigcap \mathcal{A}_{j\neq i} = \emptyset$, $\bigcup_{i=1}^{m} \mathcal{A}_i = \mathcal{K}$ *and a payoff distribution* $\mathbf{u} = [u_1, \ldots, u_K]$ *is a pure Nash equilibrium if there does not exist a player* $k \in \mathcal{K}$ *whose unilateral deviation to a different coalition (pure strategy) yields a new distribution* $\mathbf{u}' = [u'_1, \ldots, u'_K]$ *such that* $u'_k > u_k$.

In other words, in a Nash equilibrium no agent is motivated to deviate from its coalition (strategy) given that the others do not deviate. As an example, we study the forwarder's dilemma game [14] presented in Fig. 1.2. This game is intended to represent a basic wireless relay operation between two different wireless terminals. These two agents, represented by players k_1 and k_2, are supposed to operate a direct link that enables them to communicate without intermediaries. Each players wants to send a packet to its destination, d_1 and d_2 respectively, in each time step using the other player as a forwarder. We assume that each forwarding has a energy cost $0 < c \ll 1$. If player k_1 forwards (F) the player's k_2 packet, player k_2 gets a reward 1 and vice versa. Each player's utility is its reward minus the cost. Each player is allured to drop (D) the received packet for saving energy. The strategic form of this game is depicted in Tab. 1.1. In the cooperative representation of the forwarder's dilemma

[3]Robert Aumann has received in 2005 the Nobel prize in economy for his contributions to game theory, together with Thomas Schelling.

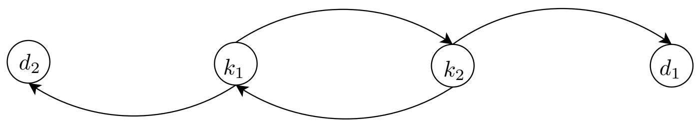


Fig. 1.2: *The network scenario of the forwarder's dilemma game.*

		k_2: F	k_2: D
k_1	F	$1-c\,,\,1-c$	$-c\,,\,1$
k_1	D	$1\,,\,-c$	$0\,,\,0$

$$\mathbf{u} = [\,u_{k_1}\,,\,u_{k_2}\,]$$

Tab. 1.1: *The strategic form in the forwarder's dilemma game. In each cell, the first value is the payoff of player k_1, whereas the second is that of k_2.*

game there are two coalitions $\psi = [\mathcal{A}_F\,,\,\mathcal{A}_D]$ and each player in $\mathcal{K} = \{k_1\,,\,k_2\}$ must choose one coalition. For instance, $\psi = [\mathcal{A}_F = \{k_1\,,\,k_2\}\,,\,\mathcal{A}_D = \emptyset]$ is equivalent to the strategy profile $(F,\,F)$ and $\psi = [\mathcal{A}_F = \{k_2\}, \mathcal{A}_D = \{k_1\}]$ corresponds to the strategy profile $(D,\,F)$, and so on.

Unilateral deviation of player k_1 from $\psi = [\mathcal{A}_F = \{k_1,\,k_2\}\,,\,\mathcal{A}_D = \emptyset]$ to the profile $\psi = [\mathcal{A}_F = \{k_2\},\,\mathcal{A}_D = \{k_1\}]$ increases its own payoff, and therefore the pure strategy profile $(F,\,F)$ is not a Nash equilibrium point. The same applies to the departure of player k_2 from $\psi = [\mathcal{A}_F = \{k_1,\,k_2\}\,,\,\mathcal{A}_D = \emptyset]$ to the pure strategy $\psi = [\mathcal{A}_F = \{k_1\}\,,\,\mathcal{A}_D = \{k_2\}]$. We can easily check the different combinations of $\psi = [\mathcal{A}_F = \{k_1\},\,\mathcal{A}_D = \{k_2\}]$, $\psi = [\mathcal{A}_F = \{k_2\},\,\mathcal{A}_D = \{k_1\}]$, and finally $\psi = [\mathcal{A}_F = \emptyset,\,\mathcal{A}_D = \{k_1,\,k_2\}]$. The unilateral move of user k_1 (resp. k_2) from the strategy profile $\psi = [\mathcal{A}_F = \emptyset,\,\mathcal{A}_D = \{k_1,\,k_2\}]$ to $\psi = [\mathcal{A}_F = \{k_1\},\,\mathcal{A}_D = \{k_2\}]$ (resp. to $\psi = [\mathcal{A}_F = \{k_2\},\,\mathcal{A}_D = \{k_1\}]$), does not yield any benefit. This game has a unique Nash equilibrium at the pure joint strategy $\psi = [\mathcal{A}_F = \emptyset, \mathcal{A}_D = \{k_1, k_2\}]$ with unsatisfactory payoff distribution $\mathbf{u} = [\,0\,,\,0\,]$. At the Nash equilibrium point either players choose the "competetive" and "egoistic" strategy D.

In many games, there are opportunities for joint deviations that are mutually beneficial for a subset of players. This led Aumann [59] to propose the idea of *strong Nash equilibrium* which ensures a more restrictive stability than the conventional Nash equilibrium. Strong Nash equilibrium reflects the unprofitability of coalition deviations. It is a strategy profile that is stable against deviations not only by single players but by all coalitions of players. A strong equilibrium is defined as a strategic profile for which no subset of players has a joint deviation that strictly benefits all of them, while all other players (in the subset) are expected to maintain their equilibrium strategies.

Definition 14 *A strategy (coalition) combination $\psi = [\mathcal{A}_1, \ldots, \mathcal{A}_m]$ where $\mathcal{A}_i \bigcap \mathcal{A}_{j\neq i} = \emptyset$ and $\bigcup_{i=1}^{m} \mathcal{A}_i = \mathcal{K}$ with payoff distribution $\mathbf{u} = [u_1, \ldots, u_K]$ is a strong Nash equilibrium if there do not exist a coalition $\mathcal{A}_i \in \psi$ whose deviation yields a new distribution $\mathbf{u}' = [u'_1, \ldots, u'_K]$ such that $u'_k \geq u_k \quad \forall\, k \in \mathcal{A}_i$ and $\exists\, k \in \mathcal{A}_i$ such that $u'_k > u_k$.*

This definition of strong equilibrium is actually slightly different from those of [59] and [60]. Def. 14 allows a coalition to deviate from a strategy profile that strictly increases the payoffs of some of its members without decreasing those of the other members, whereas the original definition allows only deviations that strictly increase the payoffs of all members of a deviating coalition. We note that if a game implements a strategy for strong equilibrium, it does not necessarily implement it for Nash equilibrium. Both interpretations of strong Nash equilibrium are prominent in the literature, and in most games the two definitions lead to the same sets of strong Nash equilibria; however, the one that we use here is slightly more appealing in the context of network-formation games (see, e.g., [61]). Network formation games involve a number of independent players that interact with each other in order to form a suited graph that connects them.

Now, we restudy the forwarder's dilemma game and try to find strong Nash equilibria profile. We will show that the game possesses strong Nash equilibria which are not equivalent to the Nash equilibrium. We pick different coalition combination and test whether there exist any coalition whose deviation satisfies its own members or not.

1. $\psi = [\mathcal{A}_F = \{k_1\}\,,\, \mathcal{A}_D = \{k_2\}]$ is not strong Nash equilibrium because the deviation of $\mathcal{A}_F$ increases its member's payoff.

2. $\psi = [\mathcal{A}_F = \{k_2\}\,,\, \mathcal{A}_D = \{k_1\}]$ is not strong Nash equilibrium because the deviation of $\mathcal{A}_F$ renders its member's payoff higher.

3. $\psi = [\mathcal{A}_F = \emptyset\,,\, \mathcal{A}_D = \{k_1\,,\, k_2\}]$ is not strong Nash equilibrium because the deviation of both players from $\mathcal{A}_D$ to $\mathcal{A}_F$ increases payoff distribution.

4. $\psi = [\mathcal{A}_F = \{k_1\,,\, k_2\}\,,\, \mathcal{A}_D = \emptyset]$ is strong Nash equilibrium because the departure of one or both players from $\mathcal{A}_F$ to $\mathcal{A}_D$ decreses at least one player's payoff.

The unique strong Nash equilibrium is the strategy profile $(F\,,\, F)$ which corresponds to coalition set of $\psi = [\mathcal{A}_F = \{k_1\,,\, k_2\}\,,\, \mathcal{A}_D = \emptyset]$, since no deviation can better off the payoff distribution vector $\mathbf{u} = [\,1 - c\,,\, 1 - c\,]$. In fact, at the strong Nash equilibrium, both players choose the "cooperative" and "altruistic" strategy of F in spite of the energy transmission cost.

In network problems, Zhong *et al.* show that using strong Nash equilibria context makes possible a collusion-resistant routing in non-cooperative wireless ad hoc networks [62]. Altman *et al.* in [63] examine a dynamic random access game with orthogonal power constraints in which the probability of transmission of a terminal in each slot depends on the amount of energy left prior to that slot. They show the existence of a strong Nash equilibrium point.

Conventional Nash equilibrium is concerned with the possibilities of only one step deviation by any player. The notion of strong Nash equilibrium requires an agreement not be subject to an improving (one step) deviation by any coalition of players given that all others coalitions be inert. This notion is stronger than Nash equilibrium, but it is not resistant to further deviation by sub-coalitions (the subsets of a coalition). Recognizing this problem, Bernheim *et al.* [60] introduced the notion of *coalition-proof Nash equilibrium*, which requires only that an agreement be immune to improving deviations which are *self-enforcing*. The definition of a self-enforcing deviation is recursive.

Definition 15 *For a singleton coalition, a deviation is self-enforcing if it maximizes the player's payoff. For a coalition of more than one player, a deviation is self-enforcing if: i) it is profitable for all its members, and ii) if there is no further self-enforcing and improving deviation available to a proper sub-coalition of players.*

Generally, a deviation by a coalition is self-enforcing if no sub-coalition has an incentive to initiate a new deviation. In the forwarder's dilemma game, the Nash equilibria is upset by a deviation of the coalition of both players k_1 and k_2. At the pure strategy Nash equilibrium where each player choose strategy D, they each obtain a payoff of 0. By jointly deviating (both choosing F instead) k_1 and k_2 each earn a payoff $1-c$. This deviation is not self-enforcing even thought the movement to the pure strategy $\psi = [\mathcal{A}_F = \{k_1\,,\,k_2\}\,,\,\mathcal{A}_D = \emptyset]$ is profitable for both players. At strong Nash pure strategy $(F\,,\,F)$, the player k_1 tempts to move to strategy $(D\,,\,F)$ to get more payoff, and player k_2 to that $(F\,,\,D)$. Thus, the strong Nash equilibrium is not immune against self-enforceability.

This notion of self-enforceability provides a useful means of distinguishing coalitional deviations that are viable from those that are not resistant to further deviations. With the concept of self-enforceability, our notion of coalition-proofness is easily formulated.

Definition 16 *In a one player game, a strategy is a coalition-proof Nash equilibrium if it maximizes the player utility. In a game with more than one player, a combination strategy is coalition-proof Nash equilibrium, if no sub-coalition has a self-enforcing deviation that makes all its members better off.*

This solution concept requires that there is no sub-coalition that can make a mutually beneficial deviation (keeping the strategies of non-members fixed) in a way that the deviation itself is stable according to the same criterion. In the forwarder's dilemma game, the strong Nash equilibrium profile $(F\,,\,F)$ is not equivalent to coalition-proof Nash equilibrium. This is due to the fact that, the deviation of $\{k_1\} \subset \mathcal{A}_F = \{k_1\,,\,k_2\}$ to the strategy $(D\,,\,F)$ increases payoff of k_1. In this game there does not exist any coalition-proof Nash equilibrium, due to the fact that all pure strategies have at least one self-enforcing deviation.

Bernheim *et al.* [60] note that for 2-person games the set of coalition-proof equilibria coincides with the set of Nash equilibria that are not Pareto dominated by any other Nash equilibrium. However in n-person games ($K \geq 3$) the equilibrium concepts are independent. At coalition-proof Nash equilibrium, the deviations are restricted to be stable themselves against further deviations by sub-coalitions. Moldovanu in [64] discusses the situations of a 3-player game, wherein coalition-proof Nash equilibrium is equivalent to the core set. The conditions under which the set of coalition-proof

Nash equilibria coincides with the set of strong Nash equilibria, are formulated by H. Konishi *et al.* in [65].

In the network engineering literature, Félegyházi *et al.* in [66] apply the concept of coalition-proof Nash equilibria to achieve a stable and fair channel allocation solution in a competitive multi-radio multi-channel wireless cognitive network. Gao *et al.* investigate multi-radio multi-channel allocation in multi-hop ad-hoc networks [67]. To better understand the concepts of self-enforceability and coalition-proof Nash equilibrium, let us introduce an intuitive *subcarrier allocation game in an OFDMA network*. Let us focus on three wireless transmitters $\mathcal{K} = \{k_1, k_2, k_3\}$ and an OFDMA base-station with two subcarriers $\mathcal{N} = \{1\,, 2\}$. Every subcarrier $n \in \mathcal{N}$ has a frequency spacing Δf. Each user $k \in \mathcal{K}$ experiences a Gaussian complex-valued channel gain $|H_{kn}|^2$ on the nth subcarrier to the base station. We assume that each subcarrier can be shared among more than one transmitter. The payoff of each player (transmitter) is defined as the achieved Shannon channel capacity. Each user $k \in \mathcal{K}$ is allowed to either spend a certain power $\overline{p}_k$ on only one choosen subcarrier, or equally divide it among both subcarriers. In the pure strategy a_1, player k transmits with the maximum power $\overline{p}_k$ on subcarrier $n = 1$ and does not transmit any information on subcarrier $n = 2$. The strategy a_2 is contrary to a_1, i.e. exclusively transmitting on subcarrier $n = 2$ with maximum power. Finally strategy a_3 equally divides its power on two subcarriers and exploits transmitting on both tones. The terminal k achieves a channel capacity:

$$C_k = \sum_{n \in \mathcal{N}} C_{kn} \tag{1.20}$$

where C_{kn} is the Shannon capacity achieved by user k on the nth subcarrier:

$$C_{kn} = \Delta f \cdot \log_2 \left(1 + \frac{|H_{kn}|^2 \, p_{kn}}{\sum_{k \neq i \in \mathcal{K}} |H_{in}|^2 \, p_{in} + \sigma_w^2} \right) \tag{1.21}$$

wherin p_{kn} represents the power allocated by terminal k over the nth subcarrier and where the interference term $\sum_{k \neq i \in \mathcal{K}} |H_{in}|^2 \, p_{in}$ is appriximated with a Gaussian random variable of equal mean and variance. Chooisng the strategy a_1 means selecting $p_{k1} = \overline{p}_k$ and $p_{k2} = 0$. For the strategy a_2, $p_{k1} = 0$ and $p_{k2} = \overline{p}_k$, and for strategy a_3, $p_{k1} = p_{k2} = \frac{\overline{p}_k}{2}$. The parameter σ_w^2 is the power of the additive white Gaussian noise (AWGN). Note that, in an OFDMA system, there is no interference between adjacent subcarriers. Hence, C_{kn} considers only intra-subcarrier noise, that occurs when the same subcarrier is shared by more terminals.

		$k_3\ (a_1)$			$k_3\ (a_2)$			$k_3\ (a_3)$	
		k_2			k_2			k_2	
	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3
a_1	8, 5, 6	11, 11, 7	9, 11, 6	12, 7, 10	15, 10, 10	11, 10, 9	9, 5, 10	12, 9, 10	9, 10, 9
k_1 a_2	12, 8, 6	8, 7, 10	9, 11, 8	11, 11, 8	7, 6, 6	8, 11, 7	13, 9, 11	8, 6, 11	8, 10, 10
a_3	15, 6, 6	14, 8, 8	13, 10, 7	14, 8, 9	15, 6, 7	13, 9, 7	14, 6, 10	14, 7, 10	12, 9, 10

$$\mathbf{u} = [\, u_{k_1}\, ,\, u_{k_2}\, ,\, u_{k_3}]$$

Tab. 1.2: *Subcarrier allocation in OFDMA network game in strategic form. The three strategies for three players k_1, k_2 and k_3 are: Transmitting with the maximum power only on subcarrier number 1 (a_1), transmitting with the maximum power only on subcarrier number 2 (a_2) and equal division of the maximum power among both subcarriers (a_3). The player's payoff is the achieved channel capacity in* kb/s.

Tab. 1.2 reports the simulation results obtained after 100 random realizations of a network with terminals distributed at a distance between 3m and 50m from the base-station. In the pure strategy matrix form of Tab. 1.2, player k_1 chooses the row, player k_2 chooses the column, and player k_3 chooses the matrix. Each payoff reports the (rounded) value of the achieved Shannon channel capacity in kb/s. We consider the following parameters for our simulations: the maximum power of each terminal k is $\overline{p}_k = 10\,\text{mW}$; the power of the ambient AWGN noise on each subcarrier is $\sigma_w^2 = 100\,\text{pW}$, and finally the carrier spacing is $\Delta f = \frac{10}{1024}$ MHz.[4] The path coefficients $|H_{kn}|^2$, corresponding to the frequency response of the multipath wireless channel, are computed using the 24-tap ITU modified vehicular-B channel model adopted by the IEEE 802.16m standard [68].

It is easy to show that the (pure) Nash equilibrium strategies of Tab. 1.2 are $(a_3\, ,\, a_3\, ,\, a_3)$ equivalent to $\psi = [\mathcal{A}_{a_1} = \emptyset\, ,\, \mathcal{A}_{a_2} = \emptyset\, ,\, \mathcal{A}_{a_3} = \mathcal{K}\,]$ and $(a_1\, ,\, a_2\, ,\, a_2)$ to $\psi = [\mathcal{A}_{a_1} = \{\, k_1 \,\}\, ,\ \ \mathcal{A}_{a_2} = \{\, k_2\, ,\, k_3 \,\}\, ,\, \mathcal{A}_{a_3} = \emptyset]$. The Nash equilibrium strategy $(a_3\, ,\, a_3\, ,\, a_3)$ is neither coalition-proof nor strong. With deviation of the coalition $\mathcal{A}_{a_3}$ to the strategy profile $(a_2\, ,\, a_1\, ,\, a_3)$ all players profit more with payoff distribution $[\,13,\, 9,\, 11\,]$. This change is no longer valid since, there exists a self-enforceability for player k_1 to transit to the strategy profile $(a_3\, ,\, a_1\, ,\, a_3)$. This transition is not favorable for players

[4] This is the carrier spacing of each subcarrier at a base station with 10 MHz bandwidth and 1024 subcarriers.

k_2 and k_3. The player k_2 is tempted to transit to the Nash equilibrium point to earn a higher payoff. Whereas, the Nash equilibrium strategy profile $(a_1\,,\,a_2\,,\,a_2)$ with payoff vector $[\,15,\,10,\,10\,]$ is a strong and coalition-proof Nash equilibrium. This is due to the fact that, in $\psi = [\mathcal{A}_{a_1} = \{\,k_1\,\},\,\mathcal{A}_{a_2} = \{\,k_2\,,\,k_3\,\},\,\mathcal{A}_{a_3} = \emptyset]$ there is no deviation and self-enforceability that can improve the payoff distribution. As can be seen, all players prefer to stay at the coalition-proof Nash equilibrium rather than the pure Nash equilibrium strategy $(a_3\,,\,a_3\,,\,a_3)$. Note that, a strong or coalition-proof Nash equilibrium does not necessarily coincide with a Nash equilibrium strategy profile, and the result of Tab. 1.2 is an exception.

In general, the existence of a pure cooperative or non-cooperative Nash equilibrium for subcarrier allocation game in OFDMA network is not guaranteed. Given different parameters approaches to quite different channel capacities and this may results a matrix form without any type of Nash equilibrium. There even might exist a Nash equilibrium which is Pareto-dominated by another strategy profile. This shows that in OFDMA networks, an appropriate resource allocation technique is needed [26].

1.7 Coordinated equilibrium

The most common solution concept in (non-cooperative) game theory, Nash equilibrium, assumes that players take mixed actions independently of each other. Cooperative games allow players to coordinate each other to find out possible equilibria and (joint) optimizations that the players can perform on their own. Unlike evolutionary games [10, Ch. 3], in coordinated games the interaction between players is implemented once, among all players by a central authority, to increase their throughput. The notion of *correlated equilibrium* was introduced by R. Aumann [69]. Correlated equilibria are defined in a context where there is an intermediator who sends random (private or public) signals to the players. An intermediator needs not have any intelligence or knowledge of the game. These signals allow players to coordinate their actions, and, in particular, to perform joint randomization over strategies. “Correlated strategies are familiar from cooperative game theory, but their applications in non-cooperative games are less understood”, says R. Auman [69]. This is because, the players of a coordination game, are not totally isolated and without a communication between them, achieving to coordinated strategy profile is not possible.

		k_2	
		W	A
k_1	W	$0\,,\,0$	$0\,,\,1-c$
	A	$1-c\,,\,0$	$-c\,,\,-c$

$$\mathbf{u} = [\,u_{k_1}\,,\,u_{k_2}\,]$$

Tab. 1.3: *The multiple access game in strategic form. The two moves for each player are: access* (A) *or wait* (W).

Let us start with an intuitive example. Consider the multiple access game [14, Table III] described in Tab. 1.3. The players k_1 and k_2 wish to send some packets to their receivers sharing a common resource, i.e., the wireless medium. They are in the sight of each other and accordingly, they interferer if transmitting at the same time. The users have two possible pure strategies: access (A) and wait (W). In this game two identical transmitters must simultaneously decide whether to access to channel or wait. The transmission of each packet has an energy cost of $0 < c \ll 1$. Each player earns a payoff 1 if it succeeds to transmit its packet without collision with the other. Waiting does not bring neither cost nor reward for the player. Each player's utility is its reward minus the cost. This game has three Nash equilibria: $(A\,,W)$, $(W\,,\,A)$ and a mixed strategy Nash equilibrium where each player transmits with the probability $1-c$ [14, Sec. 2.3, 2.4]. The utilities of Nash equilibria strategies are: $(1-c\,,0)$, $(0\,,1-c)$ and $(0\,,0)$, respectively. It is clear that the mixed strategy is not resistant to an improving deviation. In the following, we give the possibility of preplay communication to achieve a stable Nash equilibria.

In the game with "cheap conversation", each player simultaneously and publicly announces whether it decides to access or wait. Following the announcements each player makes its choice. Suppose the players agree to participate to the game binding the following agreement: each player announces A with probability $\frac{3}{4}$. If the profile of announcements is either $(A\,,W)$ or $(W\,,\,A)$, then each player plays its own announcement. Otherwise, each player plays A with probability $\frac{1}{2}$. Note that no further

		k_2	
		W	A
k_1	W	$\frac{10+32c}{64}$	$\frac{11-16c}{32}$
	A	$\frac{11-16c}{32}$	$\frac{10+32c}{64}$

$$\mathbf{u} = [\, u_{k_1} = u_{k_2} \,]$$

Tab. 1.4: *The strategic form matrix of the multiple access game with preplay agreement.*

communication is possible. The use of joint deviation requires the unanimity of all members of the deviating coalition. A player agrees to be a part of a joint deviation if given its own information the deviation is profitable. Thus, if a joint deviation is used, it is common knowledge that each deviator believes that deviation is profitable.

This tradeoff results in an expected payoff for each player of $\frac{11-16c}{32} > 0$, while in the mixed Nash equilibrium of the original game each player has an expected payoff of 0. In this coordinated Nash equilibrium of the game, the players effectively play the *correlated strategy* [69, 70] (of the original game) given in Tab. 1.4, in order to face a higher utility in strategy profiles $(A\,,\,W)$ and $(W\,,\,A)$. It is important to note that, this joint probability distribution is not the product of its marginal distributions and therefore cannot be achieved from a mixed strategy profile of the game without correlation among players.

As can be seen, the proposed correlated deviation from the mixed strategy equilibrium makes both players better off. Note that the players are allowed to bind an agreement only on the space of feasible outcomes. In the correlated multiple access game the outcome is feasible since the correlated results are in the range between the smallest and highest possible payoff. In fact, the set of correlated equilibria contains those equilibria from which no coalition has a self-enforcing deviation making all members better off.

Let us describe a more complicated correlated equilibrium. We study the near-far effect game established by G. Bacci *et al.* in [71, Fig. 6]. The basic idea of near-

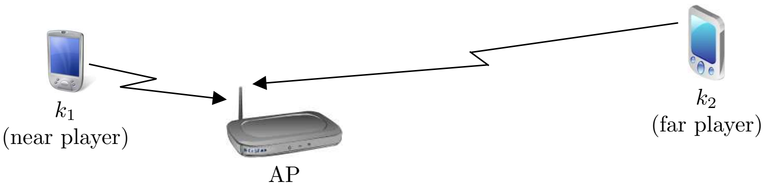


Fig. 1.3: *The network scenario in the near-far effect game.*

far effect game scheme is depicted in Fig. 1.3. Two wireless terminals k_1 and k_2, are placed close to and far from a certain access point (AP), respectively, in a code division multiple access (CDMA) network with high SINR regime. The strategy of each player is either to transmit with the maximum power $\overline{p}$, or with a weakened level $\eta\overline{p}$, where $0 < \eta < 1$. Due to the interference at the AP, the throughput (the amount of delivered information) of each player depends on the strategies chosen by both players. Transmitting with a higher power increases the BER, and this results decreasing the throughput. Each player is rewarded u if it successfully delivers its packet and a reduced δu, if it delivers a corrupted version of the packet, where $0 < \eta < \delta < 1$. If the near player k_1 decides to transmit with the power $\overline{p}$, the farther player k_2 will not be able to deliver any information to the AP.

This results in no benefit for k_2 and causes a power consumption cost equal to $-\eta c$ if k_2 chooses strategy $\eta\overline{p}$ and $-c$ otherwise, where $c \ll u$. Obviously, transmitting with power $\overline{p}$ for k_1, results in a complete information delivery. This concerns a payoff equal to reward minus power consumption cost, i.e. $u - c$, irrespective of the k_2 strategy. The packets of player k_2 are successfully delivered if it chooses the maximum power $\overline{p}$, and player k_1 that reduced $\eta\overline{p}$. On the other hand, if both players decide to transmit with reduced power $\eta\overline{p}$, the near player takes the payoff $\delta u - \eta c > 0$, whilst the farther player k_2 will not successfully deliver any packet and suffers only a power cost $-\eta c$.

The payoff matrix of the near-far effect game is depicted in Tab. 1.5. As can be seen, the unique pure strategy of this game is represented by the strategy $(\overline{p}\ ,\ \eta\overline{p})$ with benefits $u - c$ and $-\eta c$ for k_1 and k_2, respectively. This means that, at the Nash equilibrium point, the farther player is not able to send any information. On the other hand, the Pareto optimal[5] solution of the game are the strategies $(\overline{p}\ ,\ \eta\overline{p})$ and

[5]Pareto optimal is named after Vilfredo Pareto (1848-1923). He was an Italian engineer and

		k_2: $\eta\overline{p}$	k_2: $\overline{p}$
k_1	$\eta\overline{p}$	$\delta u - \eta c\,,\ -\eta c$	$\delta u - \eta c\,,\ \delta u - c$
k_1	$\overline{p}$	$u - c\,,\ -\eta c$	$u - c\,,\ -c$

$\mathbf{u} = [\,u_{k_1}\,,\ u_{k_2}\,]$

Tab. 1.5: *Payoff matrix for the near-far effect game with power control and variable throughput.*

$(\eta\overline{p}\,,\,\overline{p})$. This is an unsatisfactory outcome for the far player k_2, while the near player k_1 takes the highest possible payoff. Now, let us find the mixed strategy of the game. We denote α_1 the probability with which the near player k_1 decides to transmit with the maximum power $\overline{p}$ and α_2 the same probability for the far player k_2. The payoffs of the players k_1 and k_2 are represented by:

$$u_{k_1} = \alpha_1\Big(\,(1-\delta)\,u\ -\ (1-\eta)\,c\Big) + (\delta u - \eta c) \tag{1.22a}$$

$$u_{k_2} = \alpha_2\Big(\,(1-\alpha_1)\,\delta u\ -\ (1-\eta)\,c\Big) - \eta c \tag{1.22b}$$

Both players want to maximize their own payoff. As can be seen, u_{k_1} takes its maximum value $u-c$ with $\alpha_1 = 1$. On the other hand, with $\alpha_1 = 1$, the far player k_2 earns a negative payoff whatever $\alpha_2 \in [0,1]$. Instead, with $\alpha_1 = 0$ the near player k_1 gains $\delta u - \eta c$, and the player k_2 setting up $\alpha_2 = 1$ achieves the payoff $\delta u - c$. Thus, the best values for α_1 and α_2 are 0 and 1, respectively. The conclusion os that the mixed strategy is equivalent to the pure strategy $(\eta\overline{p}\,,\,\overline{p})$ with payoff $\mathbf{u} = [\,\delta u - \eta c\,,\ \delta u - c\,]$. In this game there is no (totally) mixed strategy and that is equal to the one of the pure Pareto optimal points.

The near player earns the highest possible payoff at the Nash equilibrium, hence, it does not leave this strategy profile. The highest possible payoff for the far player is on the contrary $\delta u - c$. We show that an appropriate agreement among players can

economist and he made several important contributions to economics.

		k_2: $\eta\overline{p}$	k_2: $\overline{p}$
k_1	$\eta\overline{p}$	$0\,,\;0$	$0\,,\;0$
	$\overline{p}$	$\kappa\,,\;\kappa\dfrac{(\delta u - c)}{(u-c)}$	$1-\kappa\,,\,(1-\kappa)\,\dfrac{(\delta u - c)}{(u-c)}$

$\mathbf{u} = [\,u_{k_1}\,,\,u_{k_2}\,]$ normalized to 1

Tab. 1.6: *The strategic form matrix of the near-far effect game with preplay agreement, and with* $\kappa = \dfrac{\delta u}{(1-\eta)\,c}$.

satisfy both of them at correlated equilibrium. Players k_1 and k_2 can guarantee an expected payoff of $\mathbf{u} = [\,u - c\,,\,\delta u - c\,]$ by playing the correlated strategy profile:

$$\frac{\delta u}{(1-\eta)\,c} \cdot (\overline{p}\,,\,\eta\overline{p}) \;+\; \left(1 - \frac{\delta u}{(1-\eta)\,c}\right) \cdot (\overline{p}\,,\,\overline{p}) \tag{1.23}$$

This is a plausible end, since both players earn their own highest possible payoff. The correlated strategy (1.23) is derived from the fact that, picking any real number κ in the expression $\kappa \cdot (\overline{p}\,,\,\eta\overline{p}) + (1-\kappa)\cdot(\overline{p}\,,\,\overline{p})$ is indifferent for the near player k_1, since it gets its own highest possible payoff, $u - c$ as well. To satisfy the far player k_2, it is enough to solve the following equation for u_{k_2}:

$$\kappa \cdot (\overline{p}\,,\,\eta\overline{p}) + (1-\kappa)\cdot(\overline{p}\,,\,\overline{p}) = [\,u - c\,,\,\delta u - c\,] \tag{1.24}$$

Supposing $\kappa = \dfrac{\delta u}{(1-\eta)\,c} < 1$, the correlated strategy (1.23) means that the near player always transmits at its highest power level $\overline{p}$, and the far player transmits at that reduced $\eta\overline{p}$ with probability $\dfrac{\delta u}{(1-\eta)\,c}$, and the maximum power $\overline{p}$ otherwise. Actually, the near and far players effectively play the matrix form game of Tab. 1.6.

Bonneau *et al.* in [72] show that the coordination among mobile users can significantly increase the performance of access to a common channel in ALOHA setting. A coordination mechanism is also considered by Bonneau *et al.* in [73] to achieve the optimal power allocation in a wireless network wherein each terminal knows only its own channel state. The concept of correlated equilibrium is also introduced in a multi-user interference channel context in [74]. Different types of coordination is deeply discussed and widely used in [70].

1.8 Dynamic learning

Until now, we have realized that the Nash equilibrium suffers from the lack of far-sighted stability, i.e., the relative results can be unsatisfactory and because of this any player can have incentive to improve its outcome by moving to another strategy. The existence of the strong and coalition-proof Nash equilibrium is not guaranteed and even if so, when the number of pure strategies is large, finding such solutions is very complicated. The challenge of finding a profitable accord among players is persistent in coordinated equilibria solution. In this section, the main question we seek an answer to is: *How can the players be led to a stable joint pure strategy gaining an acceptable payoff?* This question is important, even if multiple equilibrium points with the same payoff have been identified, since each player may autonomously decide to stay in a different strategy.

Dynamic learning [75] has been widely used in order to get rid of the anarchy derived from the conflicts between selfish decisions. Learning is a joint adaptive process for agents to converge and to get the best final response. The agents either have a common interest like a team work, or each agent has its own greedy goal. Generally, there are three learning process types: *individual learning*, *joint-action learning* and *stochastic learning*. In individual learning process, the independent agents cannot observe one another's actions; i.e. for each players the opponents are passive agents. Instead, during joint-action learning process, the notion of the "optimality" is improved by adding the observation of other concurrent learners to accomplish a stable optimal solution. The stochastic learning framework, having Markovian property and a stochastic inter-state transition rule, enables each player to observe the opponents' actions history.

In the network engineering literature, Schaar *et al.* in [76] introduce a stochastic learning process among autonomous wireless agents for the optimization of dynamic spectrum access given QoS of multimedia applications. A reconfigurable multi-hop wireless network is studied by Shiang *et al.* [77] wherein a decentralized stochastic learning process optimizes the transmission decisions of nodes aimed at supporting mission-critical applications. In [78], Lin *et al.* propose a reinforcement learning among agents of a multi-hop wireless network based on Markov decision process. Each terminal autonomously adjust transmission power in order to maximize the network utility, in a dynamic delay-sensitive environment.

Here, we study a well-known individual reinforcement learning task, namely the so-called *Q-Learning* [79]. We assume a set of players $\mathcal{K}$, and each player k has a finite set of individual actions $\mathbf{A}_k$. Each agent k individually chooses a pure joint action (strategy) to be performed $\mathbf{a}_k = (a_1, \ldots, a_K) \in \mathbf{A}_1 \times \ldots \times \mathbf{A}_K$ from the available joint strategy space. Q-learning enables the individual learners to achieve optimal coordination from repeated trials. Q-learning introduces a certain value Q as the immediate reward obtained after having moved to the new strategy. Each player individually updates a Q value for each of its actions. In each time step and after having selected the new joint action $\mathbf{a}_k$, the values of Q_k^t is individually updated. In particular, the value of $Q_k^{t+1}(\mathbf{a}_k)$ estimates the utility of performing the joint strategy $\mathbf{a}_k$ for user k. In the seminal paper of Watkins *et al.* [79], the Q value is updated by the following recursion:

$$Q_k^{t+1}(\mathbf{a}_k) \longleftarrow \left(1 - f_k^{t+1}\right) \cdot Q_k^t(\mathbf{a}_k) + f_k^{t+1} \cdot \left(u_k(\mathbf{a}_k) + \delta_k \cdot Q_k^t(\mathbf{a}_k)\right) \tag{1.25}$$

where $\delta_k \in (0,1)$ is a discount factor and $u_k(\mathbf{a}_k)$ is a reward of the joint action $\mathbf{a}_k$ for the respective player, and f_k is a function of t which is related to "learning rate". Watkins *et al.* showed that given bounded rewards, learning rate $0 \leq f_k^t < 1$, and

$$\sum_{t=1}^{\infty} f_k^t = \infty\,, \quad \text{and} \quad \sum_{t=1}^{\infty} \left(f_k^t\right)^2 < \infty \qquad \forall k \in \mathcal{K} \tag{1.26}$$

all Q_k values updating (1.25) converge a common joint pure strategy with probability one. The reward u_k is defined by a learning policy and it is not necessarily equal to the payoff defined by the game. The learning policy is greedy with respect to the Q value, i.e. the particular action $\mathbf{a}_k$ will be selected in long-run if it makes Q value better off. Q-learning is guaranteed to converge to an optimal and stable joint strategy regardless of the action selection policy. Q-learning is not applicable where the strategy space is continuous or the number of strategies is not finite. Claus *et al.* [80] establish a simplified version of the Q recursion (1.25) which updates the Q value by the following recursion:

$$Q_k^{t+1}(\mathbf{a}_k) \longleftarrow Q_k^t(\mathbf{a}_k) + \delta_k \cdot \left(u_k(\mathbf{a}_k) - Q_k^t(\mathbf{a}_k)\right) \tag{1.27}$$

For the sake of simplicity, we apply the Q recursion (1.27). In a multi learners scenario, a major challenge of Q-learning is strategy selection. When the number of strategies and players are large, the number of time step to achieve an optimal joint action exponentially increases. It is fairly clear that the best manner is to start with "exploration" of different strategies and then focus on "exploitation" of the strategies with the best value of Q. Kaelbling *et al.* in [81] recall *Boltzmann function* as an efficient strategy selection to strike a balance between exploration and exploitation. Boltzmann functions define a probability distribution among different joint actions. At each time step $t+1$, every player will individually select the joint strategy $\mathbf{a}_k$ with the probability $p(\mathbf{a}_k)$:

$$p(\mathbf{a}_k) = \frac{e^{\mathrm{E}_k(\mathbf{a}_k)\,/\,T}}{\sum\limits_{\forall\,\mathbf{a}_i \in \underset{\forall k \in \mathcal{K}}{\times} \mathbf{A}_k} e^{\mathrm{E}_k(\mathbf{a}_i)\,/\,T}} \tag{1.28}$$

The $\mathrm{E}_k(\mathbf{a}_k) = (\delta_k)^t \cdot u_k(\mathbf{a}_k)$ is the discounted reward for taking action $\mathbf{a}_k$ by the user k in time step t. The T is a function which provides a randomness component to control exploration and exploitation of the actions. Practically, the *temperature function* T is a decreasing function over time to decrease the exploration and increase exploitation. High values of T yields a small $p(\mathbf{a}_k)$ value and this encourages exploration, whereas a low T makes $Q(\mathbf{a}_k)$ more important, and this encourages exploitation. At time $t = 0$, each player randomly chooses a strategy and assign a random number to its own Q value. At time step t, after having been updated funtion T, each concurrent agents' experience consists of a sequence of stages [80]:

1) Computing $p(\mathbf{a}_k)$ for all $\mathbf{a}_k \in \underset{\forall k \in \mathcal{K}}{\times} \mathbf{A}_k$,

2) Generating a random number ξ_k^t uniformly distributed in $[0, 1]$, and then choosing the best joint strategy $\mathbf{a}_k$, i.e. the highest $p(\mathbf{a}_k)$ such that $\xi_k^t \geq p(\mathbf{a}_k)$. If $\xi_k^t < p(\mathbf{a}_k)$ for all $\mathbf{a}_k \in \underset{\forall k \in \mathcal{K}}{\times} \mathbf{A}_k$, then the learner randomly picks a strategy,

3) Updating the Q_k^t value according to (1.27). If Q_k^t grows, then the learner moves to selected joint strategy $\mathbf{a}_k$, otherwise it stays in the current joint action and do not update Q.

Despite the individual best strategy selection of the learners, this process reach a common stable joint strategy such that all players stay there forever, i.e. no player deviates from the (common) achieved joint strategy.

The theory of learning in games studies how and which equilibria might arise as a consequence of a long-run non-equilibrium process of learning. A natural question is: *Can learning algorithms find a Nash equilibrium?* The reason for asking this question is in the hope of being able to achieve Nash equilibria, as a plausible concept, via a reasonable learning algorithm in particular when there are a large number of players and strategies. At the first look, the stability of the above addressed dynamic learning approach is described as to converge to a *pure* joint strategy and it is clear that the existence of a *pure* Nash equilibrium is not guaranteed. The fact is, in general, a dynamic learning algorithm is not able to guarantee to achieve a non-cooperative or cooperative Nash equilibrium. In the literature, there are some efforts to present a dynamic learning algorithm that achieves a Nash equilibrium in dynamic and repeated games under particular constraints [82–85].

We present now some results about Q-learning in a CDMA network. In what follows, the experimental work is presented highlighting how the agents learn to increase their individual rewards by revealing their actions. As above mentioned, the strategy selection can significantly influence the number of time steps to converge. Choosing an appropriate temperature function is a heuristic search. In our experiment, we define $T = q \cdot e^{-mt}$ as our temperature function wherein m controls the rate of exponential decay and $q > 1$ encourages the exploration of different strategies in the initial time steps.

We illustrate the behavior of mobile terminals as Q-learners in a CDMA network. Our example is a *power control problem in a CDMA network* applying Q-learning and Boltzmann function. Assume a CDMA network with K mobile terminals denoted by set $\mathcal{K}$. The players wish to transmit data to a certain AP. The strategies of every player is a set of discrete power levels denoted by $\mathbf{A} = \mathbf{A}_k = [\,\Delta p\,, 2.\Delta p\,, \ldots, M.\Delta p\,]$ where Δp is our power step and $M > 1$ is an integer number. Each user has M actions to choose from, and accordingly the matrix game is made by $\times_{k\in\mathcal{K}}\mathbf{A}$ which consists of M^K joint strategies. The Shannon capacity between player k and the AP is

$$C_k = \log_2\left(1 + \frac{N_s \cdot |H_k|^2\, p_k}{\sum_{\mathcal{K}\ni i\neq k} |H_i|^2\, p_i + \sigma_w^2}\right) \tag{1.29}$$

with N_s, $|H_k|^2$ and σ_w^2 denoting (the common) spreading factor for all players, user's k path gain, and the AWGN power, respectively, and where the $p_k \in \mathbf{A}$ denotes the transmit power of user k.

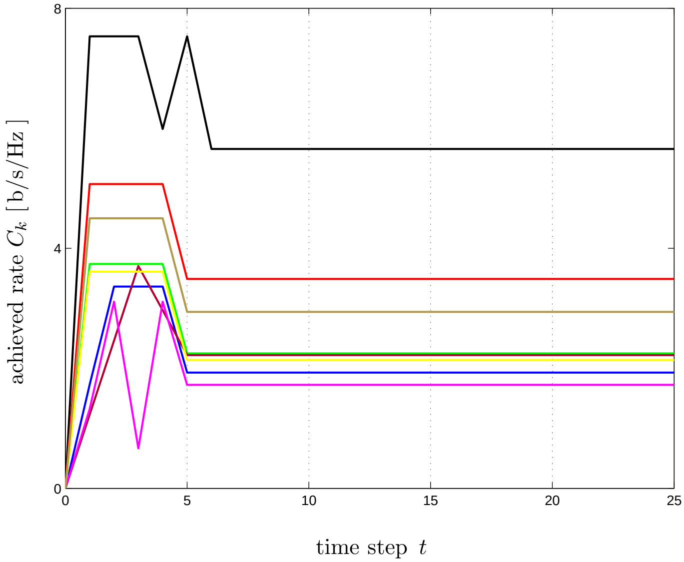


Fig. 1.4: *Achieved rates as functions of the iteration step.*

We introduce an individual work in which each player must individually choose the joint strategy at which achieves the best Shannon channel capacity. We simulate a learning process with $K = 8$ players such that each player k must choose the best p_k between $M = 5$ strategies. The power step is assumed to be $\Delta p = 100\,\text{mW}$, the power of AWGN $\sigma_w^2 = 1\,\text{nW}$ and the spreading factor $N_s = 64$. The players are uniformly located at a distance between 3 and 50m from the AP. The matrix form of this game is composed of 390625 joint strategies and there may exist different power combinations (joint strategies) which achieve the same Shannon channel capacities. Q-learning leads the players to that joint strategy $(p_1, \ldots, p_8) \in \mathbf{A}^8$ in which all players are satisfied of the proper achieved Shannon channel capacities. In the Q function of (1.27) for all players the discount factor parameter is fixed to $\delta_k = 0.09$ and the payoff function u_k is defined as:

$$u_k(\mathbf{a}_k) = C_k \qquad \forall\, k \in \mathcal{K} \tag{1.30}$$

Our experiments with different parameters show that good values of the temperature function parameters are $m = 0.001$ and $q = 50$, and we start with $Q_k^{t=0} = 0$. It is obvious to say that existing a strategy in which all $u_k(\mathbf{a}_k)$ are maximal value is not always guaranteed. Since, there is a huge conflict of interest between the players to choose different strategies.

Fig. 1.4 reports the behavior of the (reward) achievable rate C_k of $K = 8$ terminals as a function of the time step t in our scenario. The figure exhibits the convergence of all lerners to a stable joint strategy after 6 time steps. Numerical results of 500 random realization of a network show the convergence of all players to a stable joint strategy after (in average) 6 steps of the iterative Q-learning algorithm wherein each joint action is probabilistically chosen according to the distribution of Boltzmann function. Furthermore, it is experimentally observed that the sum of the achieved Shannon channel capacities is (in average) 22.4 b/s/Hz and that is 94% of the maximum possible of $\sum_{k\in\mathcal{K}} C_k$.

1.9 Summary

Cooperation can be seen as the action of obtaining some advantage by giving, sharing or allowing something. In this contribution, we aimed at mapping different coalitional game approaches into communications and networking systems. A very important boundary condition for cooperation is that each participating entity is gaining more by cooperation than they would by operating alone. It is not important that all entities contribute the same effort, gain the same amount, or even have the same gain to cost ratio, but the effect of cooperation should bring advantage or gain to each cooperating entity. One different form of cooperation is altruism, a strategy wherein one of the players may sacrifice and does not gain from the cooperation to support others. In networking, for instance, one terminal sacrifices battery power and bandwidth to act as a relay for others terminals and to increase the throughput of the whole system. In some communication systems, network protocols themselves can be seen as an implicit cooperation to achieve better performance, e.g. ALOHA system. In some communication systems, network entities establish a cooperation with each other to achieve better performance, e.g. relay communications.

Cooperative game theory is a branch of game theory which aims at studying the cooperations among individual and rational participants. Unlike non-cooperative game approaches, cooperative game concepts are centralized and they need a central authority for exchange of information and policy making process. The most challenging part of a cooperative game theoretic framework is the choice of characteristic function, since it interprets the agents perceptions of gain and satisfaction.

The main fundamental question in coalitional game theory is the question how to allocate the total generated gain by the collective of all players over the different players in the game. The distribution of payoff is described as a binding contract between the players and various criteria have been developed. The problem of the gain distribution is approached with the aid of solution concepts in coalitional game theory like core, Shapley value, kernel, and nucleolus. Core solution is the most classic solution whose result is stable against deviation of coalitions. The core solution is useful where the negotiation process is centralized and no subset of players can selfishly and privately negotiate with each other. The core set can be empty. Shapley value is the unique single-valued solution which explores the fairness in every possible prospective coalition forming. The kernel solution be understood as the set of all efficient allocations for which no pair of players want to exchange payoff. The nucleolus selects the unique imputation that successively (lexicographically) minimizes the maximal excesses. This defining property makes the nucleolus appealing as a fair single-valued solution. The kernel of a game always contains the nucleolus. The process of computing the kernel and nucleolus of arbitrary transferable utility games is hard.

The most fundamental solution concept for non-cooperative game is that of Nash equilibrium. In a Nash equilibrium no agent is motivated to deviate from its strategy given that the others do not deviate. If every player individually agrees on a certain profile of strategies without binding an agreement, then these strategies constitutes a Nash equilibrium. Nash equilibrium does not account for the possibility that groups of agents (coalitions) can change their strategies in a coordinated manner. A strategy profile is in strong Nash equilibrium if no subgroup of agents is motivated to change their strategies given that others do not change. Often the strong Nash equilibrium is too strong a solution concept, since in many games no such equilibrium exists. Coalition-proof has been suggested as a partial remedy to this problem. This solution concept requires that there is no sub-group that can make a mutually beneficial deviation (keeping the strategies of nonmembers fixed) in a way that the deviation itself is stable according to the same criterion. These solution concepts which allow coalitions to make agreements simultaneously typically suffer from incompatibility of agreements, which can give rise to empty solution sets in games of networking interest. Mixed (vs. pure) strong and coalition-proof Nash equilibrium have not been introduced.

In a game wherein there are a huge number of agents and strategies, finding a pure cooperative/non-cooperative Nash equilibrium is hard and maybe even impossible. A learning process leads participants to a common joint action with an acceptable payoff. During a learning process agents act as independent learners, i.e. they only get information about their own action choice and payoff. As such, they neglect the presence of the other agents. The learning process happens at regular time steps and is basically a signal for the agents to start an exploration phase. During each exploration phase, some agents exclude their current best action so as to give the team the opportunity to look for a possibly better joint action. This technique of reducing the action space by exclusions was only recently introduced for finding periodical policies in games of conflicting interests. There are two problems in the process of learning optimal cooperative pursuit strategy for multiple agents. One is the probability of circulation among the actions chosen by agents which make the learning, process not converging, the other is there are many conflicts among the actions chosen by agents which make the learned pursuit strategy not optimal. Q-learning with the Boltzmann action-selection strategy guarantees the convergence of multi agents to a common and optimal joint strategy after a few time step.

1.10 Discussion

This chapter has provided a unified reference for network engineers investigating the applicability of coalitional game theory to practical problems. Different approaches such as core solution, Shapley value, kernel and nucleolus, were shown to provide a strong foundation for finding possible and stable resource/cost sharing arrangements. The results confirm the apparent analogy between the definition of Nash equilibrium in non-cooperative and coalitional game theory: both strong and coalition-proof Nash equilibria reflect on unprofitability of coalition deviations rather than an individual player deviation. In a network wherein informational exchange is possible, either through a central controller or among players themselves, the concept of coordinated equilibrium arises. The results of intuitive examples show a significantly improvement in coordinated equilibrium when compared with non-cooperative schemes. When the number of agents or strategies is large, the ability of jointly reach a consensus through environmental learning guarantees convergence to the best joint action.

Chapter 2

A survey on resource allocation techniques in OFDM(A) networks

Interest in orthogonal frequency-division multiplexing (OFDM), has grow steadily, as it appears to be the most efficient air-interface for wireless communications primarily due to its inherent resistance to frequency-selective multipath fading and the flexibility it offers in radio resource allocations. One of the crucial issues in OFDM transmission is the allocation of the power resources to the available subchannels.

This chapter presents a survey on the radio resource allocation techniques in OFDM and orthogonal frequency division multiple access (OFDMA) networks. This problem goes back to 1960s and that is related to properly and efficiently allocate the radio resources, namely subcarriers and power. We start by overviewing the main open issues in OFDM. Then, we describe the problem formulation in OFDMA, and we review the existing solutions to allocate the radio resources. The goal is to discuss the fundamental concepts and relevant features of different radio resource management criteria, including water-filling, max-min fairness, proportional fairness, cross-layer optimization, utility maximization, and game theory, also including a toy example with two terminals to compare the performance of the different schemes. We conclude the survey with a review of the state-of-the-art in resource allocation for next-generation wireless networks, including multicellular systems, cognitive radio, and relay-assisted communications, and we summarize advantages and common problems of the existing solutions available in the literature. The distinguishing feature of this contribution is a tutorial-style introduction to the fundamental problems in this area of research, intended for beginners on this topic.

The use of orthogonal frequency division multiplexing (OFDM), as a modulation, and of orthogonal frequency division multiple access (OFDMA), as a channel access scheme, has grown steadily, as they appear to be the most efficient solutions for wireless communications, primarily due to their inherent resistance to frequency-selective multipath fading and the flexibility they offer to radio resource allocation [86]. One of the crucial issues in OFDM(A) transmission is the allocation of the power to the available subchannels. Even though the OFDM(A) concept is simple in its basic principle, building a practical OFDM(A) system is far from being a trivial task without a well-devised resource allocation algorithm. In the case of a multiple-access network, a typical resource allocation problem in OFDMA is based on assigning a subset of available subcarriers to simultaneously transmitting (and thus interfering) wireless terminals and (possibly jointly) adjusting the power amount over each used subcarrier in order to guarantee the minimum required quality of service (QoS). An efficient algorithm for subcarrier selection can significantly increase the signal-to-interference-plus-noise ratio (SINR), that is necessary to enhance the throughput in a dynamic scenario. Similarly, regulating the transmit power in wireless cellular networks constitutes a key degree of freedom in the management of interference, energy, and connectivity. This motivates us to revisit the relevant criteria for resource management in present and next-generation wireless networks.

This chapter aims at providing a survey on state-of-art research, providing an overview of selected topics in the context of OFDM(A) systems. We start with historical notes in the following section. Then, we provide a description of resource allocation issue in OFDM in Sect. 2.2. In Sect. 2.3, we describe different fashions of allocation of radio resources in OFDMA. We continue with exploration of three classic power allocation solutions of water-filling, max-min fairness, and weighted proportional fairness in Sects. 2.3.1,2.3.2, and 2.3.3, respectively. Sects. 2.3.4 and 2.3.5 discuss about two important resource allocation issues in multi service traffic networks: utility maximization, and cross layer. Different solutions based on game theory are reviewed in Sect. 2.4. Finally, we summarize key features of the existing solutions in Sect. 2.8.

2.1 Orthogonal frequency division modulation and multiple access

The basic principle of OFDM is to transmit data by dividing them into several interleaved bit streams, and using these to modulate several carriers. This concept helps reducing the detrimental effects of multipath fading in communication systems. In brief, OFDM is a parallel transmission scheme, where a high-rate serial data stream is split up into a set of low-rate substreams with generally equal bandwidth, each of which is modulated on a separate *subcarrier* (called also *subchannel* or *tone*). Thereby, the bandwidth of the subcarriers becomes small compared with the coherence bandwidth of the channel, so that the individual subcarriers experience flat fading, thus enabling a simple equalization. This implies that the symbol period of the substreams is made long compared to the delay spread of the time-dispersive radio channel. While each subcarrier is separately modulated by a data symbol, the overall modulation operation across all the subchannels (multicarrier modulation) results in a frequency multiplexed signal, so as to accommodate very high throughputs in severe frequency-selective scenarios.

This solution was proposed for the first time by Doelz *et al.* for the U.S. military HF communication applications in 1957 in the pioneering Collins Kineplex system [87]. This led to a few OFDM schemes in the 1960s, which were proposed by Saltzberg [88] and Chang [89]. In the late 1960s, the multicarrier concept was adopted in some military applications, such as KATHRYN [90] and ANDEFT [91]. These systems involved a large hardware complexity, since the parallel data transmission was essentially through a bank of oscillators, each tuned to a specific subcarrier. The first patent on OFDM was granted in 1970 [92]. The major contribution to the OFDM scheme came after the results of Weinstein and Ebert [93], who demonstrated that using discrete Fourier transforms (DFT) to perform the baseband modulation and demodulation considerably increases the efficiency of modulation and demodulation processing. The adoption of OFDM has been finally facilitated by the efficient implementation of fast Fourier transform (FFT) and inverse FFT (IFFT) algorithms in digital signal processing (DSP) chips.

OFDM is extremely effective in a time dispersive environment where signals can have many paths to reach their destinations, resulting in variable time delays. With classical modulations, these time delays cause one symbol to interfere with the next one(s) (giving rise to inter symbol interference, ISI) at high bit-rates. With the OFDM, all of the sinus cardinal (sinc)-shaped subchannel spectra exhibit zero crossings at all the other subcarriers' frequencies, and the subchannel spectra result to be orthogonal to each other. The orthogonality among different tones ensures that the subcarrier signals do not interfere with each other, when communicating over perfectly distortionless channels.

Although the ISI is mitigated by the guard interval between consecutive OFDM symbols and the raised-cosine filtering OFDM imposes, it is not completely eliminated. To attain perfect orthogonality between subcarriers in a time dispersive channel, Peled and Ruiz [94] introduced the notion of *cyclic prefix* (CP): the guard interval is filled with a cyclic extension of each time domain OFDM symbol, in order to overcome the inter-OFDM symbol interference due to the channel memory. The CP performs the circular convolution by the channel under the assumption that the channel impulse response is shorter than the length of the CP, thus preserving the orthogonality of subcarriers. Although adding the CP causes power and spectrum efficiency loss, this deficiency is highly compensated by the ease of receiver implementation that makes OFDM both practical and attractive to the radio link designers.

OFDM has in fact been adopted by many European and American telecommunication standards in the last few decades. In the context of wired environments, OFDM is applied for high speed digital voice services, e.g., asymmetric digital subscriber lines (ADSL) [95] and its faster version, very-high-bit-rate digital subcarrier line (VDSL) [96]. In wireless communications, the OFDM technique is the fundamental building block of the IEEE 802.16 standards and it has been considered as a solution to mitigate multipath propagation in broadband multimedia broadcasting, e.g., digital video broadcasting for terrestrial television (DVB-T) [97], digital audio broadcasting (DAB) [98], and 3G mobile communication (3GPP-LTE) [99]. To summarize, the wide interest in OFDM technique is due the following advantages:

- high spectral efficiency;
- interference suppression capability through the use of the CP;
- protection against narrowband interference and inter carrier interference (ICI);
- efficient implementation using FFT;
- flexible spectrum adaptation; and

- separated subcarrier modulation, which implies that different constellations can be applied on individual subcarriers, thus allowing for several resource allocation strategies.

Even though the concept of multicarrier transmission is simple in its basic principle, the design of practical OFDM systems is far from being a trivial task. Synchronization, channel estimation, and radio resource management are only a few examples of the numerous challenges related to multicarrier technology. As a result of continuous efforts of many researchers, most of these challenging issues have been studied and several solutions are currently available in the open literature. Besides its significant advantages, OFDM suffers from the following disadvantages:

- high peak-to-average power ratio (PAPR), which requires highly linear amplifiers and consequently high power consumption [100];
- sensitivity to Doppler effects and carrier frequency offsets [101];
- sensitivity to phase noise, and time and frequency synchronization problems [102]; and
- loss in data rate due to the guard interval insertion.

OFDM is also good from the standpoint of multiple access opportunities. Compared to single carrier systems, OFDM is a versatile modulation, that can be adopted to provide channel access scheme for multiple access systems, in that it intrinsically facilitates both time-division multiple access (TDMA) and frequency-division (or subcarrier-division) multiple access. In a multiuser scenario, the available bandwidth must be shared among several users. Each user may experience different conditions in terms of path loss and shadowing. Furthermore, each user may have different requirements in terms of QoS. An acceptable design of the network should therefore take into account the different user conditions while providing fairness, without a drastic reduction in the overall spectral efficiency.

To meet these needs, in 1998 a combination of OFDM and frequency division multiple access (FDMA), called OFDMA, was proposed by Sari and Karam for cable television (CATV) networks [103]. OFDMA is a promising multiple access scheme that has attracted interest for wireless metropolitan area networks (MANs), as it inherits the immunity to ISI and frequency selective fading of OFDM. Furthermore, in OFDMA systems different modulation schemes can be employed for different users. For instance, each user, according to its distance from the base station (BS), can invoke different orders of modulation schemes (either high- or low-order modulation) to increase its data rate.

An extension of the multiple-access technique ALOHA [86] over OFDMA was first proposed by Shen *et al.* in [104], and further discussed in [105, 106]. In [107], Qin *et al.* propose a distributed access protocol denoted as *channel-aware* ALOHA. The authors extend this idea in [108] to OFDMA systems where water-filling is performed on subcarriers. It can be easily applied to OFDMA selecting subcarriers for different users. Besides, the authors show that the algorithm can reach the Shannon capacity: the users with best channel conditions are usually transmitting if the channel is invariant over the necessary time to manage collisions.

To conclude, its implementation flexibility, the low complexity equalizer required in the transceiver, as well as the attainable high performance, make OFDMA a highly attractive candidate for high data rate communications over time-varying frequency selective multiuser radio channels. Compared to classical FDMA, OFDMA presents a higher spectral efficiency by avoiding the need for large guard bands between different users' signals. The main advantages of OFDMA are the increased flexibility in resource management and the ability for a dynamic channel assignment. OFDMA can exploit the channel state information (CSI) to provide users with the best subcarriers (in terms of channel condition between transmitter and receiver over different subcarriers) that are available, thereby leading to remarkable gains in terms of achievable data throughput. In terms of architecture complexity, OFDMA systems can now be implemented using powerful integrated circuits optimized to perform FFT operations. Because of its increasingly widespread acceptance as the modulation scheme of wireless networks of the future, it attracts a lot of research attention, in areas like resource allocation, time-domain equalization, PAPR reduction, phase noise mitigation and pulse shaping.

Thanks to its favorable features, OFDMA is widely recognized as the technique that is able to meet the requirements for fourth generation broadband wireless networks, as witnessed by the IEEE 802.16m [86] and LTE-Advanced standards [109]. In the next sections, we will focus on resource allocation techniques which includes subcarriers selection and power allocation, by leveraging on multiuser diversity and channel fading.

2.2 Resource allocation in OFDM

The research interest on resource allocation in multicarrier systems was encouraged by the successfully development of ADSL services in the 1990s [95]. This technology employs, for high-speed wireline data transmissions, a digital multitone (DMT) modulation, a particular form of frequency multitone (FMT) technology, which is a frequency-division-based transmission technique. Due to crosstalk from adjacent copper twisted pairs, the ADSL channel is characterized by strongly frequency-selective noise. This scenario is similar to that experienced in the OFDM transmission scheme, which is flexible enough to allocate individual power and modulation on different subcarriers.

In the context of OFDM, different criteria to allocate the available resources can be performed depending upon whether the network is trying to maximize the overall data rate under a total power constraint, or to minimize the overall transmit power given a fixed data rate or bit error rate (BER). The optimal OFDM adaptation algorithm, called the water-filling (WF) criterion [110] and originally derived for DMT systems, tends to allocate most information bits onto the highest signal-to-noise ratios (SNRs) carriers. Note that the number of bits determines the constellation size as follows: 1 bit corresponds to binary phase-shift keying (BPSK) modulation, 2 bits to quadrature phase-shift keying (QPSK) modulation, 4 bits to 16-quadrature amplitude modulation (16-QAM), and so on. In some situations, some subcarriers may even be left unassigned if their SNRs are too low to provide reliable data transmission.

In the literature, the problem of efficient bit allocation on the available subchannels and using the best efficient modulation (to each subcarrier) is equivalently referred to as *bit loading*, *adaptive modulation*, and *link adaptation*. In an OFDM communication, the (unique) transmitter spending power $p_n \leq \overline{p}_n$ over the nth subcarrier, with $\overline{p}_n$ being the maximum power constraint on subcarrier n, can use a number of bits b_n that is calculated using the Shannon channel capacity formula as [111, Eq. 1]:

$$b_n = \left\lfloor \log_2 \left(1 + \frac{|H_n|^2 \, p_n}{(\xi + \Gamma) \cdot \sigma_w^2} \right) \right\rfloor \tag{2.1}$$

where $\lfloor \cdot \rfloor$ is the floor operator, $|H_n|^2$ is the amplitude of the (complex) frequency response of subcarrier n, σ_w^2 is the noise power on each subcarrier, ξ is the additional amount of noise that the system can tolerate while achieving the minimum desired

BER requirement [111], even when the noise level is increased by a factor ξ, and Γ is the SNR gap (also known as the normalized SNR), used to evaluate the relative performance of a modulation scheme versus the theoretical capacity of the channel [112]. Therefore, by increasing the value of ξ, we can improve the system robustness against noise, and hence have the new operating point of the constellations at a distance of $10\log_{10}(\xi+\Gamma)$ dB from the Shannon limit.

There are many theoretical works that aim at regulating the transmit powers $\{p_n\}$ to perform the adaptive bit loading (2.1). In the following we cite some of the pioneering and well-known bit loading algorithms in the context of OFDM systems:

- Hughes-Hartogs in 1987 [113] designed a greedy algorithm to approximate the WF (e.g., see [110]) for twisted-pair channels over an additive white Gaussian noise (AWGN) channel with ISI. The goal of this discrete bit loading algorithm is the minimization of the transmit power under a BER and data rate constraints for each tone. This is accomplished by successively assigning bits to carriers, each time choosing the carrier that requires the least incremental power, until the given target rate is reached. Bingham in [114] proposes to apply sinc functions for each individual spectra instead of using quadrature amplitude shift keying (QASK) in [113]. Applying the technique proposed in [114] allows us to separate signals at the receiver using computationally efficient FFT techniques, although the high complexity burden of the proposed algorithm makes it unsuitable for a practical implementation is high-speed wireless networks.

- The principle of adaptive modulation and power over OFDM was recognized in 1989 by Kalet [115], who simulates a twisted-pair OFDM system, in which each subcarrier uses QAM to maximize the bit rate. The power distribution between the subcarriers and the number of bits per symbol per subcarrier is optimized for a given BER, showing that the proposed power allocation achieves similar results to the WF solution. Furthermore, multicarrier QAM performance is about 9 dB worse than the channel capacity, irrespectively of the channel response. Quantitative results for a twisted-pair cable show that multitone QAM transmission outperforms single-tone QAM by more than 40%. The Kalet's algorithm is often referred to as WF in the frequency domain, which is a simpler version of the technique proposed by Cimini in [116] for mobile communication channels.

- Chow *et al.* in [111] proposed an iterative bit loading algorithm which offers significant advantages over the Hughes-Hartogs algorithm [113] and the WF method [115, 116]. The simulation results over high-speed ADSL service using a required BER of 10^{-7} show a maximum degradation of only 1.3 dB in terms of SNR compared to [115]. Even though the proposed algorithm is faster than that Hughes-Hartogs one, it is not optimal in terms of number of iterations and computational load.

- Czylwik [117] in 1996 simulates an OFDM transmission system with time-variant channel functions, measured with a wideband channel sounder with fixed carrier frequency antennas. The simulation results show that the proposed subcarrier-adaptive modulation demands a total power consumption at least 5 dB lower (and reaching 15 dB lower, depending on the propagation scenario) than that required by the non-adaptation (fixed modulation) OFDM, by placing a requirement in terms of BER equal to 10^{-3}. Different modulation formats can be selected so as to minimize the BER under a constant data rate constraint.

- Fischer and Huber in 1996 [118] proposed a bit loading algorithm to reduce the computational complexity of Hughes-Hartogs and Chow algorithms. This algorithm distributes bits and transmit power to maximize the SNR over each carrier. Van-der Perre *et al.* in [119] apply [118] to simulate the performance of OFDM-based high speed wireless LANs (with data rate on the order of 100 Mb/s). Simulation results show that the proposed adaptive loading strategy improves the system performance considerably, with an SNR gain of 6 dB with respect to the fixed QPSK or 16-QAM modulations, under a BER constraint of 10^{-2}.

As a conclusion, all link adaptation studies reported here have demonstrated that a performance improvement in OFDM systems can be attained by properly adjusting power and data rate over each subcarrier, so as to exploit the channel frequency selectivity. To further increase the capacity of the system, state-of-the-art solutions always adopt *coding techniques*. In the practice, a frequency-selective radio channel may severely attenuate the data symbols transmitted on several subcarriers, leading to bit errors. By spreading the coded bits over the bandwidth of the transmitted system, an efficient coding scheme can correct for the erroneous bits and thereby exploit the wideband channel frequency diversity.

To this aim, all communication systems include forward error correction (FEC) coding techniques [86] to attain the system SNR requirements at low required BER values. In FEC schemes, only the error correction is performed, whereas in automatic repeat request (ARQ) [86] schemes, the retransmission of erroneous blocks is requested whenever the decoded data is labeled as unreliable. OFDM systems that utilize *adapting modulation and coding per subcarrier* are often referred as coded OFDM (COFDM) systems [120]. COFDM increases the data rate and outperforms solutions using only either modulation or channel coding. COFDM does not adapt the data rate of each subcarrier due to the differing SNRs, rather it uses the same high-order modulation on an all subcarriers and uses coding to correct the errors.

Recently, there has been numerous interest on the design of good error-correcting codes achieving near-Shannon performance, particularly low-density parity-check (LDPC) codes [121], that are well suited for OFDM systems [122], as they can reduce the impact of deep channel fades in both the time and the frequency domains. In high data rate wireless OFDM systems, current challenges include the design of LDPC codes with reasonable block length (and thus with feasible encoding and decoding complexity) and overhead delay [123].

In addition to the signal strength, the wireless medium may also affect the original signal through *dispersion*, which includes time dispersion (frequency selective) and frequency dispersion (time selective) fading. While OFDM is immune to the time dispersion effect at the expense of CP, it is not guaranteed whether the signals across different subchannels will not interfere to each other. Hassibbi and Hochwald in [124] pioneered to propose a linear space-time coding, called *linear dispersion coding* and also *linear constellation coding*, for high data rate communications with large number of subcarriers. The codes are designed to optimize mutual information between transmitter and receiver. To reduce the decoding complexity, [125] divides subcarriers into a number of disjoint groups based on criterions in [126] and [127] for reducing multiuser interference and PAPR, respectively. Then, to apply the same linear dispersion coding to subcarriers within each group and to transmit every information symbol over subcarriers within only one group. The idea of grouping subcarriers in smaller groups is used by references [128] and [129] which aims at minimization BER in single user and multi users, respectively. The open problem of linear pre-coding related works is the gap between achieved data rate and the outer region capacity.

2.3 Resource allocation in OFDMA

A typical case of a multiple access channel is the uplink of a cellular system. In general, in the OFDMA uplink scenario, each user receives a channel assignment and a power allocation from the BS that consists of a (usually exclusive) subset of subcarriers and power levels on each of them. In an OFDMA network, the BS must optimally allocate power and bits over different subcarriers based on instantaneous channel conditions of different active wireless terminals. The only requirement is that the fading rate is not too fast (compared to the OFDMA symbol time), as instantaneous resource allocation is impractical in the presence of rapidly-varying transmission channels of mobile terminals. Other impairments include interference management and limited resources, such as bandwidth and transmit power. This makes the link adaptation task much more challenging than in single-user systems. Recent exhaustive surveys on these topics include [130] (with emphasis on scheduling schemes), [131] (with focus on ICI mitigation), [132] (with emphasis on relay communications), and [133] (with focus on game-theoretic approaches). In the remainder of this contribution, we aim at introducing the very basics for OFDMA resource allocation by means of detailed problem formulation and numerical examples, which, to the best of our knowledge, is not available in the literature.

It is clear that, compared to a point-to-point single-user OFDM-based connection, a multiuser OFDMA link adaptation is much more complicated and hardly scalable [134]. In particular, one of the main problems with OFDMA is the large amount of feedback required from the users. Since different users can be scheduled over different subcarriers, they must feed the measurement information back about *every* subcarrier to the BS. Consider a network with K active OFDM mobile terminals and N total available subcarriers. The scheduler requires full channel state information (CSI) consisting of $K \cdot N$ complex numbers (the values of the channel frequency response at each subcarrier for every user). This feedback information represents a very large overhead if there are many users and subcarriers in the system. To reduce it, Cimini *et al.* [135] proposed to group adjacent subcarriers into *clusters* and to feed back the information about the best cluster(s) in terms of channel quality. In [136, 137], it is shown that sending back only heavily quantized CSI dramatically reduces the feedback needs without significantly sacrificing the overall performance. In this context, Svedman *et al.* [138] showed that a suitable cluster size, which highly impacts on the performance in terms of achieved downlink throughput, must be selected according to the average channel delay spread of the users.

In this context, let us start to review the main concepts and the main categories behind bit and power loading in OFDMA-based transmissions. Generally, a resource allocation algorithm can either be *centralized* or *distributed*. In centralized schemes, such as [16, 139], the algorithm is run by a central unit (in an infrastructure networks, typically the BS) that is aware of the demands and of the channel conditions of all mobile terminals, as described in the previous paragraph. In a distributed model (such as [140]), each mobile terminal tries to accomplish its own (minimum) QoS autonomously, sometimes resorting to cross-layer approaches (e.g., [141]), to reduce the total power consumption and to support different services and traffic classes, mostly for the downlink of an OFDMA system. In general, centralized techniques show better performance at the expense of a higher signaling between terminals and the central unit, and lower scalability than distributed techniques.

Another typical classification of resource allocation techniques for OFDMA networks is based on the objective of the optimization problem. The solutions available in the literature mainly fall into two different categories: *margin-adaptive* and *rate-adaptive* methods. The goal of margin adaptive schemes [142] is to minimize the total transmit power expenditure given a set of fixed user data rates and BER requirements. Algorithms based on the rate-adaptive criterion [143] aim on the contrary at achieving the maximum total (continuous) sum-rate over all users subject to different QoS constraints, e.g., power expenditure. Note that, unlike some broadband systems, e.g., based on code division multiple access (CDMA), ultra-wideband (UWB), and multicarrier CDMA (MC-CDMA), in which the whole bandwidth is shared by all active wireless terminals, OFDMA-based networks do not consider resource allocation strategies based on the *mean-BER minimization*, in which the robustness of the system is enhanced by allocating bits and powers to subcarriers to minimize the error rate of an entire symbol. This scheme is not of major interest for OFDMA systems, since, as will be seen in the next subsection, in (almost) all OFDMA resource allocation techniques each subcarrier is not permitted to be assigned to more than one user. This means that a well devised algorithm to maximize the total data rate also results in minimizing each user's BER.

The first resource allocation strategy presented here is the minimization of the OFDMA system power expenditure for a given target data rate, solving the following margin-adaptive optimization problem:

$$\min_{\mathbf{p},\,\mathcal{N}} \sum_{k=1}^{K} \sum_{n\in\mathcal{N}_k} p_{kn} \tag{2.2a}$$

$$\text{s.t.} \quad \sum_{n\in\mathcal{N}_k} R_{kn} \geq \underline{R}_k \qquad \forall\, k \in \mathcal{K} \tag{2.2b}$$

$$\text{and} \quad \sum_{n\in\mathcal{N}_k} p_{kn} \leq \overline{p}_k \qquad \forall\, k \in \mathcal{K} \tag{2.2c}$$

$$\text{and} \quad \mathcal{N}_k \cap \mathcal{N}_m = \emptyset \qquad \forall\, k,\, m \in \mathcal{K}, k \neq m, \tag{2.2d}$$

where $k \in \mathcal{K} = [1, \ldots, K]$ denotes the index of the wireless terminal which transmits with powers $\mathbf{p}_k = \left[p_{k1}, \ldots, p_{kn}, \ldots, p_{kN}\right]$ over the N subcarriers, which are represented by the set $\mathcal{N} = [1, \ldots, n, \ldots, N]$, and $\mathbf{p} = [\mathbf{p}_1, \ldots, \mathbf{p}_k, \ldots, \mathbf{p}_K]$. Let $\mathcal{N}_k \subset \mathcal{N}$ be the set of subcarriers assigned to user k, and R_{kn} be the channel capacity that can be achieved by user k over the nth subcarrier. The sets of assigned subcarriers are disjoint, as explicitly stated by (2.2d): this means that each subcarrier is not allowed to be shared by more than one terminal. Each user k wishes to attain its target rate $\underline{R}_k$, as specified in (2.2b), under the constraint $\overline{p}_k$ on its total transmit power, as formulated in (2.2c). It is clear that, for each terminal k and every $n \notin \mathcal{N}_k$, we have $p_{kn} = 0$, and accordingly, $R_{kn} = 0$. The overall data rate of each user is obtained by the Shannon capacity formula as:

$$R_k = \sum_{n\in\mathcal{N}_k} R_{kn} = \sum_{n\in\mathcal{N}_k} \log_2\left(1 + \frac{\left|H_{kn}\right|^2 p_{kn}}{\sigma_w^2}\right) \tag{2.3}$$

in which $\left|H_{kn}\right|^2$ denotes the amplitude of the Gaussian-complex path gain experienced by user k on subcarrier n, and, similarly to Sect. 2.2, σ_w^2 is the power of the AWGN zero mean Gaussian noise on each subcarrier.

Two levels of decomposition are necessary to turn this NP-hard problem into the set of subproblems, subcarrier allocation and power control [144]. In fact, the exclusive assignment of subcarriers to users is a way to reduce the complexity computation of the optimization problem (2.2a), as the rate R_{kn} can be computed using (2.3). On the other hand, as users are not allowed to share a common subcarrier, the allocation process boils down to a combinatorial optimization problem, for which no optimal greedy solution exists. Kivanc *et al.* [145] developed a computationally inexpensive method for OFDMA resource assignment which achieves a comparable performance with respect to the solution of the NP-hard problem (2.2) in terms of transmission power and bandwidth efficiency at a reduced computational complexity. However, this approach does not provide a fair apportionment among users, so that some of them may be dominant in terms of resource occupancy even when the minimum rate requirement is not satisfied for the others.

In addition to this limitation, the margin-adaptive formulation that focuses on the minimization of transmit powers is often of lower interest compared to the maximization of the data rates. For this reason, the most common optimization problem for OFDMA systems is the rate-adaptive one, that aims at maximizing the bit rate as follows:

$$\max_{\mathbf{p},\,\mathcal{N}} \sum_{k=1}^{K} \sum_{n\in\mathcal{N}_k} R_{kn} \tag{2.4a}$$

$$\text{s.t.} \quad \sum_{n\in\mathcal{N}_k} R_{kn} \geq \underline{R}_k \qquad \forall\, k \in \mathcal{K} \tag{2.4b}$$

$$\text{and} \quad \sum_{n\in\mathcal{N}_k} p_{kn} \leq \overline{p}_k \qquad \forall\, k \in \mathcal{K} \tag{2.4c}$$

$$\text{and} \quad \mathcal{N}_k \cap \mathcal{N}_m = \emptyset \qquad \forall\, k,\, m \in \mathcal{K}, k \neq m. \tag{2.4d}$$

The objective of this problem is to distribute bits and power among different subcarriers in such a way that the overall data rate of the system is maximized. Most algorithms focus on the downlink scenario, with constraints on the total power transmitted by the radio BS. In the uplink scenario, restrictions apply on an individual basis to each user terminal, and the simplest solution to maximize the channel capacity of mobile devices under a power constraint is the WF criterion [146], described in the following subsection.

2.3.1 The water-filling solution

Cheng and Verdú in [147] pioneered the application of the WF solution in an uplink OFDMA network scenario, and derived the capacity region and the optimal power allocation of individual users. In the rate-adaptive optimization, the channel capacity is obtained by maximizing the right-hand side of (2.3) with respect to (2.4c), i.e.,

$$\max_{\mathbf{p}\,,\mathcal{N}} \left\{ \sum_{k=1}^{K} \sum_{n\in\mathcal{N}_k} \log_2\left(1 + \frac{|H_{kn}|^2\, p_{kn}}{\sigma_w^2}\right) \right\}. \tag{2.5}$$

Since the objective function in (2.5) is convex in the variables $\{\mathbf{p}_k\}$, the optimum power allocation under the convex constraints of overall transmit power can be found using Lagrangian methods [148]. The optimal strategy to satisfy (2.4a) is such that each subcarrier $n \in \mathcal{N}$ is assigned to the user with the largest channel gain in a centralized way as follows:

$$k \longleftarrow \arg\max_{\ell \in \mathcal{K}} |H_{\ell n}|^2 . \tag{2.6}$$

The resulting optimal power allocation for user k is given by:

$$p_{kn} = \left[\frac{1}{\lambda_k} - \frac{\sigma_w^2}{|H_{kn}|^2}\right]^+ \tag{2.7}$$

where $[x]^+ = \max\{x, 0\}$, and λ_k is the Lagrangian parameter ("water-level"), chosen such that the sum of the allocated powers satisfies the total power constraint $\overline{p}_k$:

$$\lambda_k = |\mathcal{N}_k| \cdot \left(\overline{p}_k + \sum_{n \in \mathcal{N}_k} \frac{\sigma_w^2}{|H_{kn}|^2}\right)^{-1} . \tag{2.8}$$

To conclude, the WF is a greedy (centralized) power allocation scheme that increases the channel capacity by assigning every subcarrier to the user with the best path gain, and by distributing the power according to (2.7). Note that the WF solution is highly unfair, since only users with the best channel gains receive an acceptable channel capacity, while users with bad channel conditions (e.g., far users) achieve very low data rates. More information-theoretic discussions on related topics can be found in [149]. To derive fair resource allocation schemes, we resort to other techniques, described in the following subsections.

2.3.2 The max-min fairness criterion

In an OFDMA network, one possible approach to overcome the unfairness of WF is described in [150]. This alternative formulation aims at maximizing the minimum data rate across users, thus enforcing the notion of *max-min rate-maximization fairness* that avoids the starvation of some users.

Definition 17 *A feasible*[1] *rate vector* $\mathbf{R} = [R_1, \ldots, R_k, \ldots, R_K]$ *is defined to be* max-min fair *if any rate* R_k *cannot be increased without decreasing some other rate* R_m*,* $m \neq k$*, which is smaller than or equal to* R_k*.* ■

[1] A rate allocation $\mathbf{R}$ is *feasible* if the network resources are enough to provide every user k in

Roughly speaking, in the max-min power control the objective is to optimize the performance of the worst link amongst all users for a fixed QoS-based power control approach. The idea behind the max-min fair approach is to treat all users as fairly as possible, by making all rates as large as possible [151]. The work of Rhee and Cioffi in [150] is an extension of [152], which is a dual problem of minimizing the total transmit power for given data rate requirements. The problem is formulated as the following convex optimization problem [150]:

$$\max_{\mathbf{p},\,\mathcal{N}} \min_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}_k} R_{kn} \tag{2.9a}$$

$$\text{s.t.} \quad \sum_{n\in\mathcal{N}_k} p_{kn} \leq \overline{p}_k \qquad \forall\, k \in \mathcal{K} \tag{2.9b}$$

$$\text{and} \quad \mathcal{N}_k \cap \mathcal{N}_m = \emptyset \qquad \forall\, k,\, m \in \mathcal{K}, k \neq m. \tag{2.9c}$$

The Lagrangian relaxation [148] algorithm proposed in [150, 152] approaches the solution to (2.9a) by slowly increasing the power level for each user. By elaborating on a simple iterative algorithm to compute a suboptimal max-min fair rate vector proposed by Bertsekas and Gallager in [153, p. 527], we can easily extend it for an OFDMA network as follows:

1) *Zero initialization:* Supposing $K \ll N$, the algorithm starts with an all-zero data rate vector, i.e., $R_k = 0$ and $\mathcal{N}_k = \emptyset \quad \forall\, k \in \mathcal{K}$.

2) *Round-robin step:* Assign every user $k \in \mathcal{K}$ the subcarrier n whose channel gain $|H_{kn}|^2$ is the highest among the remaining ones, using a uniform power $\overline{p}_k/N$ as:

$$n \longleftarrow \arg\max_{m\in\mathcal{N}} |H_{km}|^2; \tag{2.10a}$$

$$\mathcal{N}_k = \mathcal{N}_k \cup \{n\}; \tag{2.10b}$$

$$\mathcal{N} \leftarrow \mathcal{N} \setminus \{n\}; \tag{2.10c}$$

$$R_k = R_{kn}^{1/N}, \tag{2.10d}$$

the network with rate R_k. To the best of the authors' knowledge, the algorithms available in the literature do not propose criteria to assess the a-priori feasibility of a certain vector $\mathbf{R}$. The remainder of this paper is thus based on the assumption that the network resources can guarantee achievable rates R_k, e.g., based upon the Shannon capacity [111] and some performance gaps, such as those mentioned in Sect. 2.2 for the single-user scenario.

where

$$R_{kn}^{1/N} = \log_2\left(1 + \frac{|H_{kn}|^2\,\overline{p}_k}{N\sigma_w^2}\right). \tag{2.11}$$

At this point, every user $k \in \mathcal{K}$ is assigned exactly one subcarrier.

3) *Best user rate update:* Find the user k with the smallest attained data rate, i.e., $k \leftarrow \arg\min_{\ell\in\mathcal{K}} R_\ell$, and then assign to it the subcarrier $n \in \mathcal{N}$ with the best channel condition $|H_{kn}|^2$, and update its data rate as:

$$k \leftarrow \arg\min_{\ell\in\mathcal{K}} R_\ell; \tag{2.12a}$$

$$n \longleftarrow \arg\max_{m\in\mathcal{N}} |H_{km}|^2; \tag{2.12b}$$

$$\mathcal{N}_k = \mathcal{N}_k \cup \{n\}; \tag{2.12c}$$

$$\mathcal{N} \leftarrow \mathcal{N} \setminus \{n\}; \tag{2.12d}$$

$$R_k = R_k + R_{kn}^{1/N}. \tag{2.12e}$$

4) *Exit condition:* If there exists some unassigned subcarrier, then go back to step 3, else exit the algorithm.

As can be seen by inspecting the steps of the algorithm, the rationale behind max-min fairness solution, in contrast to the WF result, is to assign more power to users exhibiting poor channel conditions (step 3) so that they can achieve a data rate comparable to that of other users with better channel quality. It is worthwhile to note that the max-min fair rate allocation is unique when the number of resources and flows, i.e., subcarriers and wireless terminals, are both finite [151]. Unfortunately, due to the nonlinear nature of the integer problem (2.9), the algorithm proposed in [150, 152] is computationally very expensive.

In [150, Eq. 2], the formulation (2.9a) is extended to:

$$\max_{\mathbf{p},\,\mathcal{N}}\ \min_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}_k} t_{kn}\, R_{kn} \tag{2.13a}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}_k} p_{kn} \leq \overline{p}_k \qquad \forall\, k \in \mathcal{K} \tag{2.13b}$$

$$\text{and} \sum_{k=1}^{K} t_{kn} \leq 1 \qquad \forall\, n \in \mathcal{N}, \tag{2.13c}$$

wherein the positive coefficient $t_{kn} \in [0, 1]$ introduces the percentage of time each subcarrier n is used by a given user k. With t_{kn}, each subchannel can be shared by different users in a TDMA fashion. Clearly, the assumption behind this approach is that the users' channel responses do not change significantly over a timing interval. However, practical solutions, such as those reported in [150], assume $K \ll N$ and that no subchannel is shared among users, i.e., t_{kn} is a binary value and $\sum_{k\in\mathcal{K}} t_{kn} = 1 \ \forall \ n \in \mathcal{N}$, or, equivalently, (2.9c). In addition, determining the best values for $t_{kn} \in (0, 1)$ and indicating a time-sharing allocation policy is not always feasible for $K > N$, as reported in [154].

In addition, to achieve a max-min fairness data rate vector, Kelly [155] suggested to formulate the problem as:

$$\max_{\mathbf{p},\,\mathcal{N}} \sum_{k=1}^{K} - \left(- \log_2 \left(\frac{R_k}{\beta} \right) \right)^{\rho} \tag{2.14}$$

wherein $\rho > 1$ is a constant parameter, and β is a positive constant, measured in bits/s, satisfying $R_k < \beta \ll \infty \ \forall \ k \in \mathcal{K}$. Thus, the collection of utility functions (2.14) provides a priority to smaller data rates, which increases as ρ increases, and becomes absolute as $\rho \to \infty$. Furthermore, instead of choosing the best user in step 3, (2.12a), an alternative criterion is defined in [155]:

$$k \longleftarrow \arg\max_{\ell\in\mathcal{K}} \left\{ \frac{1}{R_\ell} \cdot \left(\log_2 \left(\frac{R_\ell}{\beta} \right) \right)^{\rho-1} \right\} \tag{2.15}$$

to find the best user k for subcarrier n. Note that, for $\rho \to \infty$, the condition (2.15) becomes:

$$k \longleftarrow \arg\min_{\ell\in\mathcal{K}} R_\ell \tag{2.16}$$

which coincides with the original strategy of max-min fairness to allocate a subcarrier to the user with the minimum achieved data rate.

Although the max-min criterion gives priority to the weakest users, thus balancing the near-far effect, this solution cannot be used in the practice, because, in general, the number of allocated bits may not correspond to any practical modulation scheme [156]. Furthermore, the results show that under the max-min fair solution, some users may consume significantly more bandwidth than others [157], at the cost of a reduction in the overall throughput of the network.

2.3.3 The weighted proportional fairness criterion

Achieving traffic fairness and efficiency either in the energy or in the spectral domains are two conflicting goals. Hence, the optimization of the radio resource utilization tends to penalize terminals with low SINRs, irrespectively of their traffic level performance. The max-min fairness scheme described in Sect. 2.3.2 is however inappropriate when different users have different priorities. Generally, the problem is how to balance between fairness and utilization of the resources. This led Kelly *et al.* to formulate in [158] the notion of *weighted proportional fairness.* Under a proportional maximization rate constraint, the rate of each user should adhere to a set of predetermined proportionality constants which make a concrete way of assigning priorities to the users as follows:

$$R_1 : \cdots : R_k : \cdots : R_K = \varphi_1 : \cdots : \varphi_k : \cdots : \varphi_K \tag{2.17}$$

where $\{\varphi_k\}$'s are the proportion constants. In the practice, φ_k can be interpreted as the amount user k is willing to pay per unit time. At the end, user k receives in return a data rate R_k which is proportional to φ_k.

Definition 18 *A vector data rate* $\mathbf{R} = [R_1, \ldots, R_K]$ *is* proportional fair *if it is feasible and, for any other feasible rate vector* $\mathbf{R}^{'} = \left[R_1^{'}, \ldots, R_K^{'}\right]$*, the aggregate of proportional changes is non-positive, i.e.:*

$$\sum_{k=1}^{K} \varphi_k \frac{R_k^{'} - R_k}{R_k} \leq 0. \tag{2.18}$$

■

This method is also useful for service level differentiation, which allows for flexible allocation mechanisms to different classes of users with separable constraints. The proportional-fair objective of (2.18) is continuously differentiable, monotonically increasing, and strictly concave, therewith admitting a convex optimization formulation [148]. In [158], Kelly *et al.* suggested an algorithm that converges to the proportionally fair rate vector, using the maximization of the sum of the (logarithmic) long-run average data rates provided to the users, based on the Kuhn-Tucker conditions for the problem (2.4a). Otherwise stated, a proportional-fairness rate allocation can be achieved by formulating the problem as [158]

$$\max_{\mathbf{p},\,\mathcal{N}} \sum_{k=1}^{K} \varphi_k \log_2\left(R_k\right) \tag{2.19}$$

over all feasible rate allocations. Thus, since the logarithm function is strictly concave, proportional-fair rates are unique [159, Sect. 6.7]. Note that the logarithmic utility function indicates that users with low average rates benefit more in terms of utility from being scheduled than users with high average rates. The iterative algorithm to compute proportionally max-min fair rate vectors is similar to that for max-min fairness, except for the choice of the best user for each unassigned subcarrier n in (2.12a) and (2.12b) (step 3). In this case, it follows the following criterion instead:

$$k \longleftarrow \arg\max_{\ell\in\mathcal{K}} \varphi_\ell \frac{R_{\ell n}^{1/N}}{R_\ell} \tag{2.20}$$

where $R_{\ell n}^{1/N}$ is computed according to (2.11). The rationale behind this approach is the following. Using (2.20), users compete for resources not directly based on their channel conditions, as happens in Sect. 2.3.2, but according to the combination of priorities φ_ℓ and rates *normalized* by their respective average throughputs, $R_{\ell n}^{1/N}/R_\ell$. In other words, each subcarrier is assigned to a user when its channel, weighted by its priority, is near its own peak in the frequency domain, thus trading off multiuser diversity and fairness.

The update of the data rate R_k can be done in different ways. A low-complexity update equation that also bears low memory requirements is defined in [159, Sect. 6.7], by keeping track of the average throughput R_k of each user in an exponentially-weighted time window of length T_c as follows:

$$\begin{cases} R_k = \left(1-\frac{1}{T_c}\right) R_k + \frac{1}{T_c} R_{kn}^{1/N} & , \\ R_k = \left(1-\frac{1}{T_c}\right) R_k & k \neq m, \end{cases} \tag{2.21}$$

where k is the index of the preferred user for the next updating round and m is the selected user for the current round, both selected following (2.16). The update (2.21) is an exponentially weighted filter that, instead of using (2.12e), includes all historical rates in the average rate. Note that using a very large time-scale T_c, (2.21) is equivalent to maximization problem (2.14) [159, Sect. 6.7].

In the literature of OFDMA resource allocation, some other instantaneous sum-rate maximization methods with proportional rate constraints have been studied (e.g., [160–162]). In terms of problem formulation, the main emphasis of these works is on the maximization of the data rates with instantaneous proportional rate constraints, exclusive subcarrier assignment, and constrained total transmit power. The solution is achieved by resorting to integer programming methods, with time complexity (i.e., number of time steps in the iterative algorithm) on the order of $\mathcal{O}\left(NK\log_2 N\right)$ or higher.

The notion of weighted proportional fairness has been extended by Mo and Walrand in [163], observing some particular transmission control protocol (TCP)-based network traffics, in which the total throughput of weighted proportional fairness is not optimal in terms of spectral efficiency. To overcome this drawback, the problem is then formulated using the following definition.

Definition 19 *Let α be a non negative constant, and $\boldsymbol{\varphi} = [\varphi_1, \ldots, \varphi_K]$ be a positive weight vector. A vector data rate $\mathbf{R} = [R_1, \ldots, R_K]$ is $(\boldsymbol{\varphi},\, \alpha)$* proportional fair *if it is feasible and, for any other feasible rate vector $\mathbf{R}' = \left[R_1', \ldots, R_K'\right]$,*

$$\sum_{k=1}^{K} \varphi_k \frac{R_k' - R_k}{{R_k}^{\alpha}} \leq 0. \tag{2.22}$$

■

Obviously, if $\alpha = 1$, (2.22) reduces to the weighted proportional fairness introduced in (2.18). If $\alpha \to \infty$, $\mathbf{R}$ approaches the max-min rate vector [163, Lemma 3]. In other words, this generalization includes arbitrarily close approximation of max-min fairness. Unfortunately, the challenge of choosing the best value of α makes this framework (almost) impractical. Further examinations clarify that $(\boldsymbol{\varphi},\, \alpha > 1)$ proportional fairness maximizes [163, Lemma 2]:

$$\sum_{k=1}^{K} \varphi_k \left(1-\alpha\right)^{-1} {R_k}^{1-\alpha} \tag{2.23}$$

over all feasible data rate vectors.

Mathematically, (2.22) is a twice continuously differentiable and strictly concave function. Algorithms for computing $(\boldsymbol{\varphi},\, \alpha)$ proportionally fair rates have been developed in [163], where each transmitter adapts its window size based on the total delay between the transmission of a packet and the reception of its acknowledgment. The main drawback of the proportionally fair rate allocation is that utility (maximization) functions are commonly assumed as concave. Lee *et al.* [164] showed that, if the abovementioned algorithms developed for concave utility functions are applied to non concave utility functions, the system can be unstable and cause excessive congestion in the network. Since the rate adaptive functions of some real-time applications are not concave (e.g., a multimedia communication) [165], they cannot be dealt with in this kind of systems.

2.3.4 Utility maximization

Max-min fairness (Sect. 2.3.2) and weighted proportional fairness (Sect. 2.3.3) consider the same QoS requirements among network users with a strictly concave rate adaptive function. As mentioned above, in some systems, e.g., multimedia applications, the rate maximization functions are not concave. Furthermore, in such contexts we are not able to formulate real-time constraints, e.g., in terms of delay. In their seminal work [166], Cao and Zegura overcome these disadvantages by introducing the concept of *utility maximization* in terms of application-layer performance, whose aim is to provide individual QoS requirements for each user with a (not necessarily concave) function for rate maximization. More in general, a utility function is a function that can be used to mathematically describe the QoS characteristics of an application, thus allowing the system designers to put the emphasis on specific QoS parameters of the network. Unlike rate-adaptive formulations, in which the objective, as described in the previous subsections, is the sum-rate maximization with constraints in terms of power expenditure, the utility maximization approach can guarantee the application-specific demand which can be characterized by bandwidth, delay, delay jitter, or time spent to complete data deliveries, just to mention a few examples.

In other words, this framework allows for more general resource allocation problems, that can be formulated in different ways according to the goal of the system. For instance, a power control scheme for optimal uplink SNR assignment can be expressed in a centralized way as follows:

$$\max_{\mathbf{p},\,\mathcal{N}} \sum_{k=1}^{K} \sum_{n\in\mathcal{N}_k} u_k(\gamma_{kn}) \tag{2.24a}$$

$$\text{s.t.} \quad \sum_{n \in \mathcal{N}_k} p_{kn} \leq \overline{p}_k \qquad \forall\, k \in \mathcal{K} \tag{2.24b}$$

$$\text{and} \quad \mathcal{N}_k \cap \mathcal{N}_m = \emptyset \qquad \forall\, k,\, m \in \mathcal{K}, k \neq m. \tag{2.24c}$$

where

$$\gamma_{kn} = \frac{|H_{kn}|^2 \, p_{kn}}{\sigma_w^2} \tag{2.25}$$

denotes the SNR of the user k on the nth carrier (used by user k only in an exclusive fashion) as measured at the BS, and $u_k(\cdot)$ is user k's individual maximization function that is a function of each user's relevant parameters. The maximization function can also be represented as a greedy function for each user as follows:

$$\max_{\mathbf{p}_k,\, \mathcal{N}_k} \sum_{n \in \mathcal{N}_k} u_k(\gamma_{kn}) \qquad \forall\, k \in \mathcal{K} \tag{2.26}$$

that makes the power control a distributed problem, in which each user k seeks the optimal vectors $\mathbf{p}_k$ and $\mathcal{N}_k$ that maximize its own sum-utility (2.26). However, note that selecting the set $\mathcal{N}_k$ by each user while meeting the exclusive assignment of the subcarriers, i.e., (2.24c), implies a certain amount of feedback information among the users, which, although less demanding in terms of feedback rate than the problem (2.24), makes this scheme not completely distributed.

In the literature, many utility-based resource allocation formulations appeared in the last few years. As already mentioned, [166] proposed the use of a utility function to maximize the performance of the application layer. The drawback in [166] is a high delay in the communication network among users. Cho and Chang *et al.* in [167] extend this formulation to address the limitation in terms of delay, by proposing a control-theoretic utility max-min flow control algorithm, and showing that the algorithm converges to a utility max-min fair rate vector by using Dewey and Jury's stability criterion [168]. Among the others, Huang *et al.* in [169, 170] introduced scheduling and radio resource allocation algorithms in OFDMA-based wireless networks in the downlink and the uplink direction, respectively, using a dual formulation, and showing a complexity $\mathcal{O}\left(KN + N\log_2 N\right)$. In particular, [169] looks at joint scheduling and resource allocation for the downlink by considering several practical constraints largely ignored in the previous literature (e.g., self-noise). Reference [170] aims at maximizing the achieved data rates taking into account the queue length of each user, using an algorithm that can also be applied to downlink transmissions. Zhou *et al.* [171] solve a scheduling and resource allocation problem in an OFDMA system using an approach based on utility functions, that eventually results in a discrete optimization problem with a non-differentiable non-convex objective with minimum data rate constraints. The idea is to transform the discrete problem into a suitable weighted max-min fairness problem which is easier to be implemented. In [172], Kim and Lee present a general utility-based framework for joint uplink/downlink optimization, where the user's satisfaction is modeled by two different utility functions, one for the uplink, and another one for the downlink. The resource allocation is formulated as a maximization problem with an objective based on the sessions' utility functions and allocation probabilities as scheduling constraints that are solved via dual optimization techniques. To investigate radio resource allocation in OFDMA with heterogenous traffic classes, reference [173] defines a utility function as a sinusoidal function that depends on minimum and maximum data rates.

References [174–176] address the problem of energy efficiency maximization subject to power constraints according to the circuit power consumed. Xiong *et al.* in [175] devise a joint uplink/downlink water-filled energy efficient resource allocation under users priority constraints. The WF based iterative algorithm proposed by [175] converges faster than that of [174], while the spectral efficiency of the algorithm proposed by [174] is higher than that of [175].

To summarize, even though the utility maximization approach has made advances in dealing with heterogeneous resource allocation issues, it also exhibits serious limitations. As already mentioned, there exists a tradeoff between average throughput and fairness in the system. Sometimes there also exists a conflict between the QoS balance and the utility maximization. If users select utility functions based on their actual QoS requirements, then the optimal achieved data rate may result in a totally unfair resource allocation within the network. Applying advanced optimization methods of geometric programming [148], majorization theory [177], and fractional programming [178] may achieve an admissible tradeoff between fairness and overall throughput [176, 179, 180]. However, depending on the problem formulation, it is impossible to achieve the desired network performance if the resource allocation scheme operates on the link layer only. To further generalize the problem formulation, and thus to increase its potential, it is worth resorting to the cross-layer approach described in the next subsection.

2.3.5 Cross-layer optimization

So far, we talked about the design of an OFDMA system based on classical link-level approach. The wireless link level primarily addresses two challenges that arise from the physical medium: channel fading and multiple access interference (MAI). Advances in link design for wireless channels have led to different modulation and channel coding schemes that provide increased robustness to MAI and multipath and, thereby, enhance the radio band capacity. While OFDMA provides a powerful physical layer engine for broadband communications, applying it without thorough application level considerations may lead to poor results. In high-speed data networks, in which the traffic is in fact highly diverse (i.e., with distinct QoS parameters), and channel conditions that may vary dramatically over a short time scale, the traditional (decoupled) layer design cannot meet such requirements. For instance, if the medium access control (MAC) layer does not interact with the upper layers, it cannot obtain information regarding the type of service and the associated QoS parameters. As a consequence, the MAC has no ability to adjust itself to the variable characteristics of the traffic.

An OFDMA radio allocation module can be designed to be both channel-aware and application-aware through *cross-layer* interactions [181] that break the traditional layered paradigm of communication by relying on the concept of joint optimization across multiple layers. The cross-layer approach allows different layers to be grouped and/or assumes the existence of protocols that work with more than one layer, thus optimizing the protocol stack. With cross-layer techniques, decision making can be more accurate, bringing forth several benefits to the performance of the network. Bohge *et al.* in [182] provides basic definition and knowledge of cross-layer optimization in the context of OFDM and in the downlink direction of OFDMA systems.

In the context of cross-layer design, many joint scheduling-routing-flow control algorithms have been proposed, including multiuser techniques such as: maximization of the rate delivered on the radio channel [183]; a fair allocation of resources among users belonging to the same traffic class [184]; shaping the dynamics of traffic sources by limiting the delay of data packets in the queues [185]; and maximization of the QoS at the application layer [186]. Sometimes the difference between cross-layer and utility maximization schemes blur away, since cross-layer schemes may require to improve their performance by applying a non-concave utility function, that may consist of parameters from different layers. The common idea behind cross-layer schemes is to properly maintain packet queues to dynamically adapt packet transmission as well as rate allocation. Some pioneering works in the field of resource allocation in OFDMA using cross-layer design have appeared in [187–189].

Jiang *et al.* in [190, Ch. 6] present a more general framework of cross-layer OFDMA resource allocation, with [169, 170] as special cases (Sect. 2.3.4). References [141, 191, 192] propose some feasible solutions to maximize the throughput for the downlink of an OFDMA system under QoS constraints, also reducing the computational complexity. This is achieved using the method of Lagrangian relaxation [148], that is effective to provide users with very low SINRs with good performance. However, in the case the channel conditions and the QoS requirements vary significantly between successive frames, a new set of Lagrangian multipliers must be found in each frame, that may reveal to be impractical.

2.4 OFDMA resource allocation based on game theory

In the utility maximization (Sect. 2.3.4) and cross-layer (Sect. 2.3.5) schemes, different utility functions apply for different users. Sometimes the interests of wireless terminals are not aligned, so that they compete for the scarce wireless resources, namely bandwidth and power. Each user's interest could also be in conflict with others'. In this situation, the wireless terminals can decide to behave in either an altruistic or a selfish manner. In both cases, the related problems can be formulated applying *game theory* [10], which considers the users as *players* in a *game*. In particular, in an OFDMA network, there are multiple interacting users which occupy a fraction of the whole bandwidth, using a fraction of their available transmit power on each subcarrier based not only on their decisions, but also on the interests of any other mobile terminal in the network. This kind of interactions is just the main field of application of game theory, which thus represents an effective analytical tool not only to extend the optimization methods described in the subsections above (see [133] and references therein), but also to address the problem of *Pareto optimality* [10]. In resource allocation problems, one of the major challenges is in fact to achieve a Pareto-optimal rate vector, i.e., a rate allocation such that each user is provided with a certain performance, and any allocation other than that will degrade the performance of at least one user in the network. Interestingly, Pareto-optimal solutions can be investigated by means of game-theoretic formulations.

In the context of game theory, depending on the interaction rules, there exist various types of games. For instance, if the users are allowed to exchange their proper interests and information before the game starts in order to form coalitions and coordinate their actions, the game is said to be *cooperative* (and thus studied by *coalitional game theory*). If coordination among users is not present, the game is said to be *non-cooperative*, and modeled according to *non-cooperative game theory*. In both frameworks, the players act according to their *strategies*. The strategy of a player can be a single move or a set of moves during the game. For games in wireless communications, each transmitter represents a player whose strategy space covers the choices of modulation level, coding rate, transmit power, transmission frequency, just to mention a few examples. Another factor that identify different types of games is the number of times the users interact. If users play the game over multiple rounds, the game is said to be a *repeated game*. Contexts where the users only interact once are referred to as *static games* [10, 22].

2.4.1 Non-cooperative solutions

Non-cooperative game theory has been vastly applied to wireless communication problems, and much progress has been made on distributed power control in Gaussian interference channels. In [193], Wu *et al.* investigates a joint power and (exclusive) subcarrier assignment scheme in single-cell uplink OFDMA systems based on non-cooperative game theory, using the sum-capacity as the utility function to be maximized. This game bears a unique Nash equilibrium (NE), which is a stable outcome of the game (i.e., a stable resource apportionment across users) in which no player has incentive to *unilaterally* (i.e., non-cooperatively) deviate from [10]. In [140], Yu *et al.* apply a different convex utility function to the same scenario, aiming at maximizing the power efficiency of the network. In the utility function, a (transmit) power pricing factor [194] is introduced to overcome the near-far effect, reaching a (nearly) Pareto-optimal NE point. The fairness of both approaches [140, 193] is experimentally showed among a small number of users.

Kwon *et al.* in [195] aim at maximizing the weighted sum-rate of the users in the uplink of a multicell OFDMA scenario. This objective, together with power and rate constraints, defines the non-cooperative game. The simulation results show that the performance of the proposed algorithm strictly depends on the power pricing coefficient, which represents the cost imposed on each BS for the co-channel interference generated by it as well as its power consumption.

Han *et al.* in [16] analyze the previous scenario to maximize the data rates under a constraint in terms of maximum transmit power, showing that the pure non-cooperative game may have some undesirable NE points with low system and individual performance. The authors suggest to introduce a centralized "virtual referee" whose role is to prevent users with high co-channel interference from sharing one subcarrier, or to reduce the demanded transmission rates that prove to be unfeasible. Even though the results significantly outperforms the WF solution in terms of reduced transmit power and increased data rate, the proposed algorithm suffers from high computational complexity. Tan *et al.* in [196] experimentally show that their non-cooperative game-based algorithm achieves a good performance in terms of total data rate, computational complexity, and fairness among users.

The problem of energy-efficient resource allocation for a multicell OFDMA system is studied in [197, 198]. In [197], the authors devise a non-cooperative potential game [199] aimed at maximizing the users' energy efficiency, which proves to bring performance improvements in terms of goodput (error-free delivery) for each unit of energy. In [198], the same purpose is accomplished by a centralized subcarrier allocation procedure and a distributed non-cooperative power control game. The simulation results in a realistic multicell network scenario show that the proposed algorithm achieves an acceptable performance and computational complexity burden.

Non-cooperative game theory is also flexible enough to investigate resource allocation problems for contexts different from the data detection phase, popularly considered in the literature. For instance, in [200] Bacci *et al.* formulate a non-cooperative game to regulate the transmit powers in an OFDMA uplink during the initial, contention-based network association.

2.4.2 Cooperative solutions

Recently, several other methods which use various heuristics based on cooperative (coalitional) game theory [10, 22] have been proposed to address the problem of fair resource allocation for OFDMA systems, using either centralized or distributed algorithms. The Nash bargaining solution (NBS) [10] is the most refined technique applied to wireless resource allocation problems in an OFDMA network. The NBS proves the existence and uniqueness of an NE point of the following convex utility function:

$$\max_{\mathbf{p},\,\mathcal{N}} \prod_{k=1}^{K} (R_k - \underline{R}_k) \tag{2.27a}$$

$$\text{s.t.} \quad R_k = \sum_{n\in\mathcal{N}_k} R_{kn} \geq \underline{R}_k \qquad \forall\, k \in \mathcal{K} \tag{2.27b}$$

$$\text{and} \quad \sum_{n\in\mathcal{N}_k} p_{kn} \leq \overline{p}_k \qquad \forall\, k \in \mathcal{K} \tag{2.27c}$$

$$\text{and} \quad \mathcal{N}_k \cap \mathcal{N}_m = \emptyset \qquad \forall\, k,\, m \in \mathcal{K}, k \neq m, \tag{2.27d}$$

In other words, the goal is to maximize the product of the excesses of the transmitters' rates over their own minimum demands $\underline{R}_k$. The NBS guarantees each user to achieve its own demand, thus providing an individual rationality to the resource allocation. The important result of applying NBS is that the final rate allocation vector is Pareto optimal. Taking into consideration the strictly concave increasing property of the logarithm function, we can transform (2.27a) into:

$$\max_{\mathbf{p},\,\mathcal{N}} \sum_{k=1}^{K} \log_2 (R_k - \underline{R}_k) \tag{2.28}$$

Clearly, when $\underline{R}_k = 0$, the NBS fairness scheme reduces to the weighted proportional one, with $\varphi_k = 1$ [201].

Han *et al.* in [139] introduce a distributed algorithm for an OFDMA uplink based on the NBS and the Hungarian method [202] to maximize the overall system rate under individual power and rate constraints. The underlying idea is that once the minimum demands are provided for all users, the rest of the resources are allocated proportionally to different users according to their own conditions. The proposed algorithm shows a complexity $\mathcal{O}\left(K^2 N \log_2 N + K^4\right)$, without considering the (expensive) computational load to solve the (convex) equations of the NBS. In [203], Lee *et al.* solve two subproblems of exclusive subcarrier assignment and power control in an OFDMA network aiming at maximizing the NBS fairness. The simulation results show an overall end-to-end rate between the nodes comparable to that achieved in [139].

One main drawback of applying NBS in resource allocation problems is that this scheme guarantees minimum requirements of the users, but it does not impose any upper bound constraint. In fact, the achieved data rate may be much higher than the initial demands and this is unsatisfactory from the wireless network provider viewpoint. One of the most prominent alternatives to the NBS is the Raiffa-Kalai-Smorodinsky bargaining solution (RBS), defined by Raiffa [13] and characterized by Kalai and Smorodinsky [204]. The RBS requires that a user's payoff data rate should be proportional not only to its minimal rate, but also to its maximal one. Whereas the NBS takes into account the individual gains, RBS emphasizes the importance of one's gain and others' losses. For an OFDMA resource allocation problem, the RBS bargaining outcome is the solution to:

$$\max_{\mathbf{p},\,\mathcal{N}} \prod_{k=1}^{K} \left(R_k - \underline{R}_k + \frac{1}{K-1} \sum_{m\in\mathcal{K}, m\neq k} \left(\overline{R}_m - R_m\right) \right) \tag{2.29a}$$

$$\text{s.t.} \quad \underline{R}_k \leq R_k \leq \overline{R}_k \qquad \forall\, k \in \mathcal{K} \tag{2.29b}$$

$$\text{and} \quad \sum_{n\in\mathcal{N}_k} p_{kn} \leq \overline{p}_k \qquad \forall\, k \in \mathcal{K} \tag{2.29c}$$

$$\text{and} \quad \mathcal{N}_k \cap \mathcal{N}_m = \emptyset \qquad \forall\, k,\, m \in \mathcal{K}, k \neq m, \tag{2.29d}$$

wherein $\overline{R}_k$ denotes the upper bound of the transmission rate of the each user. When applying RBS, if the channel quality of a terminal improves, it will get a better capacity without any reduction to that of the other users (individual monotonicity). The existence and uniqueness of RBS can be shown, but a Pareto optimal NE point is not always attained for more than two players, as Roth stated in [205]. By using again the properties of the logarithm function, the utility maximization (2.29a) can be equivalently investigated using the following objective function:

$$\max_{\mathbf{p},\,\mathcal{N}} \sum_{k=1}^{K} \log_2 \left(\frac{R_k - \underline{R}_k}{\overline{R}_k - \underline{R}_k} \right) \tag{2.30}$$

Using this formulation, the RBS is a point at which each individual's gain is proportional to its maximum gain. When $\underline{R}_k = 0\ \forall\, k \in \mathcal{K}$ and $R_1 : \cdots : R_K = \overline{R}_1 : \cdots : \overline{R}_K$, the RBS achieves the same results of the max-min fairness criterion. In RBS formulation, the achieved data rate vector satisfies:

$$\frac{R_1 - \underline{R}_1}{\overline{R}_1 - \underline{R}_1} = \cdots = \frac{R_k - \underline{R}_k}{\overline{R}_k - \underline{R}_k} = \cdots \tag{2.31}$$

In [18], Chee *et al.* propose a centralized algorithm for the OFDMA downlink scenario based on RBS. The results show a good performance only when the gap between the maximum and the minimum rate is (very) large. Even though the subcarriers are assigned in an exclusive manner, the computational complexity of this algorithm is $\mathcal{O}\left(KN + K^2\right)$. Reference [206] investigates the problem of time-space resource allocation in a MIMO-OFDMA network in the downlink direction with aim at maximization data rate of each terminal, without specifying the complexity of the iterative algorithm to solve the NBS convex equation. In Chapter 3, we will attempt to improve the fairness of the solution and to reduce the complexity in the uplink direction of OFDMA-based networks, by deriving a coalition-based algorithm to provide each terminal with *exactly* the desired rate, so as to satisfy both wireless terminals and the network service provider.

Auction methods are another cooperative game scheme which has recently drawn attention in the resource allocation research literature. In [19], Noh proposes a distributed and iterative auction-based algorithm in the OFDMA uplink scenario with incomplete information. The time complexity of the algorithm is experimentally equal to $\mathcal{O}\left(KN \log_2 K\right)$. However, the simulation parameters are not realistic (three users and three subcarriers), and it is thus hard to estimate the computational complexity when using real-world network parameters. Alavi *et al.* in [207] propose an auction-based algorithm to achieve near a proportionally fairness data rate vector, although the computational complexity is not specified. Reference [208] propose a joint downlink/uplink subcarrier allocation (with fixed power) based on stable matching game to maximize data rate of each terminal in downlink and uplink directions, simultaneously.

2.5 A toy example with two terminals

In this section, we apply the different problem formulations introduced above to a simplified scenario, namely a network populated by just two terminals ($K = 2$). For the reader's convenience, we report the optimization formulas for this specific case:

$$\text{Max rate:} \quad \arg\max_{\mathcal{U}} \left(R_1 + R_2\right) \tag{2.32a}$$

$$\text{Max-min rate:} \quad \arg\max_{\mathcal{U}} \min_{k=1,2} R_k \tag{2.32b}$$

$$\text{Proportional fairness:} \quad \arg\max_{\mathcal{U}} \left(R_1 + R_2\right) \quad \text{s.t. } R_1 : R_2 = \varphi_1 : \varphi_2 \tag{2.32c}$$

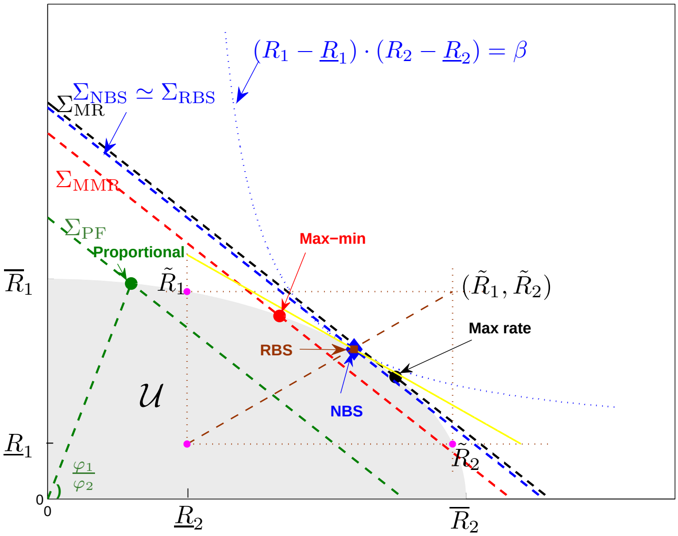


Fig. 2.1: *Two-user rate optimization.*

$$\text{NBS fairness:} \quad \arg\max_{\mathcal{U}} \, (R_1 - \underline{R}_1) \cdot (R_2 - \underline{R}_2) \tag{2.32d}$$

RBS fairness:

$$\arg\max_{\mathcal{U}} \left(R_1 - \underline{R}_1 + \left(\overline{R}_2 - R_2\right)\right) \cdot \left(R_2 - \underline{R}_2 + \left(\overline{R}_1 - R_1\right)\right) \tag{2.32e}$$

The (network) overall rate (the sum-rate) can be maximized using the max rate optimization formula (2.32a). However, although globally efficient, this solution is unfair: users with bad channel quality (e.g. cell-edge users) may be completely excluded from communication. On the contrary, max-min rate optimization protects terminals with a low data rate, and to this aim the lowest data rate is maximized according to (2.32b). In this case, a user with bad channel quality limits the system performance, and, if the channel quality of one user improves, all users will achieve a higher data rate. To maximize the sum-rate while balancing the ratios among the rates, we can use the formulation (2.32c).

To better visualize the problem, Fig. 2.1 reports the case in which user 2 experiences a better propagation channel than user 1. This is confirmed by the gray area $\mathcal{U}$

representing the feasible ranges for R_1 and R_2, that can be computed by assuming that both terminal receivers treat co-channel interference as noise [209]. As can be seen, $\mathcal{U}$ is a convex area, with $\overline{R}_2 > \overline{R}_1$, due to user 2's better channel conditions. By numerically solving (2.32a), we can find the max rate solution, represented by the black circle in Fig. 2.1. Geometrically, it can be obtained by identifying the point at which the Pareto boundary, given by the contour of $\mathcal{U}$, osculates a straight line with slope -1, depicted by the black dashed line labeled[2] with Σ_{MR}. Note that such line, given by all pairs (R_1, R_2) such that $R_1 + R_2 = \Sigma_{\mathrm{MR}}$, is the only constant-sum-rate line that is tangent to the Pareto boundary: all other lines such that $R_1 + R_2 = \xi < \Sigma_{\mathrm{MR}}$ intersect the Pareto boundary in two points, whereas all lines such that $R_1 + R_2 = \xi > \Sigma_{\mathrm{MR}}$ do not intersect it. Using numerical methods and setting $\varphi_1/\varphi_2 = 3.2$ in this example, we can also find the solutions to (2.32b) and (2.32c), represented by the red and the green circles, respectively. It is worth noting that the such points intersect the red and green dashed lines, corresponding to $R_1 + R_2 = \Sigma_{\mathrm{MMR}}$ and $R_1 + R_2 = \Sigma_{\mathrm{PF}}$, respectively, confirming that the sum rate achieved by such formulations is of course lower than that given by (2.32a), as $\Sigma_{\mathrm{MMR}} < \Sigma_{\mathrm{MR}}$ and $\Sigma_{\mathrm{PF}} < \Sigma_{\mathrm{MR}}$. This result is valid in general, and can be met with equality only under special settings of the network.

In addition to the proportional fair method, we can use two cooperative game-based solutions, namely NBS and RBS, to introduce fairness into our resource allocation problem. The NBS, formulated by the convex formula (2.32d), can be seen as a general case of the weighted proportional fairness, in which all users are guaranteed to receive some resources $\underline{R}_k$. The RBS solution, represented by (2.32e), is a generalization of the max-min solution, in which the achieved rates are bounded between a minimum, $\underline{R}_k$, and a maximum, $\overline{R}_k$, demanded rates. Similarly to the max-min allocation, if the channel quality of a user improves, in the RBS solution he/she will get a higher data rate wihtout any reduction for the other users' rates. In the cooperative-game formulations, we consider a minimum demanded $(\underline{R}_1, \underline{R}_2)$ and a maximum constraint $\left(\overline{R}_1, \overline{R}_2\right)$ as disagreement points. NBS and RBS solutions must then satisfy $\underline{R}_k \leq R_k$ and $\underline{R}_k \leq R_k \leq \overline{R}_k$, respectively. If the demanded rates, ignored in max rate, max-min rate, and proportional fairness solutions, are not met, in NBS and

[2]For the sake of graphical presentation, all line labels $\Sigma_{(\cdot)}$ correspond to the lines whose points are such that $R_1 + R_2 = \Sigma_{(\cdot)}$.

RBS solutions a user would leave the negotiation (hence the name "disagreement point"). As a consequence, the feasible regions for max rate, max-min rate, and proportional fairness solutions is $\mathcal{U}$, whereas it is $\mathcal{U} \bigcap \{[\underline{R}_1,\, \underline{R}_2] \leq \mathbf{R}\}$ for NBS and $\mathcal{U} \bigcap \left\{[\underline{R}_1,\, \underline{R}_2] \leq \mathbf{R} \leq \left[\overline{R}_1,\, \overline{R}_2\right]\right\}$ for RBS, respectively, with $\mathbf{R} = [R_1, R_2]$.

The NBS and RBS solutions can be found numerically in this example, and are depicted by the blue diamond and the brown square in Fig. 2.1, respectively. As can be seen, the difference between the NBS and RBS pairs is negligible. Although this does not always hold in general, here is due to having only $K = 2$ sources, and placing the same maximum achievable rate $\overline{R}$ for both solutions. Similarly to the max-rate case, we can obtain such points graphically [11, Ch. 35]. The NBS point can be identified as the point of tangency between the Pareto boundary of $\mathcal{U}$ and the hyperbola $(R_1 - \underline{R}_1) \cdot (R_2 - \underline{R}_2) = \beta$, where $\beta > 0$ is chosen properly to ensure only one intersection between the two curves. Note that, if we draw the tangent line to $\mathcal{U}$ at the NBS point, the length of the segment between the NBS solution and the vertical line drawn through $\underline{R}_2$ is equal to the length of the segment between the NBS solution and the horizontal line drawn through $\underline{R}_1$ (see Fig. 2.1, yellow segments).

To obtain the RBS point graphically, we need to identify the "utopian point" $(\tilde{R}_1, \tilde{R}_2)$, where $\tilde{R}_k$ is the maximum achievable rate by user k when the other user demands its minimum one $\underline{R}_m$ (see Fig. 2.1). This point is named utopian, as both terminals cannot achieve such rates simultaneously, as confirmed by the feasible region $\mathcal{U}$. The RBS solution is thus the intersection between the Pareto boundary of $\mathcal{U}$ and the segment connecting the utopian point $(\tilde{R}_1, \tilde{R}_2)$ and the disagreement point $(\underline{R}_1, \underline{R}_2)$. To measure the global efficiency of the cooperative solutions, we can draw the constant-sum-rate lines Σ_{NBS} and Σ_{RBS}, depicted by the blue and brown dashed lines, respectively. As expected, the sum-rate achieved by both solutions is nearly the same, and lower than that provided by the max-rate solution, although in this particular example the gap is significantly reduced with respect to the max-min and proportional-fair solutions. Note this is true in general, as NBS and RBS outperform max-min and proportional-fair solutions in terms of achieved sum-rate [11, Ch. 35].

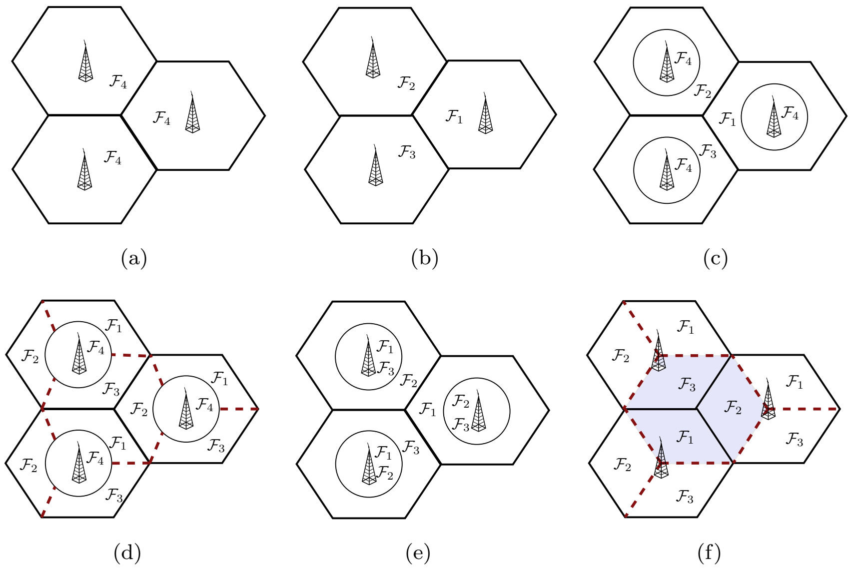


Fig. 2.2: *Different frequency reuse schemes.*

2.6 Toward 4th generation of wireless networks

2.6.1 Multicellular networks

Until now, we focused on a single-cell OFDMA network scenario wherein the BS and the mobile terminals use one set of frequency bands. In a multi-cell network scenario, the same frequency band can be reused by cells which are physically separated far enough to endure mutual interference. A BS that uses the same frequency band begets *co-channel interference*. The allocation of frequency bands to BSs is called *frequency reuse* which has a significant impact on system performance. Existing frequency reuse schemes can be divided into three classes: static frequency channel allocation, dynamic channel allocation, and combined channel allocation. In static frequency reuse, the partitions of frequency band do not adapt to traffic dynamics and interference conditions experienced by users. A subcarrier under a deep fade for one user at a given time may not be in a deep fade for other users. Therefore, every subcarrier may have a good channel response for some users in a multiuser environment. Unevenly loaded traffic results in unbalanced performance over the cells, which leads to degraded overall system performance. Instead of predicting and averaging, dynamic subcarrier allocation takes advantage of multiuser channel and traffic diversity to adjust the channel allocation over time. Although dynamic subcarrier requires higher computation complexity and signaling overhead during operation, its ability to utilize real-time system information leads to higher spectrum efficiency. Basically, dynamic radio resource allocation in a multi-cell network can be performed in centralized or distributed manner. In centralized schemes, the terminals and BSs are responsible for gathering traffic/channel information and feed back the information to the controller and then participate in protocol implementation. Combine channel allocation can be reputed as the combination of static and dynamic channel allocation, where some of the channels are fixed for each BS and others are dynamically assigned to cells.

Static frequency reuse approaches are based on fractional frequency reuse (FFR) where frequency bands are divided into a number of segments. Fractional frequency reuse is divided into two schemes: *strict fractional frequency reuse (FFR)*, and *soft frequency reuse (SFR)*. Typically, in an FFR approach, each segment is reserved for a certain reuse factor and is associated with a particular transmission power profile. When (strict) "frequency reuse 1" (called also universal frequency reuse) is supported, all BSs operate on the same frequency channel (Fig. 2.2(a)). In this case, to maximize frequency efficiency, decreasing the inter-cell interference is a major concern. Since a link will experience co-channel interference from signals from neighboring cells, the SINR for a link between the bth BS and the kth terminal via the nth subcarrier is defined as:

$$\gamma_{kn}^{b} = \frac{h_{kn}^{b} p_{kn}}{\sum_{b \neq c \in \mathcal{B}} h_{kn}^{c} p_{kn} + \sigma_w^2} \tag{2.33}$$

wherein $\mathcal{B}$ denotes the set of cells (BSs). In domain of frequency reuse 1, the result of contribution [210] written by Gjendemsjø *et al.* show that in a two-cells setup scenario, a binary power allocation policy which assigns either full power to both cells or shuts down one cell significantly increases the whole data rate in downlink direction under power constraints per-BS. Venturino *et al.* in [211] use a distributed approach based on optimization problem with aim at maximization downlink rate region subject to power constraints per BS. Yu *et al.* in [212] applied the idea of "interference pricing" which measures the impact of each terminal's interference on its neighbors cells and then BSs dynamically allocate radio resources based on the exchange of these measures.

The scheme called (strict) "frequency reuse 3" divides the frequency band into 3 sub-band and allocates one sub-band to a given cell, so that adjacent cells use different frequency band as illustrated in Fig. 2.2(b). A "hybrid frequency allocation" is proposed which is, in general, a mix of reuse 1 and 3 approaches. A hybrid approach can be applied to avoid interference at cell edges. For example, suppose we have three cells covering a certain area, and there are four frequency segments. Then, frequency segment $\mathcal{F}_4$ can be reserved for cell-interior users (with less interference from other sectors), and frequency segments $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3$, for cell-edge users (more interference from other sectors) in cells 1, 2, 3, respectively (Fig. 2.2(c)). As a result, we have 1/3 reuse for far users and 1/1 reuse for close users. Reference [213] proposes an algorithm to apply hybrid frequency allocation in an OFDMA-based network that maximizes the total cell throughput. Reference [214] introduces an FFR optimization technique wherein the edge-region of each is divided into three sectors as illustrated in Fig. 2.2(d). The optimal configuration of the proposed algorithm is based on maximizing the average sector data rate subject to a minimum cell-edge data rate. In a SFR deployment, the cell-interior users are allowed to share frequency bands with edge-users in other cells (Fig. 2.2(e)). Typically, cell-interior users in SFR transmit at lower power than cell-edge users [215]. While SFR outperforms FFR in terms of spectral efficiency, it results in more interference to users [216]. Some studies in the literature (e.g., [217, 218]) suggest that power control does not always yield significant performance gain in OFDM systems compared to the complexity it adds to the operations of the system. Accordingly, these studies adopt a simple binary power control model in which either a subcarrier can be assigned to a terminal with maximum power or not. In the scheme proposed by Kwon *et al.* in [217], resource allocation is managed independently at each "pseudo-cell" composed of the major-interfering sectors belonging to the neighboring cells (Fig. 2.2(f)) in order to reduce the signaling and computation overhead. For each subcarrier, the transmission power is fixed, while varying the transmission rate by using adaptive modulation. The subcarriers are dynamically divided into different groups according to their load condition and for each group it is applied either frequency reuse 1 or frequency reuse N_s, where N_s denotes the number of sectors within one pseudo-cell.

For OFDMA networks, centralized FFR approaches available in references [219,220] show results with equal/unequal power levels and adaptive power levels as well. These works consider fixed partitioning of radio resources for cell center and edge users.

The parameters such as distance and SINR to partition cell center and edge regions are used as fixed thresholds. References [221–226] investigate the problem of game-theoretic radio resource allocation in OFDMA-based multi-cell networks. Reference [131] surveys different techniques for ICI mitigation in OFDMA-based multicellular networks.

There are few open issues in applying existing channel assignment approaches to OFDMA-based multi-cell networks. First, traditional frequency assignment assumes a predefined SINR threshold, which is rather suitable for homogenous services, but cannot be adopted for multimedia services. Secondly, traditional frequency assignment deals with flat fading channels and consequently overhead and computational complexity due to the measurement and signaling is associated to one frequency band. However, an OFDMA network needs to exchange information on all of the subcarriers which dramatically increases the complexity of measurements. As a result, fully centralized schemes are often too heavy for implementation as all the interference information on all channels has to be gathered and calculated at a central controller. On the other hand, fully distributed schemes have difficulties dealing with uneven and instantaneous loaded traffic.

2.6.2 Pico- and Femto-cell networks

One of the key expectations for the future wireless system is to provide ubiquitous high data rate coverage. But with the traditional cellular architecture, increasing the capacity together with the coverage requires the deployment of a large number of BSs, which is very costly. Multi-tier networks consist of a conventional cellular network overlaid with low-power and small range micro- and pico-BS (e.g. femto-cells, distributed antennas, or wired/wireless relays) which offer a cost effective way to enhance cellular system performance. Small BSs act as access points which extend network coverage and enhance capacity without incurring in the cost of backhaul connections. Femto-cells can be installed by end-users to improve the indoor coverage and capacity in residential and office environments. On the other hand, a pico-cell can be installed by a wireless service provider in public spaces or large buildings. In multicellular networks, macro-, micro-, and pico-cells suffer from a problem of co-channel interference between them in the neighboring cells. In radio resource allocation the problem of co-channel interference must be taken into consideration to evaluate the improvement of system throughput and economical feasibility by employing the different types of BSs. The coverage of a pico-/femto-cell becomes smaller when it is closer to a high power macro BS [227]. Under the pre-allocated frequency band within a macro-cell through the FFR optimally, a radio resource management allocates sub-bands the small BSs efficiently to consider macro-cell having a priority over micro-/femto-cells and total/edge throughputs.

References [228–230] propose spatial reuse schemes to mitigate the co-channel interference for OFDMA femto-cell networks and increasing femto-cells throughput as well. Ko *et al.* in [231] present a self-organizing femto-cell networks where users optimize their performance in a distributed manner. References [232, 233] investigate spectrum allocation techniques for femto-cells, based on Markov modeling and Q-learning [79] , respectively. Contributions [234–236] propose game theoretic algorithms to adjust the transmit powers of femto-BSs to mitigate interference, improve total capacity, and approach fairness among users. An efficient power allocation to femto-cells to cover specific terminals is presented by [237]. References [227, 238] propose algorithms to adjust pico BSs power as a tradeoff between cell coverage and cell throughput. Reference [239] devises a cross layer design for joint resource allocation and admission control in a two-tier OFDMA based network. The admission control is optimally designed based on Markov decision and the power of femto BSs is efficiently allocated using non-cooperative game theory.

2.6.3 Relay assisted networks

Cooperative communication with intermediate relay stations is an emerging technology to improve the performance of a wireless communication system. A relay is used to improve the transmission quality between a source node and a destination, and it can operate in either the amplify-and-forward (AF), the decode-and-forward (DF), or compress-and-forward (CF) mode. In the AF mode, the relay simply retransmits its received signal, including the interference and its local additive noise. A DF relay decodes its received signal before retransmission. Since the interference generated in the source-to-relay transmission is eliminated, the DF mode improves the effective SINR considerably, but at the cost of an increased hardware complexity. When the relay is not able to perfectly decode the received signal, the CF strategy is used to estimate the transmitted signal by the source node.

The most important factor which impacts on performance of relay-aided communications is radio resource management, i.e., how different subcarriers of OFDMA sources/relays should take part into a relayed transmission. References [240, 241] study the capacity of relaying in OFDMA applying AF and DF strategies. They also propose algorithm to subcarrier assignment with fixed-power. The algorithm proposed by Hammerström *et al.* in [242] provides a power allocation on the different subcarriers at the AF relay and the transmitter node, in a dual-hop OFDM relay communication scenario. The goal of the proposed power allocation is the maximization of the channel capacity with respect to the separate constraints on the transmitted power on the relay and source. In [243] and [244] L. Vandendorpe *et al.* propose some power allocation techniques in an OFDMA dual-hop relay communication applying the DF strategy. In particular, [243] places a constraint on the sum of the power consumption by the relay and sender, whereas [244] uses two individual constraints. References [245–248] investigate the resource allocation problem in an OFDMA-based point-to-point communication assisted by multiple relays to maximize the data rate under a power constraint.

Relaying is one of the enabling techniques for the next generation wireless networks. The first commercial relay-assisted OFDMA network has been standardized by IEEE 802.16j [249]. In single-cell scenarios, references [250, 251] evaluate the performance of a relay-assisted OFDMA network for a specific setup with three relay stations in a cell with and without intracell frequency reuse. The numerical results show that the relay-assisted system significantly outperforms the conventional cellular system with respect to system capacity and coverage. References [252, 253] present radio resource approaches in the downlink and uplink direction, respectively, assisted by multiple relays. The optimization is formulated as a spectrally efficient maximization, and the results show good performance in terms of spectral efficiency and fairness. Game theoretic frameworks for the best relay selection policy and efficient spectral usage in relayed OFDMA networks are presented in [254–257]. These schemes significantly increase the system performance and achieve a fair achieved data rate vector.

In the multicellular environment, the deployment of relay stations in the co-channel cells can be jointly optimized to maximize the overall spectral efficiency. Joung *et al.* in [258] introduce a power efficient radio resource allocation algorithm in relay multicellular networks in the downlink direction. Reference [259] present a dynamic FFR, like Fig. 2.2(e), in a multicellular relay-assisted network in the downlink direction to mitigate ICI. For the maximization system data rate in multicellular relay assisted OFDMA-based networks, different resource management approaches are presented by [260–266] under the constraint on the total power consumption. For further details, in the context of relay assisted OFDMA networks, the reader is referred to the survey provided by Salem *et al.* [132].

2.6.4 Cognitive radio

IEEE 802.22 [267], a continuously developing standard, employs OFDMA for the physical layer with cognitive radio technology in the MAC layer, thus opening new topics of radio resource management for OFDMA-based cognitive radio. In a cognitive radio system, a secondary user (SU) identifies available or unused licensed parts of the spectrum and exploit them with the goal of maximizing the throughput while minimizing the interference to primary users (PUs). The inherent FFT operation and the capability of assigning disjoint subsets of subcarriers to different secondary users make OFDMA adaptive for spectrum sensing in frequency domain and spectrum shaping, respectively [268]. The SUs can change the assigned subcarriers, the transmit powers, and the modulation and coding over each subcarrier according their channel gains. This flexibility helps OFDMA terminals to adapt to the environment with goal of minimizing BER and maximizing throughput. The limiting factor of the system performance is ICI between PUs and SUs. The interference can be limited not only by a proper radio resource allocation, but also usingthe channel state information. In [269], Weiss *et al.* provide a quantitative evaluation of the mutual interference between SUs and PUs that is caused by non-orthogonality of their respective transmitted signals. The "adjacent channel interference" can be mitigated by providing a flexible guard bands between PUs and SUs. They show that, although cyclic prefix and postfix help to reduce interference, they result in a reduced system throughput. To this aim, the authors propose dynamic deactivation of subcarriers lying adjacently to PU allocated sub-bands.

Generally, the resource allocation problem in OFDMA-based cognitive radio systems is formulated as the maximization of the total transmission rates of SUs by adjusting the power of selected subcarrier while the interference introduced to the PU must be kept below a tolerable threshold, and the total power of subcarrier does not exceed the total power constraint. An SU may not able to detect existence a PU when there is a large distance between them, but they can interfere when the SU is transmitting. In order to avoid unacceptable interference to the PUs that may not be detected by a SU, the SU should limit its transmit power even when no PU is detected [270]. As a result, the traditional WF approach is not suitable to maximize the capacity of an SU in an OFDMA-based cognitive radio system. For this purpose, [271–273] reformulate the WF problem and propose iterative algorithms based on convexity programming. Mao *et al.* in [274] introduce an iterative energy efficient WF-based power allocation which converges after few time steps. To maximize the sum rate in OFDMA-based cognitive radio multicellular networks, [275–278] propose algorithms based on convex geometric programming to optimize allocation of radio resources. Ma *et al.* in [276] propose a (reconfigured) WF solution to achieve weighted sum rate of SUs over multiple cells. The centralized and iterative algorithm proposed by [277] outperforms the one introduced in [275] in terms of computational complexity and spectral efficiency. The computational complexity of algorithm MLWF proposed by [278] is lower than that of ELCI proposed by [277]. Wang *et al.* in [279] propose an algorithm to achieve a proportional fairness data rate vector with a good sum data rate.

Applying game theory in the context of cognitive radio systems where OFDMA terminals can be useful to sense the environment and adaptively adjust their transmission power over the best selected subcarriers. To this aim, both non-cooperative (e.g., in [280, 281]) and cooperative (e.g., [282]) approaches are used to address this problem.

2.7 Summary

In this chapter, we have presented a survey on the state-of-the-art techniques to apportion the resources in both single-user systems, based on the OFDM modulation, and multiuser networks, based on the OFDMA channel access scheme. While resource allocation in OFDM systems is a mature field of investigation, with current trends of research attempting to further increase the efficiency towards near-Shannon performance, similar techniques for OFDMA still represent a hot topic of research. Many optimization have been proposed in the literature, focusing on both energy- and spectral-efficient approaches, using the margin- and the rate-adaptive formulations, respectively. The latter optimization problem has received most attention, appearing to be the most appealing one from both the centralized and the distributed point of view, and many practical solutions have been proposed in the last two decades. Important issues such as fairness and algorithmic complexity have also been included into the loop. To improve the performance of the proposed schemes, cross-layer formulations and several optimization tools, including game theory, have been recently adopted. Nevertheless, many drawbacks that limit the application of state-of-the-art algorithms to practical contexts still hold, mainly due to the high computational complexity and the weak scalability of the proposed techniques, that often make real-time solutions intractable problems, as they also require a considerable amount of feedback information across the network nodes. In this respect, we have tried to summarize the most relevant state-of-the-art techniques, also outlining current open problems in this active area of research, and including an overview of current challenges for next-generation networks, such as macro/micro multicellular planning, relaying and cognitive communications.

2.8 Discussion

It is a matter of controversy whether the OFDMA resource allocation techniques in the literature are actually usable in the practice. All the mentioned schemes, which represent, to the author's knowledge, the most relevant algorithms for OFDMA resource allocation with cooperative game theory, exhibit a good trade-off between overall system rate and fairness. The fairness schemes in the solutions based on cooperative game theory are extended approaches of that in classic solutions of: max-min fairness, and weighted proportional fairness schemes. Unfortunately, they also present a number of common problems:

1) In almost all algorithms the utility function is restricted to either be convex or strictly concave;

2) Most algorithms are based on non-linear programming, which is computationally intensive and hardly scalable when considering thousands of subcarriers and tens of users. Thus, they are not suitable for a cost-effective real-time implementation by network designers;

3) Although the resource apportionment turns out to be fair from the users point of view, the achieved QoS may be much larger than demanded. This implies a waste

of network resources from a network service provider perspective, which is often overlooked by previous works;

4) To reduce the computational complexity, each subcarrier is allocated to mobile terminals in an exclusive manner, although this may limit the number of concurrent connections in the uplink channel;

5) To reduce the computational complexity, the power constraint is usually defined as the overall energy consumption of each user over all subcarriers rather than individual limitation on each subcarrier, and this may result in impractical spectral power distribution.

We reviewed the concepts of coalitional game theory in Chapter 1. In Chapter 3 we will introduce an algorithm based on cooperative games to overcome most of the above mentioned disadvantages of the existing schemes. We aim at designing a low-complexity algorithm that achieves each users QoS requirement in terms of target transmit rates, with the best utilization of the network resources, so as to satisfy both the users and the network service provider.

Chapter 3

A resource allocation cooperative game in OFDMA

Following what discussed in Chapter 2, various attempts have been made to standardize a certain protocol for resource allocation in OFDMA, but all have fallen into disuse largely because of their over-complexity, and unfairness from the network service provider point of view. We also showed that cooperative game theory is a suitable tool to face resource allocation problems, especially when altruism and fairness play crucial roles.

The focus of the first part of this thesis is to introduce a scheme for resource allocation in OFDMA based on cooperative games, wherein applicability and fairness are target criterions. This chapter investigates a fair adaptive resource management criterion for the uplink of an OFDMA network populated by mobile users with constraints in terms of target data rates. We aim at fulfilling each users QoS requirement in terms of target transmit rates *exactly* with the best utilization of the network resources, so as to satisfy both the users and the wireless service provider. We also aim at designing a low-complexity algorithm that allows a centralized solution for the joint power and bandwidth allocation for OFDMA uplink channels to be achieved in a few steps using typical network parameters. In our approach, we allow every subcarrier to be possibly *shared* among more than one user, and we add a constraint on the maximum number of used subcarriers per terminal. This is achieved by dividing the available bandwidth into a number of disjoint blocks of consecutive subcarriers, and forcing each terminal to use at most one subcarrier per block. The motivation of this is twofold: we wish to *i*) increase the signal-to-interference-plus-noise ratio (SINR) on the used subcarriers, which also simplifies channel estimation; and *ii*) exploit frequency diversity across carriers used by one user to increase the performance of forward error correction (FEC) techniques.

In Sect. 3.1 we propose two methods to allot subcarriers to mobile terminals, in a possibly shared assignment. Next, the inherent optimization problem is tackled with the analytical tools of cooperative game theory aiming at accomplishment of data rate demanded exactly. The definition of the players, coalitions and utility function are formally discussed, and we prove the existence of the core set solution by means of the analytical tools of cooperative game theory. To accomplish data rate demanded we propose a utility function which is neither convex nor concave. Sect. 3.2 proposes a dynamic learning algorithm based on Markov modeling to achieve optimum transmit power over each subcarrier at each individual wireless terminal. Simulation results in Sect. 3.3 show that the average number of operations of the proposed algorithm is much lower than $K \cdot N$, where N and K are the number of subcarriers and users. We also show that the transmit power is comparable to the remarkable existing power effective literature results. Low-complexity, efficient use of available spectrum, and low power consumption bring promise to usability of the proposed scheme in each time slot at physical layer in the 4G of cellular networks.

3.1 Problem formulation

Let us consider the uplink of a single-cell infrastructure OFDMA system with total bandwidth W, subdivided in N subcarriers with frequency spacing $\Delta f = W/N$. The cell is populated by K mobile terminals, each terminal $k \in \mathcal{K} = [1, \ldots, K]$ experiencing a complex-valued channel gain H_{kn} on the nth subcarrier to the base station and having a data rate requirement $R_k^\star$ (in bit/s). We assume that fulfilling such constraints simultaneously by all terminals is feasible. To exploit frequency diversity, the subcarriers set $\mathcal{N} = [1, \ldots, N]$ is grouped in B blocks of N/B contiguous subcarriers $\mathcal{N}^{(b)} = \left[\frac{N}{B}(b-1)+1, \ldots, \frac{N}{B}b\right] \subset \mathcal{N}$, with $1 \leq b \leq B$, as shown in Fig. 3.1. Each terminal is allowed to take at most one subcarrier per each subblock.

Our resource allocation strategy consists in finding a vector of transmit powers $\mathbf{p}_k$, where $\mathbf{p}_k = \left[p_{k1}, \ldots, p_{kN}\right]$, with p_{kn} representing the power allocated by terminal k over its nth subcarrier, that allows the QoS constraint $R_k^\star$ to be satisfied. We decouple the problem into the subsequential resolution of subchannel assignment and (subsequent) power allocation.

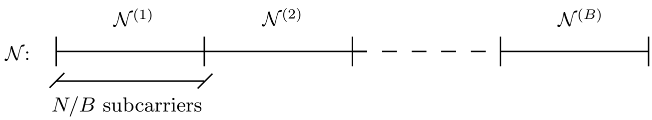


Fig. 3.1: *Block partitioning of the available bandwidth.*

3.1.1 Subchannel assignment

We describe here two different options to perform this function:

Best-carrier assignment

For every subblock $\mathcal{N}^{(b)}$, every terminal $k \in \mathcal{K}$ is assigned its best subcarrier $n_k^{(b)} = \arg\max_{n \in \mathcal{N}^{(b)}} |H_{kn}|^2$. The probability of assigning the same subcarrier to multiple mobile terminals is non-null.

Vacant-carrier assignment

In a sequential manner, for every subblock $\mathcal{N}^{(b)}$, every terminal $k \in \mathcal{K}$ is assigned its best subcarrier $n_k^{(b)} = \arg\max_{n \in \mathcal{N}^{(b)}} |H_{kn}|^2$. But, if $k \leq N/B$, we would like to ensure exclusive use of each subcarrier $n \in \mathcal{N}^{(b)}$ to better exploit the available bandwidth W (i.e., to reduce the multiple access interference). So, if $n_k^{(b)}$ has been already assigned to some other terminal $\ell < k$, then terminal k is assigned the nearest vacant (unassigned) subcarrier to $n_k^{(b)}$ within the channel coherence bandwidth. Clearly, this is not considered if $k > N/B$, so that terminal k is assigned its best subcarrier in the subblock anyway. Note that the ordering of $\mathcal{K}$ has a negligible impact on system performance when N is sufficiently high and, as usual, $N \gg K$ (e.g., 2048 subcarriers in LTE).

Both assignment strategies can be easily extended to the case in which each terminal is allowed to have a different number of assigned subcarriers (different B for each mobile terminal), based on its own data rate requirement $R_k^\star$, without any change in the strategy that we describe below. For the sake of simplicity, we consider the same B for all terminals.

3.1.2 Power allocation

To derive a stable solution to the power allocation subproblem, we consider it as a coalitional game, in which each subchannel $n_k^{(b)} \in \mathcal{N}$ is identified as a player in the game. To model the coalitional game, we build K coalitions $\psi = [\mathcal{A}_1, \ldots, \mathcal{A}_K]$, to be assigned to the K terminals. Each coalition $\mathcal{A}_k$, $k \in \mathcal{K}$, contains the B players $n_k^{(b)}$: $\mathcal{A}_k = \left[n_k^{(1)}, \ldots, n_k^{(B)}\right]$. Note that i) the members of each coalition are fixed, since one player cannot move from one coalition to another; and ii) since a subcarrier $n \in \mathcal{N}$ can be shared among multiple users, there exist virtual copies of it belonging to different coalitions. For the sake of notation, we will identify with a generic $n \in \mathcal{A}_k$ any of the subcarriers assigned to terminal k. The strategy of each player $n \in \mathcal{A}_k$ is represented by the optimal power expenditure $p_{kn} \leq \overline{p}_{kn}$. Note that i) if $n \notin \mathcal{A}_k$, $p_{kn} = 0$; and ii) if $n \in \mathcal{A}_k$, we can also have $p_{kn} = 0$, which means that the kth terminal does not transmit on the nth subcarrier, and it thus bears an actual number of active subcarriers $B'_k < B$.

The system under investigation aims at fulfilling the QoS requirement of every terminal k in terms of target rate $R_k^\star$. For simplicity, we estimate the achieved data rate as the Shannon capacity R_k of terminal k, that can be approached by using suitable channel coding techniques [283]:

$$R_k = \sum_{n \in \mathcal{N}} R_{kn} \tag{3.1}$$

where R_{kn} is the Shannon capacity achieved by terminal k on subcarrier n:

$$R_{kn} = \Delta f \cdot \log_2 \left(1 + \frac{|H_{kn}|^2 \, p_{kn}}{\sum_{m \neq k} |H_{mn}|^2 \, p_{mn} + \sigma_w^2} \right). \tag{3.2}$$

Clearly, $R_{kn} = 0$ if $n \notin \mathcal{A}_k$, since $p_{kn} = 0$. If $n \in \mathcal{A}_k$, R_{kn} depends on the received SINR at the base station on subcarrier n, which is a function of the strategy (i.e., the transmit power) chosen by player n (i.e., one of the B subcarriers assigned to the kth terminal), of the transmit power of other terminals on the same subcarrier (if $n \notin \mathcal{A}_m$, $p_{mn} = 0$), of the corresponding channel gains, and of the power of the additive white Gaussian noise (AWGN) σ_w^2. Note that, in an OFDMA system, there is no interference between adjacent subcarriers. Hence, R_{kn} considers only intra-subcarrier noise, that occurs when the same subcarrier is shared by more terminals. Each player $n \in \mathcal{A}_k$ causes interference only to its virtual copies, i.e. to the players of other coalitions such that $n_m^{(b')} = n \in \mathcal{A}_m$, with $m \neq k$ and for any b', $1 \leq b' \leq B$.

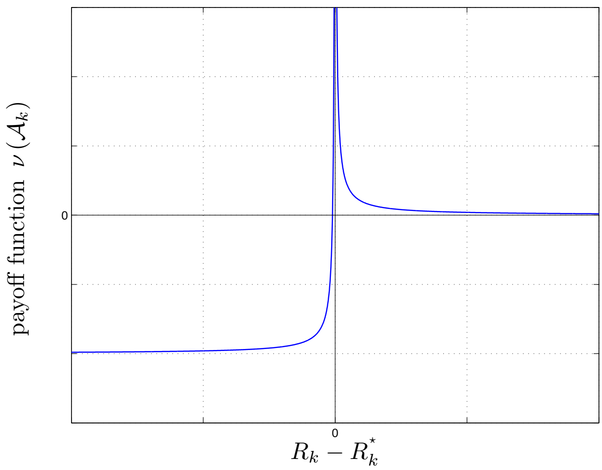


Fig. 3.2: *Shape of the utility as a function of the Shannon capacity.*

The network service provider are satisfied at most when each mobile terminal k achieves its own data rate requirement *exactly*: $R_k = R_k^\star$. In view of this goal, we can force all players in each coalition $\mathcal{A}_k$ to select their strategies (i.e., the power allocation for terminal k over the available bandwidth W) so as to maximize a utility function for the kth coalition $\mathcal{A}_k$, defined as:

$$\nu(\mathcal{A}_k) = \frac{1}{|R_k/R_k^\star - 1|} - \beta \cdot \mathrm{u}\left(1 - R_k/R_k^\star\right) \tag{3.3}$$

where $\mathrm{u}(\cdot)$ is the unit step function, with $\mathrm{u}(y) = 1$ if $y \geq 0$ and $\mathrm{u}(y) = 0$ otherwise (see Fig. 3.2).

If $R_k = R_k^\star$, $\mathcal{A}_k$ earns the highest possible payoff $\nu(\mathcal{A}_k) = +\infty$. If $R_k > R_k^\star$, $\mathcal{A}_k$ gets a positive payoff, whereas it obtains a negative payoff if $R_k < R_k^\star$. The factor β is a positive constant (much) greater than zero that ensures $\nu(\mathcal{A}_k)$ to be negative when $R_k < R_k^\star$. This is expedient to let the players distinguish a capacity R_k that is lower/upper than $R_k^\star$ only by knowing their own coalition's payoff. Note that, in practice, $+\infty$ can be represented by the largest countable number available (e.g., $2^{64} - 1$) in a given computational platform.

The payoff of each coalition is a real number and, in our formulation, the most important parameter is the gain of each coalition, whereas the outcome of each player does not matter at all. Therefore, this game is a transferable utility (TU) one [10,22].

The specific shape of our utility function (3.3) is actually immaterial, and was chosen to ensure fast convergence of the iterative algorithm that will be introduced later on. We could have considered any utility function which increases as its argument moves from $\pm\infty$ to 0, just to make sure that, for any $R_k \neq R_k^{\star}$, each coalition has an incentive to move towards $R_k = R_k^{\star}$.

To provide further insight into the problem, we investigate now some properties of the proposed game $\mathcal{G}$. As a first step, we note that the players in $\mathcal{G} = (\mathcal{K} = \bigcup_{k\in\mathcal{K}} \mathcal{A}_k, \nu)$ with the utility function (3.3), do *not* tend to form the grand coalition. This is because every player $n \in \mathcal{A}_k$ can not leave its coalition $\mathcal{A}_k$: the members of each coalition are fixed and do not change during the game. This may appear inappropriate to the notion of a coalitional game. However, our assumption is fairly common in economic problems like the study of a bargaining game between two corporations when each corporation has its own business branches [284]. In this case the members (branches) of each coalition (corporation) are fixed.

A relevant result for our game is the following:

Theorem 3 *The core of the game $\mathcal{G} = (\mathcal{K} = \bigcup_{k\in\mathcal{K}} \mathcal{A}_k, \nu)$ with utility function (3.3) is not empty.*

Proof The number of coalitions and the number of players in each coalition are both fixed. Since each player belongs just to one coalition, the unique balanced collection of weights $(\mu_{\mathcal{A}})_{\mathcal{A}\in\psi}$ is $\mu_{\mathcal{A}} = 1 \ \ \forall\, \mathcal{A} \in \psi$. To conclude the proof, we must verify that $\sum_{\mathcal{A}\in\psi} \nu(\mathcal{A}) \leq \max_{\psi\in\boldsymbol{\Psi}} \sum_{\mathcal{A}\in\psi} \nu(\mathcal{A})$. Since the target rates of all terminals are assumed to be feasible, then every coalition expects R_k to approach $R_k^{\star}$. Therefore, every coalition is allowed to earn the highest possible payoff. ■

In the following section, we will show how the fundamental properties of our game lead to a practical allocation algorithm.

3.2 The best-response algorithm

We are interested in answering questions like: *How do the players set their proper transmit powers?* Dynamic learning models provide a framework for analyzing the way the players may set their proper strategies. A player adopts a certain power amount if and only if this matches its coalition's interests, and this goal can be achieved through a best-response iterative algorithm [285] based on Markov modeling [286]. Each player takes its own decisions individually, myopically, and concurrently with the others, so as to lead its own coalition's payoff toward $+\infty$ ($R_k = R_k^\star$). At each (discrete) time step of the algorithm, the (autonomous) players simultaneously adjust their transmit powers based on a model to increase the payoff of their own coalitions. Although this leads to interference when virtual copies of the same subcarriers simultaneously change their powers, we show that this dynamic myopic procedure guarantees the maximum payoff to each coalition.

The process starts up at time step $t = 0$ with an arbitrary assignment of the transmit powers $p_{kn}^{t=0}$ to all $K \cdot B$ players in the game (that are grouped in K coalitions with players $n \in \mathcal{A}_k$ with $n = n_k^{(b)}$, $1 \leq b \leq B$). At the generic time step t, our system is in the state $\omega^t = (\psi^t, \boldsymbol{\nu}^t)$, where ψ^t is the set $[\mathcal{A}_1^t, \ldots, \mathcal{A}_K^t]$, and $\boldsymbol{\nu}^t = [\nu(\mathcal{A}_1^t), \ldots, \nu(\mathcal{A}_K^t)] \in \mathbb{R}^K$ contains the payoffs of the coalitions in ψ^t. The evolution of the Markov chain is then dictated by the strategy of the game. The strategy of each player $n \in \mathcal{A}_k$ is to find the best power amount p_{kn}^t that leads to an increase in the payoff $\nu(\mathcal{A}_k^t)$ of its own coalition $\mathcal{A}_k$. In practice, player $n \in \mathcal{A}_k$ decides whether to change its power allocation, making its coalition better off, or to keep transmitting at the same power level (e.g., when its coalition's payoff is infinite). The following pseudocode shows how each player $n \in \mathcal{A}_k$ takes its decision at time step t:

```
if ν(A_k^t) = +∞, then p_kn^{t+1} = p_kn^t, exit;
else //setting correct power range
   if ν(A_k^t) ≤ 0, then p̃_kn = p_kn^t, p̃_kn^max = p̄_kn;
                    else p̃_kn = 0,    p̃_kn^max = p_kn^t;
repeat
   p̂_kn = p̃_kn; //saving tentative power
   compute  ν(Ã_k); //tentative payoff
   Δp̃_kn = unif[0, Δp̄_kn]; //random power step
   p̃_kn = p̃_kn + Δp̃_kn; //tentative power
 until (ν(Ã_k) > ν(A_k^t)) or (p̃_kn > p̃_kn^max)
 if (ν(Ã_k) > ν(A_k^t)), then p_kn^{t+1} = p̂_kn; //accept
                          else p_kn^{t+1} = p_kn^t; //discard
```

In this algorithm, $\nu(\tilde{\mathcal{A}}_k)$ is the "trial" value of the current payoff of the coalition when the tentative power $\tilde{p}_{kn}$ is adopted: it is computed with $p_{mn} = p_{mn}^t$ for all $n \in \mathcal{N}$ and for any $m \neq k$, and $p_{kn} = \tilde{p}_{kn}$. At each step of the update process, the power step $\Delta\tilde{p}_{kn}$ is the particular outcome (value) of a random variable uniformly distributed between 0 and $\overline{\Delta p}_{kn}$, with $\overline{\Delta p}_{kn} \ll \overline{p}_{kn}$. As better detailed in Sect. 3.3, optimal values for $\overline{\Delta p}_{kn}$ can be found in order to minimize the algorithm computational load, based on experimental results. If $\nu\left(\mathcal{A}_k^t\right) \leq 0$, then $R_k < R_k^\star$, and the best strategy for player $n \in \mathcal{A}_k$ is to increase its current transmit power so as to increase its coalition's payoff. As a result of the random power stepping, the tentative power is a random number in the interval $[p_{kn}^t, \overline{p}_{kn}]$. Player $n \in \mathcal{A}_k$ accepts this value if and only if the coalition payoff $\nu\left(\mathcal{A}_k^t\right)$ increases, otherwise it ends up transmitting at its previous value. If $0 < \nu\left(\mathcal{A}_k^t\right) < \infty$, player $n \in \mathcal{A}_k$'s best strategy is on the contrary to decrease p_{kn}^t, and thus the tentative (random) transmit power belongs to the interval $[0, p_{kn}^t]$. At the end of each time step t, the base station computes the payoff $\nu\left(\mathcal{A}_k\right), \forall k \in \mathcal{K}$ with updated power amounts. As shown in the pseudocode, a uniformly distributed random power stepping is adopted to increase the probability of picking the best adjustment value, and thus both to reduce the convergence time of the algorithm and to possibly minimize the overall power consumption. As is apparent, the convergence speed of the algorithms depends not only on the parameters of the network, but also on the choice of the maximum update step $\overline{\Delta p}_{kn}$.

As already stated, two copies $n \in \mathcal{A}_k$ and $n \in \mathcal{A}_m$ (the virtual copies of the same subcarrier n) may happen to wish to adjust their transmit powers in a conflicting (and thus incompatible) way. If we assume that each player just follows the decision rules listed in the pseudocode above, then the probability of conflicting decisions will be high. To reduce the occurrence of this event, we modify our algorithm by requesting each player *not* to update its transmit power at every step of the game with a probability $\lambda \in [0, 1]$. At each time step t, every player $n \in \mathcal{A}_k$ selects a random number ξ_{kn}^t uniformly distributed in $[0, 1]$. If $\xi_{kn}^t > \lambda$, then the player applies the algorithm and (possibly) update p_{kn}^{t+1}, otherwise $p_{kn}^{t+1} = p_{kn}^t$ (i.e., during time step t, it skips the update process, and the value of p_{kn}^t is kept). If λ is close to 1, then the probability of conflicting decisions tends to 0, but the algorithm will have a large convergence time, since the probability of updates is low. In addition to the conflicts described above, another potentially disruptive condition may arise between different subcarriers belonging to the same coalition: if both (myopic) players simultaneously increase their powers $p_{kn}^t > 0$ and $p_{kn'}^t > 0$, it may occur that $R_k > R_k^\star$. To optimize the update mechanism and to cope with both negative kinds of events, we could consider a variable and adaptive threshold λ_{kn}^t for each virtual copy of the same subcarrier (each player). However, to reduce the complexity of the algorithm, we assume $\lambda_{kn}^t = \lambda > 0$ for all the players (i.e., virtual copies of the subcarriers). As better detailed in Sect. 3.3, the optimal value of λ must be selected as a suited trade-off. Note that the value of λ is common knowledge among the players at every step of the algorithm. Nevertheless, interference between concurrent, conflicting decisions may prevent the coalitions from achieving the expected payoff. If all coalitions earn less than the previous time step, all players assign the previous power amount for the next time step. There may exist network configurations in which the iterative algorithm is not guaranteed to converge. To account for these situations, we place a maximum number of operations Θ, beyond which the algorithm is stopped, and the sum of the users demands is thus labeled as unfeasible.

We show now that our proposed algorithm reaches a stable state at which no player attempts to change its own transmit power. Moreover we show that the stable state corresponds to the core apportionment of the game. We model the evolution of the algorithm as the output of a finite-state Markov chain with state space $\Omega = \{\omega = (\psi, \boldsymbol{\nu}) | \psi \in \boldsymbol{\Psi}, \boldsymbol{\nu} \in \mathbb{R}^K\}$. For all time steps t, $\psi^t = \psi$ belongs to the subset of all possible disjoint coalitions $\boldsymbol{\Psi}$ with exactly B members, and remains fixed for the whole duration of the algorithm. The time evolution of the algorithm as a Markov chain is due to the time variability of $\boldsymbol{\nu}^t$, which depends on the power levels p_{kn}^t chosen by the players in the coalitions collected by ψ^t. We use this notation for the sake of convenience, to emphasize that $\boldsymbol{\nu}^t$ is directly connected to ψ^t.

The Markov process asymptotically tends towards a stable coalition structure state, where no player has any incentive to change its power. In other words, all coalitions get their maximum payoffs. Our algorithm guarantees that, when $t \to \infty$, this Markov chain tends towards a singleton steady state with probability 1.

Definition 20 ([286]) *A set $\Phi \subset \Omega$ is an* ergodic set *if, for any $\omega \in \Phi$ and $\omega' \notin \Phi$, the probability of reaching the state ω' starting from ω is zero. Once the Markov chain falls into a state belonging to an ergodic set, it never leaves that set, and it wavers between the states in that ergodic set from then on. The probability of reaching any state in the ergodic set is strictly positive.*

Lemma 4 ([286]) *In any finite Markov chain, no matter which state the process* *starts from, the probability of ending up into an ergodic set tends to* 1 *as time tends to infinity.*

Definition 21 ([286]) *Singleton ergodic sets are called* absorbing states.

If Φ is an absorbing state and $\omega \in \Phi$, the probability of ending up into state ω when beginning from ω is one. In fact, absorbing states individually represent points of equilibrium.

Lemma 5 *The state* $\omega = (\psi, \boldsymbol{\nu})$ *is an absorbing state of the best-response process if and only if*

$$\nu\left(\mathcal{A}_k\right) = +\infty \quad \forall \mathcal{A}_k \in \psi \tag{3.4}$$

Proof This condition ensures that no player has any incentive to change its power amount. If this condition is met, then no coalition can get a higher payoff by deviating from state $\omega = (\psi, \boldsymbol{\nu})$. Since all the target rates are feasible, this condition is also necessary. ■

Theorem 4 *The best-response process has at least one absorbing state.*

Proof Since the best-response algorithm is a Markov process, Lemma 4 ensures that the best-response process reaches an ergodic set Φ. To conclude the proof, it is enough to show that Φ is singleton. Suppose that the number of states in the ergodic set is $|\Phi| > 1$. Then all players revise their strategies without conflicting decisions with a non-null probability. As a consequence, the Markov process moves to a new state, in which all coalitions' payoff are higher than those achieved in the previous state. This means that the probability of going back to the previous state is null, which contradicts the notion of an ergodic set. ■

Note that Theorem 4 does not ensure the uniqueness of the ergodic set in the best-response process. There may exist some different combinations of the power allocation for the players to reach to a steady state. It means that the game possesses multiple equilibria. The major finding of Theorem 4 is that, according to the way the players adjust their strategies, the best-response process leads to one of the steady states, in which no player has any incentive to revise its power allocation.

Theorem 5 *The set of payoffs associated to an absorbing state of the best-response process coincides with the set of core allocation:*

i) if $\omega = (\psi, \boldsymbol{\nu})$ is an absorbing state, then $\boldsymbol{\nu}$ is a core allocation.

ii) if $\boldsymbol{\nu}$ is a core allocation, then all $\omega = (\psi, \boldsymbol{\nu})$ are absorbing states.

Proof Part i) Suppose $\omega = (\psi, \boldsymbol{\nu})$ is an absorbing state but $\boldsymbol{\nu}$ is not a core allocation. In this case, there exist some coalitions that can obtain a higher payoff. This is contradictory, since the game reaches an absorbing state when every coalition gets the maximum payoff.

Part ii) If $\boldsymbol{\nu}$ is a core allocation, then no coalition can earn by letting its member change their powers. This implies that the state will not move to a new state, and thus the current state is absorbing. ■

Coalitional games aim at identifying the best coalitions of the agents and a fair distribution of the payoff among the agents. Interestingly, in this game the absorbing state coincides with one of the Nash equilibria [10] of the game. Suppose there are $K = 2$ mobiles connected to a base station with $N = 1$ subcarrier only. In this case, the $K = K \cdot N = 2$ copies of the subcarrier, each constituting a coalition, are engaged in a 2×2 game. Every player has two strategies: either $p_k = 0$ or $p_k = \overline{p}_k$. It is straightforward to verify that, in this game, a mixed (vs. pure) Nash equilibrium exists which satisfies the stability of the static game. With due attention to the notation, we can extend this result to a general case.

Theorem 6 *The set of absorbing states in the best-response process and the set of Nash equilibria of the static game are asymptotically (in the long run) equivalent.*

Proof Let us consider the coalitions in the best-response process as players in a static game. Lemma 4 ensures that this process reaches an ergodic set in the long run. According to Theorem 4, this set is singleton, and thus its member is an absorbing state. Hence, no coalition (i.e., no player in the static game) has any incentive to revise its strategy. In static games, this is the definition of a Nash equilibrium. ■

We can now conclude that the absorbing state is an extension of the Nash equilibrium, since the coalitions bind agreements with each other as economic agents and earn a vector value rather than a real number. Once the coalitions reach the absorbing state, their payoff is the highest possible ($+\infty$), and no coalition is willing to revise its current strategy. In general, as follows from Theorem 6, the Nash equilibrium of the game is Pareto-optimal (efficient), since no other strategy can achieve a payoff greater than $+\infty$.

3.3 Numerical results

In this section, we evaluate the performance of the best-response algorithm presented in Sect. 3.2. We consider some cases with different numbers of mobile terminals, target data rates, and subcarriers, showing that our suggested scheme reaches a steady state after a few steps only. To increase the convergence speed of the algorithm, we introduce a tolerance parameter ε in our utility function, such that, if $\left|R_k/R_k^\star - 1\right| < \varepsilon$, then we assume that the payoff is $+\infty$. We can possibly set an asymmetric range $[\varepsilon_1, \varepsilon_2]$ such that $\varepsilon_1 \le (R_k/R_k^\star - 1) \le \varepsilon_2$, so as to favor solutions with $R_k > R_k^\star$.

We consider the following parameters for our simulations: the maximum power of each terminal k on each subcarrier n is $\overline{p}_{kn} = \overline{p} = 3\,\mu\text{W}$; the power of the ambient AWGN noise on each subcarrier is $\sigma_w^2 = 100\,\text{nW}$, and the constant number in (3.3) is $\beta = 5000$. We also set $\Theta = 10K \cdot N$ as the stopping criterion of the iterative algorithm, where K and N depend on the network parameters of the simulation. The path coefficients $|H_{kn}|^2$, corresponding to the frequency response of the multipath wireless channel at the carrier frequency $n\Delta f$, are computed using the 24-tap ITU modified vehicular-B channel model adopted by the IEEE 802.16m standard [68]. To account for the large-scale path loss, we assumed the terminals to be uniformly distributed between 3 and 100 m. Based on numerical optimizations, the parameter λ that reduces the probability of conflicting decisions among members of different coalitions for different number of terminals, subcarriers, and signal bandwidth, is $\lambda = 0.97$. The initial power allocation is $p_{kn} = 0\ \forall k \in \mathcal{K}$ and $\forall n \in \mathcal{N}$. This experimentally provides the minimal power consumption at the steady state, and in most cases the minimum number of steps of the algorithm.

Fig. 3.3 reports the behavior of the achievable rate R_k as a function of the time step t in a network with $K = 10$ terminals, $N = 1024$ subcarriers, and bandwidth $W = 10\,\text{MHz}$ using the vacant-carrier assignment scheme. The target rates, reported in Fig. 3.3 with solid markers on the right axis, are assigned randomly to each terminal using a uniform distribution in the range $[100, 250]$ kb/s. Further parameters are:

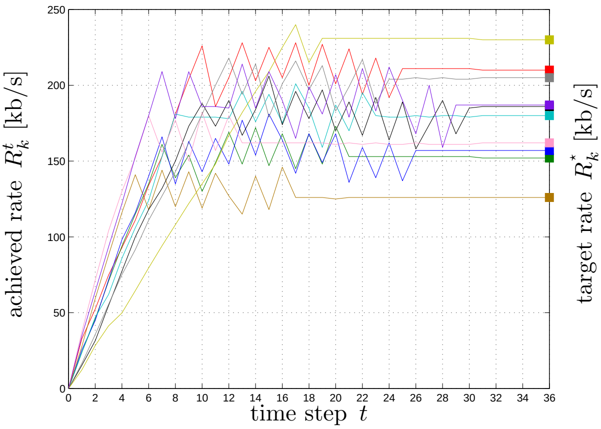


Fig. 3.3: *Achieved rates as functions of the iteration step.*

tolerance $\varepsilon_1 = 0, \varepsilon_2 = 0.01$, power update step $\overline{\Delta p}_{kn} = \overline{p}_{kn}/25 = 120\,\text{nW}$, and number of subblocks $B = 32$. Numerical results show the convergence of R_k to the respective target rates $R_k^\star$ after 31 steps of the best-response algorithm.

In the remainder of this section, we will evaluate by simulation the average performance of our proposed algorithm in terms of power expenditure and computational burden using realistic system parameters and extensive simulation campaigns. Note that we are not able to compare our technique with the joint resource allocation techniques available in the literature and reviewed in Chapter 2.4, mainly due to the unfeasible algorithmic complexity of the implementation of the latter when using tens of terminals, hundreds of subcarriers, and high data rates (on the order of Mb/s). As a consequence, in the following we will compare our measured results with the theoretical performance provided by the literature.

Figs. 3.4 and 3.5 report the simulation results obtained after 500 random realizations of a network with $R_k^\star = R^\star = 200\,\text{kb/s}\ \forall k \in \mathcal{K}$, $N = 1024$, $W = 10\,\text{MHz}$, and $\varepsilon_1 = 0, \varepsilon_2 = 0.04$ again with the vacant-carrier assignment strategy. Solid lines represent the case $\overline{\Delta p}_{kn} = \overline{p}_{kn}/5 = 600\,\text{nW}$, whereas dashed lines depict the case $\overline{\Delta p}_{kn} = \overline{p}_{kn}/25 = 120\,\text{nW}$. Circles, squares, upper triangles and lower triangles correspond to $B = \{8, 16, 32, 64\}$, respectively.

Fig. 3.4 shows the average normalized power expenditure ζ_k at the steady state as a function of K, computed by averaging $\zeta_k = \frac{1}{N}\sum_{n\in\mathcal{N}} \frac{p_{kn}}{\overline{p}_{kn}}$ over all terminals.

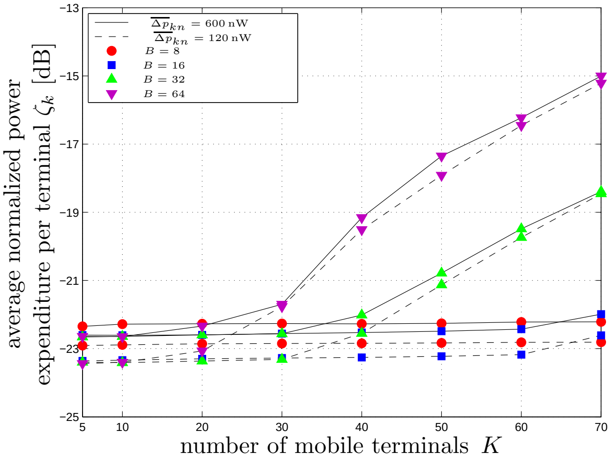


Fig. 3.4: *Average normalized power expenditure as a function of* K*, with* $W = 10\,\mathrm{MHz}$*,* $N = 1024$*, and* $R_k^\star = R^\star = 200\,\mathrm{kb/s}\ \forall k \in \mathcal{K}$ *in the case of vacant-carrier assignment model.*

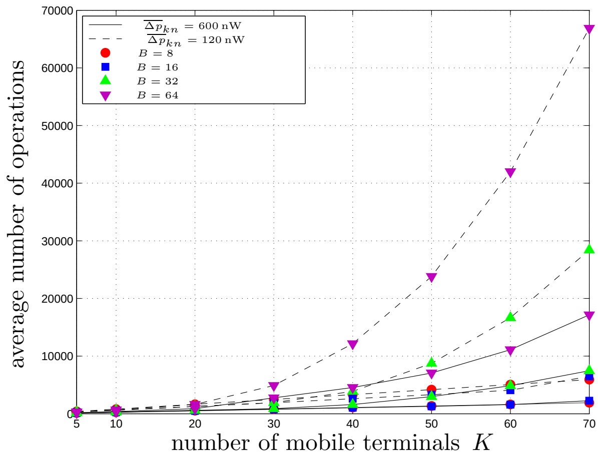


Fig. 3.5: *Experimental average number of operations as a function of* K*, with* $W = 10\,\mathrm{MHz}$*,* $N = 1024$*, and* $R_k^\star = R^\star = 200\,\mathrm{kb/s}\ \forall k \in \mathcal{K}$ *in the case of vacant-carrier assignment model.*

This serves as a measure for the average total power consumption normalized to the maximum power expenditure available to each terminal. As can be noticed, ζ_k increases for $K \geq N/B$, since the number of shared subcarriers increases and the terminals must spend more power to overcome the intra-subcarrier noise. Interestingly, the power expenditure of the proposed centralized algorithm shows higher efficiency than the distributed and cross-layer schemes available in the literature (e.g., see [16, 17, 287, 288]). For instance, when considering 500 random realizations of a system with bandwidth $W = 10\,\mathrm{MHz}$ and $N = 1024$ subcarriers, and using the vacant-carrier assignment model, we find that, in the case of a total sum-rate demand of 20 Mb/s (i.e., with a spectral efficiency of 2 b/s/Hz) and $R_k^\star = R^\star = 200\,\mathrm{kb/s}$ (i.e., $K = 100$ terminals), the maximum power consumption per user is $31\,\mu\mathrm{W}$ and the average power consumption of the system is 0.53 mW. In the multicell scenario of [16], the average power expenditure for each cell is 8 mW when the achievable data rate is 40 Mb/s. When considering the cross-layer algorithm proposed in [17], the average power expenditure per mobile terminal is 0.4 W with maximal spectral efficiency of 2 b/s/Hz, whereas the average power expenditure per mobile terminal required by the energy-efficient techniques proposed in [288] is 0.4 and 1.2 W when the achieved data rate is equal to 40 and 140 kb/s, respectively.

Fig. 3.5 shows the computational complexity of our algorithm expressed in terms of the average number of operations per terminal required to reach the steady state as a function of the number of terminals K with the vacant-carrier assignment model. The number of operations is measured experimentally by counting the number of steps required by the subchannel assignment plus the total number of trials required to update the transmit power according to the best-response algorithm. As can be seen, the complexity increases as B increases. This can be justified since increasing B increases the number of players $K \cdot B$, which yields an increase in the number of conflicting decisions. Note that the proposed algorithm is able to provide a spectral efficiency higher than 1 b/s/Hz, which occurs, for instance, when we assume more than $K = 50$ users with rates $R_k^\star = 200\,\mathrm{kb/s}$ over a bandwidth $W = 10\,\mathrm{MHz}$ in the proposed scenario, with a linear computational burden at the base station using appropriate values for the parameters. In this particular example, a good tradeoff between performance and complexity is $B = \{8, 16\}$ and $\overline{\Delta p_{kn}} = 600\,\mathrm{nW}$. Using these values, the number of operations of the proposed algorithm is experimentally *lower* than the product $K \cdot N$, and so considerably *lower* than the complexity of

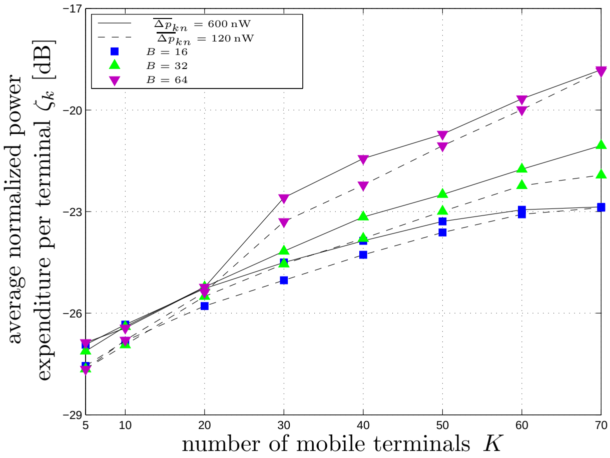


Fig. 3.6: *Average normalized power expenditure as a function of K, with $W = 10\,\text{MHz}$, $N = 1024$, and $R_k^\star = R^\star = 200\,\text{kb/s}$ $\forall k \in \mathcal{K}$ in the case of best-carrier assignment model.*

the schemes available in the literature (e.g., see [18, 19, 139]). Our experiments with different data rate demands show that a smaller data rate reduces also the number of operations significantly. To further reduce the number of operations, we can also increase the tolerance parameters (e.g., with $\varepsilon_2 = 0.1$, we experience a complexity reduction on the order of $20 \div 30\%$). Note also that the spectral efficiency achieved by the proposed fair resource allocation method, while showing a linear computational burden, is comparable with that provided by sum-rate maximizing algorithms (e.g., see [289]). In the practice, a reasonable value for the maximum spectral efficiency achieved by the network in the region of linear complexity in all simulated scenarios (not reported here for the sake of brevity) is slightly lower than $2\,\text{b/s/Hz}$. For higher spectral efficiencies, no parameter selections can achieve the optimal resource allocation with linear complexity, and the number of operations appears to increase exponentially with the number of mobile terminals. However, note that the solutions can be found in most cases.

Figs. 3.6 and 3.7 depict the simulation results of a network with $R_k^\star = R^\star = 200\,\text{kb/s}$ $\forall k \in \mathcal{K}$, $N = 1024$, $W = 10\,\text{MHz}$, and $\varepsilon_1 = 0, \varepsilon_2 = 0.04$ using the best-carrier assignment model. Solid lines represent the case $\overline{\Delta p}_{kn} = \overline{p}_{kn}/5 = 600\,\text{nW}$, whereas dashed lines depict the case $\overline{\Delta p}_{kn} = \overline{p}_{kn}/25 = 120\,\text{nW}$. Squares, upper triangles

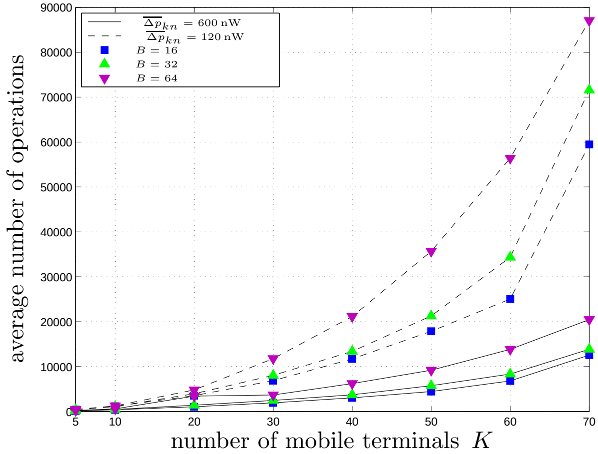


Fig. 3.7: *Experimental average number of operations as a function of K, with $W = 10\,\mathrm{MHz}$, $N = 1024$, and $R_k^\star = R^\star = 200\,\mathrm{kb/s}\ \forall k \in \mathcal{K}$ in the case of best-carrier assignment model.*

and lower triangles correspond to $B = \{16, 32, 64\}$, respectively. Fig. 3.6 shows the average normalized power expenditure ζ_k at the steady state as a function of K. As can be seen, the average power expenditure using the best-carrier assignment model is lower than with the vacant-carrier assignment, since the terminals having better channel conditions spend less power.

A drawback of the best-carrier assignment is an increased complexity of the algorithm. Fig. 3.7 shows the average number of operations per terminal required to reach the steady state as a function of the number of terminals K. As can be seen, the best-carrier assignment model has a computational complexity higher than vacant-carrier assignment model, since the number of shared subcarriers in the best-carrier assignment model is larger than in the vacant-carrier assignment, which increases the probability of interference between simultaneous decisions in the best-reply algorithm. Note that, using the best-carrier assignment model, the case $B = 8$ appears to be computationally expensive.

Fig. 3.8 shows the average number of operations per terminal in the case of a network with parameters $R_k^\star = R^\star = 500\,\mathrm{kb/s}\ \forall k \in \mathcal{K}$, $N = 512$, $W = 10\,\mathrm{MHz}$, and $\varepsilon_1 = 0, \varepsilon_2 = 0.04$ using vacant-carrier assignment model. Solid and dashed lines represents the cases $\overline{\Delta p}_{kn} = 3\,\mu\mathrm{W}$ and $\overline{\Delta p}_{kn} = 600\,\mathrm{nW}$, respectively, whereas circles, squares,

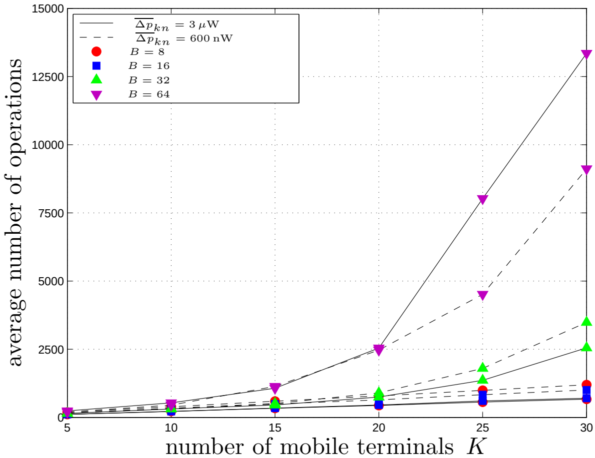


Fig. 3.8: *Experimental average number of operations as a function of* K, *with* $W = 10\,\mathrm{MHz}$, $N = 512$, *and* $R_k^\star = R^\star = 500\,\mathrm{kb/s}\ \forall k \in \mathcal{K}$ *in the case of vacant-carrier assignment model.*

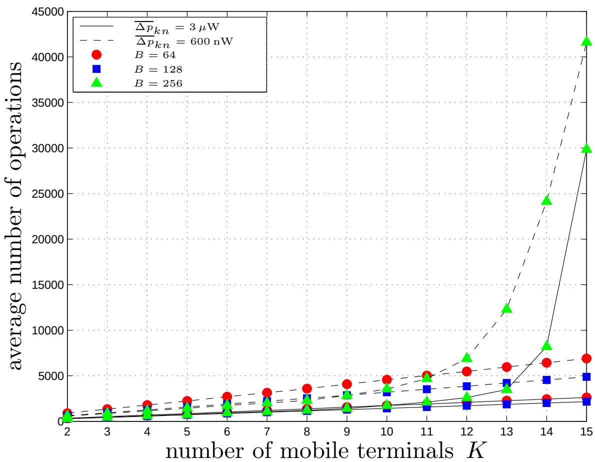


Fig. 3.9: *Experimental average number of operations as a function of* K, *with* $W = 20\,\mathrm{MHz}$, $N = 2048$, *and* $R_k^\star = R^\star = 2\,\mathrm{Mb/s}\ \forall k \in \mathcal{K}$ *in the case of vacant-carrier assignment model.*

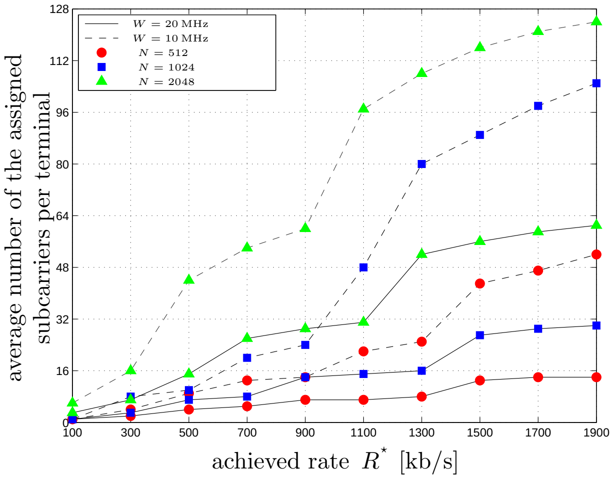


Fig. 3.10: *Average number of assigned subcarriers as a function of the achieved rate, with* $\overline{\Delta p}_{kn} = 600\,\mathrm{nW}$ *in the case of vacant-carrier assignment model.*

upper triangles and lower triangles depict $B = \{8, 16, 32, 64\}$, respectively. Even in this case, with more severe requirements in terms of target data rates, the number of operations is shown to be *lower* than $K \cdot N$, again using spectral efficiencies higher than $1\,\mathrm{b/s/Hz}$.

Finally, Fig. 3.9 shows the average number of operations per terminal in the case of a network with parameters $W = 20\,\mathrm{MHz}$, $N = 2048$, $R_k^\star = 2\,\mathrm{Mb/s}$, $\varepsilon_1 = 0$, and $\varepsilon_2 = 0.04$ with vacant-carrier assignment model. Solid and dashed lines represents the cases $\overline{\Delta p}_{kn} = 3\,\mu\mathrm{W}$ and $\overline{\Delta p}_{kn} = 600\,\mathrm{nW}$, respectively, whereas circles, squares and upper triangles depict $B = \{64, 128, 256\}$, respectively. The complexity is again *lower* than $K \cdot N$.

As can be seen in Fig. 3.5, 3.7, 3.8, and 3.9, due to the random behavior of the proposed algorithm, there is a strict relation between the average number of operations, the network parameters, and the algorithm parameters (including the channel assignment model). Depending on the parameter selection, we see different shapes (linear or exponential behavior) for the average number of operations. Thus, estimating the analytical complexity function for the best-response algorithm is hard to do. However, for *all* tested scenarios (not reported here for the sake of brevity), there exist properly tuned values (such as $B, \overline{\Delta p}_{kn}$) that provide an average number of operations for the proposed algorithm that are *lower* than the product $K \cdot N$, even with high data rate demands like in the cases of Figs. 3.8 and 3.9. The parameter that most impacts on the number of operations is B. Our experiments show that, for the optimal parameter selection (i.e., when the number of operations scales linearly with N and K), the average number of used subcarriers per terminal (i.e., those which bear $p_{kn} > 0$) is approximately $B/2$ when the vacant-carrier model is adopted. This rule-of-thumb can be used as a design criterion for the proposed algorithm. Let us consider Fig. 3.10, that reports the average number of assigned subcarriers to each mobile terminal as a function of the achieved rate $R^\star$, in the linear complexity regime and using $\overline{\Delta p}_{kn} = 600\,\text{nW}$. Dashed and solid lines depict the cases $W = \{10, 20\}\,\text{MHz}$, respectively, whereas circles, squares and upper triangles represent $N = \{512, 1024, 2048\}$, respectively. For instance, when $W = 20\,\text{MHz}$, $N = 512$, and $R^\star = 500\,\text{kb/s}$, the average number of used subcarriers is 4. If we look back at Fig. 3.8, we can verify that the linear complexity can be achieved using $B = 8$. Note that the number of assigned subcarriers in the case of $W = 10\,\text{MHz}$ is higher than in the case $W = 20\,\text{MHz}$, since the subcarrier spacing is halved.

3.4 Discussion

This chapter described a computationally inexpensive centralized algorithm based on coalitional game theory to address the issue of fair optimal resource allocation (in terms of subcarrier assignment and power control) for the uplink of an infrastructure OFDMA wireless network. The scheme derived here is designed to meet the required data rates exactly, thus ensuring a fair performance apportionment to both users and service providers, with the best utilization of the network resources (minimum power expenditure and good spectral efficiency). The proposed algorithm can be analyzed as a Markov model that converges to an absorbing state with unitary probability in the long run. Our criterion also allows us to tradeoff system performance and computational burden of the algorithm, based on the number of subblocks used to apportion the available bandwidth and the data rate requirements of the terminals. Simulations show that the target rates are achieved with a low complexity procedure, even in the case of populated networks and stringent QoS requirements. The (greedy) best-carrier assignment rule results into a higher complexity but a lower power expenditure compared to the case with full use of the available subcarriers. The presented coalition-based strategy appears to be a good tradeoff between computational complexity and power efficiency in comparison with the schemes available in the literature, and achieves a spectral efficiency larger than 1 b/s/Hz.

Chapter 4

Power trading coordination in smart grids

In traditional power distribution models, consumers acquire power from the central distribution unit, while "micro-grids" in a smart power grid can also trade power between themselves. In this paper, we investigate the problem of power trading coordination among such micro-grids. Each micro-grid has a surplus or a deficit quantity of power to transfer or to acquire, respectively. A coalitional game theory based algorithm is devised to form a set of coalitions. The coordination among micro-grids determines the amount of power to transfer over each transmission line in order to serve all micro-grids in demand by the supplier micro-grids and the central distribution unit with the purpose of minimizing the amount of dissipated power during generation and transfer. We propose two dynamic learning processes: one to form a coalition structure and one to provide the formed coalitions with the highest power saving. Numerical results show that dissipated power in the proposed cooperative smart grid is only 10% of that in traditional power distribution networks.

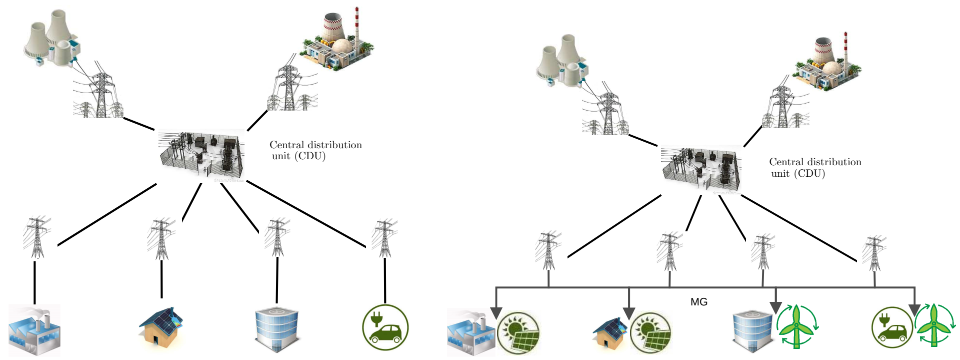
The remainder of the paper is structured as follows. After a brief motivation in the next section, Sect. 4.2 studies the circuit of transmission lines in a smart power grid. Sect. 4.3 consists of two subsections which contains the smart power network model, and models it as a coalitional game, respectively. The two subsections in Sect. 4.4 propose an innovative algorithm to form coalition and to maximize the performance in terms of power saving, respectively. Sect. 4.5 presents some simulation results and finally Sect. 4.6 concludes the paper.

4.1 Motivation

Interest in use of renewable electricity sources, the need for more efficient power distribution systems, lower energy delivery costs, and the improvement of reliability converge in the concept of the smart grid technology. The smart grid technology is an excellent candidate for exploiting information and communications technology (ICT) advantages at every line-powered device. The smart grid may be considered as the technological enabler of a variety of future applications that probably would not be available otherwise. The intelligence of smart grid relies upon the real-time communication either to or from the devices installed in homes and energy providers within the distribution and transmission grids.

While “macro-grids” were traditionally viewed as a technology used in remote area power supplies at a high-voltage, a “micro-grid” (MG) is introduced as a collective of geographically proximate, electrically connected loads and generators based on renewable energy technologies at a medium-voltage [290]. In general, an MG may or may not be connected to the wider electricity grid. Fig. 4.1 shows conceptual differences between the traditional grid, which is hierarchical, and a grid including MGs. In traditional electricity transmission, distribution networks act like the branches of the tree, interconnecting loads and the long-distance transmission network. The MG concept offers a path to autonomous, intelligent low-emissions electricity systems, by creating a localized smart grid that allows advanced and distributed control while being compatible with traditional electricity infrastructure. With such promise, MGs are of growing interest to grid operators, as a way of enhancing the performance of electricity systems [291]. However, realizing MGs is not without challenges, among which we will investigate the problem of coordination between MG electricity generators in order to minimize of the power dissipation over the transmission lines. The minimization of energy dissipation is and will be an important focus in electricity markets considering that power dissipated is accompanied with the cost of energy. Refs. [292–295] investigate the problem of power loss reduction in smart grids. Refs. [292, 293] focus on the optimization of various electricity parameters during peak load times.

Integration of renewable and distributed energy resources encompassing large scale at the transmission level, medium scale at the distribution level and small scale on commercial or residential building can present challenges for the controllability of



(a) Traditional power distribution (b) Smart power grid distribution

Fig. 4.1: *Traditional vs. smart power grid electricity distribution.*

these resources and for operation of the electricity system. Energy storage systems, both electrically and for thermally based, can alleviate such problems by decoupling the saving and delivery of energy. Smart grids can help through automation of control of generation and demand response to ensure balancing of supply and demand [296]. The potential economic impact of deploying power storage units and the possibility of having groups of storage and control units are studied in [297, 298].

Game theory [10] is a potential mathematical tool to model smart grid [298–300]. Non-cooperative game theory can model the distributed operations in smart power grids and cooperative/coalitional game theory can model the cooperation among nodes [301, 302]. To the best of our knowledge, in the existing literature there are only two game theory based algorithms with aim at minimization of power dissipated. Refs. [294, 295] propose coalitional game theory based algorithms which form a set of disjoint coalitions and enable MGs in each coalition to trade power among themselves with aim at minimization the overall power dissipated. The authors in [294] maximize the utility function of the grand coalition (coalition of all MGs), while the authors in [295] maximize Shapley values [48] of MGs. The results of [294] show that the proposed algorithm improves the performance reaching up to 31% (with 30 MGs) compared to the traditional transmission grids. Unfortunately, the performance of the proposed algorithm in [295] is evaluated with non realistic smart grid network area $10 \times 10\,\mathrm{km}^2$. It is obvious that in small areas the amount of power dissipated is less than in a realistic network area $100 \times 100\,\mathrm{km}^2$. In addition, in Refs. [294,295], the circuit of transmission lines is not realistic (We explain why in Sect. 4.2).

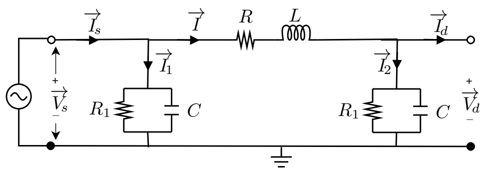
In this work, we investigate the problem of power trading coordination between MGs in smart power grids. Each MG has either a surplus or deficit quantity of power. Each MG with a surplus amount of power can transfer to the central distribution unit (CDU), and meanwhile it can serve MGs with deficit power. Each MG with a deficit amount of power can receive from the CDU and from MGs with surplus power. With the aim of power loss minimization during power generation and transfer, we will introduce a power trading coordination strategy among MGs. We formulate the problem using coalitional game theory in which MGs form a set of not necessarily disjoint and possibly singleton coalitions. MGs in each coalition can trade power between themselves and with the CDU. Each non-singleton coalition consists of one MG with surplus power which will serve the needed MG(s) with deficit power. On the other hand, the micro-grid in a singleton coalition only trades directly with the CDU.

For achieving the best power distribution over transmission lines, we propose two dynamic learning algorithms: 1) coalition formation dynamic learning, and 2) power loss minimization dynamic learning. The dynamic learning process 1) is nested in 2) one. In other words, the dynamic learning process 2) iteratively executes learning process 1) until it achieves a fixed point at which the performance can no longer improve. Coalition formation dynamic learning achieves a coalition formation structure and then the complementary power loss minimization dynamic learning leads the MGs to the maximum performance in terms of power saving. We show the stability (the convergence to a fixed-point) of both dynamic processes using the Kakutani fixed point theorem [303]. As our evaluation shows, our approach enables MGs to come to a power trading coordination among themselves that yields a significant power saving compared to the traditional power trading.

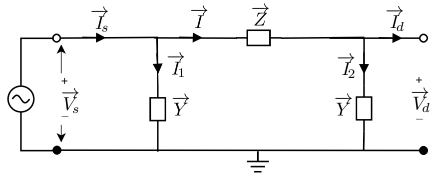
4.2 Transmission line model

Fig. 4.2(a) shows the single-phase equivalent[1] circuit of a medium-length (up to 200km) transmission line [290]. On an electricity transmission line from the sending-

[1] This circuit is suitable for analysing its symmetrical three-phase operation.



(a) Transmission circuit



(b) π-equivalent circuit

Fig. 4.2: *Single-phase circuit of a medium-length transmission line.*

end s to the receiving-end d, the variables of interest are the (complex-valued) voltages and currents per phase $\overrightarrow{V_s}$, $\overrightarrow{V_d}$, $\overrightarrow{I_s}$, and $\overrightarrow{I_d}$ at the end-line terminals. The variables V_s and I_s denote the amplitudes of $\overrightarrow{V_s}$ and $\overrightarrow{I_s}$, respectively, and likewise for V_d and I_d. As the transmission line model in Fig. 4.2(a) has many parameters, it is convenient to replace each line with its π-equivalent shown in Fig. 4.2(b) where the impedance $\overrightarrow{Z} = R + i\omega L \;\; \Omega$ and the admittance $\overrightarrow{Y} = 1/R_1 + i\omega C \;\; \Omega^{-1}$ with i as the imaginary unit and $\omega = 2\pi f$ as the (sinusoidal) angular frequency in electrical radians at the frequency f.

Suppose the sending-end wants to supply a power amount P_s [W] and transfers it toward the receiving-end. A fraction of P_s will be dissipated during generation and transfer and the receiving-end d can load a power amount $P_d < P_s$. The real power loss is calculated by the following formula:

$$D_{sd} = P_s - P_d = V_s.I_s.\cos\theta_s - V_d.I_d.\cos\theta_d \quad [\mathrm{W}] \tag{4.1}$$

where the parameter θ_s is the "power factor angle", in the range of $(-90°, +90°)$, and that is the phase difference between $\overrightarrow{V_s}$ and $\overrightarrow{I_s}$, and likewise for θ_d. So, the received power at the receiving-end d from the transmission line $s \to d$ can be formulated as $P_d = P_s - D_{sd}$.

In a transmission line, the amplitude of the voltage at sending-end V_s is known and we suppose $\overrightarrow{V_s}$ to be the reference vector, i.e, $\overrightarrow{V_s} = V_s\angle 0°$. We suppose also, as usual, that the power factor at the sending-end, $\cos\theta_s$, is known with the assumption that the current waveform comes delayed after the voltage waveform, i.e., $\overrightarrow{I_s} = I_s\angle -\theta_s$. For supplying and transferring P_s amount of power, the sending-end regulates the amplitude of the current as $I_s = \frac{P_s}{V_s.\cos\theta_s}$ and synchronizes its phase at $-\theta_s$. Knowing the parameters of the transmission line in the nominal π-circuit in Fig. 4.2(b), the voltage and the current at the receiving-end are calculated by the following equations:

$$\begin{aligned}\overrightarrow{V_d} &= \left(1+\overrightarrow{Y}.\overrightarrow{Z}\right).V_s - \overrightarrow{Z}.\overrightarrow{I_s}\\ \overrightarrow{I_d} &= -\overrightarrow{Y}.\left(2+\overrightarrow{Y}.\overrightarrow{Z}\right).V_s + \left(1+\overrightarrow{Y}.\overrightarrow{Z}\right).\overrightarrow{I_s}\end{aligned} \tag{4.2}$$

Now, we can calculate the receiving-end's parameters V_d, I_d, and θ_d using (4.2) and then the amount of power loss is calculated as in (4.1). It is simple to show that the power loss equation (4.1) is a concave function of I_s when $L = C = 1/R_1 = 0$ (short-length transmission line model [290] as considered in Refs. [294, 295]), otherwise its shape strictly depends upon the values of the parameters.

4.3 System model and Problem formulation

4.3.1 System model

We study a smart power grid consisting of a single CDU which is connected to one or more main power plants at a high-voltage. The CDU is connected to K MGs denoted by the set $\mathcal{K}$. At any given time frame [299], each MG $k \in \mathcal{K}$ has a Q_k residual power load which is defined as the difference between the generated power and the overall demand. A positive quantity $Q_k > 0$ determines the surplus power that the MG can transfer to other MGs or to the CDU, whereas a negative quantity $Q_k < 0$ determines the deficit power the MG needs to acquire from other MGs or from the CDU. When $Q_k = 0$, the MG k meets its demanded power and it will not interact with any other MG or the CDU. We divide MGs into three groups: "suppliers" ($Q_k > 0$), "demanders" ($Q_k < 0$), and "inactives" ($Q_k = 0$) denoted by $\mathcal{K}^+$, $\mathcal{K}^-$, and $\mathcal{K}^0$, respectively, such that $\mathcal{K} = \mathcal{K}^+ \cup \mathcal{K}^- \cup \mathcal{K}^0$. In the rest of this paper, we assume $\mathcal{K}^0 = \emptyset$, $|\mathcal{K}^+| \geq 1$, and $|\mathcal{K}^-| \geq 1$.

Our goal is to develop an efficient power transfer policy between active MGs and the CDU themselves to minimize the overall power dissipation in the network. We will propose an algorithm which assigns to each supplier $s \in \mathcal{K}^+$ a subset of demanders in $\mathcal{K}^-$ and determines different fractions of Q_s to be transferred to each assigned demander and to the CDU. Doing so, traded power may be transferred on short distance lines. As values of each transmission line components (resistor, inductor, and capacitor) are an inverse function of the distance, the amount of dissipated power will be much lower than that in a traditional distribution. However, a MG can decide to

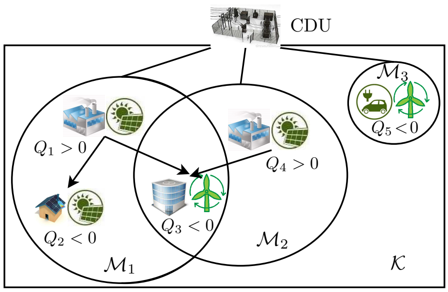


Fig. 4.3: *A sample of formed coalitions for cooperative power distribution.*

act as a non-cooperative MG which trades only with the CDU. Each MG in demand can be assigned to more suppliers to acquire the whole or a fraction of its own needed power. Two suppliers will not belong to the same coalition, since they do not trade power between themselves.

4.3.2 Problem formulation

To study the cooperative behavior of the MGs, we use a coalitional game theory framework [301]. A coalitional game is defined as $\mathcal{G} = (\mathcal{K}, \nu)$ where $\mathcal{K}$ is the player's set (the active MGs), and $\nu : 2^{\mathcal{K}} \longrightarrow \mathbb{R}$ is the characteristic function of each coalition (subset of $\mathcal{K}$) that assigns a real number representing the benefit earned by the coalition. We will propose an algorithm which forms a set of not necessarily disjoint and possibly singleton coalitions denoted by $\boldsymbol{\mathcal{M}} = \{\mathcal{M}_1, \ldots, \mathcal{M}_M\}$. The (unique) MG in a singleton coalition will trade only with the CDU. A non-singleton formed coalition consists of *only* one supplier and one or more demanders, i.e., the unique supplier MG provides the whole or a fraction of the overall power demanded by the assigned demander(s). Each demander in $\mathcal{K}^-$ may belong to more coalitions, i.e., the whole or a fraction of its needed power can be provided by more suppliers. Then, all MGs can trade with the CDU for the rest, if any. Fig. 4.3 shows a sample wherein two cooperative coalitions and one non-cooperative singleton coalition are formed. For instance, MG1 can decide to transfer 10% of its (surplus) quantity to MG2 and 30% to MG3 and the rest to the CDU. MG4 can decide to transfer 50% of its quantity to MG3 and the rest to the CDU. The rest of deficit quantities of MG3 and MG4 will be provided by the CDU. MG5 will be completely served by the CDU.

A coalition structure will be formed only if (i) surplus power quantities of all suppliers are completely loaded, and (ii) deficit power quantities of all demanders are completely served, as the following equations state:

$$\begin{cases} \sum_{d \in \mathcal{K}^-} P_{sd} + P_{s0} = Q_s \quad \forall s \in \mathcal{K}^+ & (4.3a) \\ \sum_{s \in \mathcal{K}^+} (P_{sd} - D_{sd}) \; + \; (P_{0d} - D_{0d}) = -Q_d \quad \forall d \in \mathcal{K}^- & (4.3b) \end{cases}$$

where the subscript 0 denotes the CDU index. The parameter P_{ij} denotes the amount of loaded power by the sending-end i as $P_{ij} = V_{ij}.I_{ij}.\cos\theta_{ij}$ to transfer over the transmission line $i \to j$. We suppose that the power network is *meshed*, i.e. each sending-end can regulate a different voltage, current, and power factor angle over each connected transmission line. The parameter D_{ij} is the amount of power dissipated, calculated as in (4.1) while the sending-end terminal i loads P_{ij} over the transmission line $i \to j$. Condition (4.3a) guarantees that the power quantity of each supplier $s \in \mathcal{K}^+$ is completely loaded over the connected transmission lines and condition (4.3b) guarantees that each demander $d \in \mathcal{K}^-$ receives the whole deficit quantity (demanded) power.

We divide the MGs in each formed coalition $\mathcal{M}_m$ into two subsets of suppliers and demanders denoted by $\mathcal{M}_m^+$ and $\mathcal{M}_m^-$, respectively. For a singleton coalition $\mathcal{M}_m$, either $\mathcal{M}_m^+$ or $\mathcal{M}_m^-$ is empty. In each non-singleton coalition $\mathcal{M}_m$, $\mathcal{M}_m^+$ is singleton since the supplier MG is unique. For each coalition $\mathcal{M}_m$ with $\mathcal{M}_m^+ = \{s\}$, we define the characteristic function as the inverse of total dissipated power over the distribution lines incurred by the power generation and transfer, as:

$$\nu\left(\mathcal{M}_m\right) = \left(\sum_{d \in \mathcal{M}_m^-} D_{sd} + D_{s0} + \sum_{d \in \mathcal{M}_m^-} D_{0d} \right)^{-1} \tag{4.4}$$

wherein the power minus accounts for the maximization problem, term D_{sd} refers to the power dissipated incurred by the generation and transfer of P_{sd} from the supplier s to each $d \in \mathcal{M}_m^-$, the term D_{s0} refers to the power dissipated incurred by the transfer of the power amount P_{s0} from the supplier s to the CDU, and D_{0d} refers to the power dissipated incurred by the power transfer from the CDU to the demanders in $\mathcal{M}_m^-$. For a singleton coalition which consists of one non-cooperative supplier MG, terms D_{sd} and D_{0d} are equal to zero and term D_{s0} is calculated with setting $P_{s0} = Q_s$.

On the other hand, for a singleton coalition which consists of one demander MG, terms D_{sd} and D_{s0} are equal to zero and the term D_{0d} is calculated with setting $P_{0d} = -Q_d + D_{0d}$, i.e., the demanded power $-Q_d$ will be served only by the CDU. We assign $\nu\left(\mathcal{M}_m\right) = -\infty$ when the power distribution conditions (4.3) do not hold. We assume the unit of currency as the price of unit amount of power. It is worthwhile to note that in the proposed framework it could also be considered coefficient minus instead of power minus in (4.4), since the power loss is neither a concave nor convex function.

For the grand coalition, the characteristic function is the inverse of the total amount of power loss in the entire network as the following formula:

$$\nu\left(\mathcal{K}\right) = \left(\sum_{s\in\mathcal{K}^+}\sum_{d\in\mathcal{K}^-} D_{sd} + \sum_{s\in\mathcal{K}^+} D_{s0} + \sum_{d\in\mathcal{K}^-} D_{0d}\right)^{-1} \tag{4.5}$$

We assign $\nu\left(\mathcal{K}\right) = -\infty$ when the power distribution conditions (4.3) do not hold.

A central question in a coalitional game is how to divide the earnings among the members of the formed coalition. The payoff of each MG (player in the game) can show the power of influence of the MG and it is obvious that a higher payoff is an incentive to cooperate more efficiently. The Shapley value [48] assigns a unique outcome to each MG. Let us denote ϕ_k as the Shapley value of MG $k \in \mathcal{K}$ in the game. For each MG $k \in \mathcal{K}$:

$$\phi_k = \sum_{\substack{\forall\, \mathcal{M}_m \subseteq \mathcal{K} \\ \text{s.t. } k \in \mathcal{M}_m}} \frac{(|\mathcal{M}_m| - 1)! \cdot (K - |\mathcal{M}_m|)!}{K!} \left(\nu\left(\mathcal{M}_m\right) - \nu\left(\mathcal{M}_m \backslash \{k\}\right)\right) \tag{4.6}$$

The expression $\nu\left(\mathcal{M}_m\right) - \nu\left(\mathcal{M}_m \backslash \{k\}\right)$ is the marginal payoff of the MG k to the coalition $\mathcal{M}_m$. The Shapley value can be interpreted as the marginal contribution an MG makes, averaged across all permutations of MGs that may occur.

4.4 Best response algorithm

In this section we provide an answer to the question: *How do the MGs form the best coalition structure with aim at minimization of the amount of power dissipated?* Dynamic learning models provide a framework for analyzing the way players set their proper strategies. To address this question, we propose an “adaptive learning algorithm" [304] in which the learners use the same learning algorithm. Typically, these types of algorithms are constructed to iteratively play a game with an opponent, and, by playing this game, to converge a solution. Forming a coalition structure by MGs is equivalent to distribute Q_k $\forall\, k \in \mathcal{K}$ over the transmission lines satisfying conditions (4.3). Obviously the solution is not unique. In the next subsections, we first propose a dynamic learning process to form a coalition structure where each formed coalition earns a positive bounded payoff and each MG earns a bounded Shapley value. We then introduce another complementary dynamic process which iteratively executes the coalition formation dynamic learning process in order to choose the best coalition structure with minimum power dissipated.

4.4.1 Coalition formation dynamic learning process

To realize a coalition formation structure, we use dynamic learning. Our goal is to determine the best amount of power loaded at the sending-end sides of the uni-directional transmission lines denoted by $\mathcal{L} = \{l_{sd}\} \cup \{l_{s0}\} \cup \{l_{0d}\}$, $\forall\, s \in \mathcal{K}^+$ and $\forall\, d \in \mathcal{K}^-$ where the symbol l_{ij} denotes the transmission line from the sending-end i to the receiving-end j. We denote $L = |\mathcal{L}| = |\mathcal{K}^+| \cdot |\mathcal{K}^-| + |\mathcal{K}^+| + |\mathcal{K}^-|$ as the number of individuals. We denote the parameters of the sending-end i over the transmission line l_{ij} (to the receiving-end j) by I_{ij}, V_{ij}, P_{ij}, and θ_{ij}. We will propose an algorithm to determine the best value of P_{ij}. The values of V_{ij} and θ_{ij} are known at the sending-end i and it will regulate the electricity current as $I_{ij} = \frac{P_{ij}}{V_{ij}\cos\theta_{ij}}$ to load P_{ij} toward the receiving-end j.

In our dynamic learning process, each transmission line $l_{ij} \in \mathcal{L}$ is an individual learner which, during learning process learns about the best amount of the power P_{ij} to be loaded by the sending-end i to transfer over the transmission line l_{ij}. By doing so: 1) each supplier $s \in \mathcal{K}^+$ will be aware of the amount of power to load over the transmission lines toward all demanders and the CDU, i.e., over l_{s0} and $\{l_{sd}\}$ $\forall\, d \in \mathcal{K}^-$, and 2) the CDU will be aware of the amount of power to load over the transmission lines toward all demanders, i.e., over $\{l_{0d}\}$ $\forall\, d \in \mathcal{K}^-$. If one supplier s loads a power $P_{sd} > 0$ over the transmission line l_{sd}, the MGs s and d will belong to the same coalition. If one supplier s loads $P_{s0} = Q_s$ over l_{s0}, the supplier s will form a singleton coalition. One demander $d \in \mathcal{K}^-$ will form a singleton coalition if it receives the whole $-Q_d$ from the CDU, i.e., $P_{0d} - D_{0d} = -Q_d$.

For each P_{ij}, let us denote the maximum power constraint and the best power amount by $\bar{P}_{ij}$ and P^*_{ij}, respectively. We set $\bar{P}_{sd} = \min\{Q_s, -\beta Q_d\}$, $\bar{P}_{s0} = Q_s$, and $\bar{P}_{0d} = -\beta Q_d$ with $\beta > 1$ for $\forall\, s \in \mathcal{K}^+$ and $\forall\, d \in \mathcal{K}^-$. About $\bar{P}_{s0}$, it is obvious that loaded power by a supplier cannot be greater than its residual power load. With respect to $\bar{P}_{0d}$, whenever the CDU wants to completely serve one demander d, it must load a power amount $P_{0d} > -Q_d$ to overcome power loss over the transmission line $0 \to d$ and so to guarantee received power $-Q_d$ at the proper demander side. Now, the setting of $\bar{P}_{sd}$ is obvious. As is apparent, choosing an appropriate value for β strictly depends upon the parameters of the transmission lines and the value of Q_ks. In our simulation in Sect. 4.5, we set $\beta = 2$, i.e., the CDU should load at most $-2Q_d$ for completely serving the proper demander (the maximum power loss over the line $0 \to d$ is $-Q_d$). Instead of using the parameter β, the appropriate solution is to answer this question: How much must be the load power P_s at the sending-end s, if it wants to guarantee a power amount P_d at the receiving-end d? Multiplication the both sides of $\overrightarrow{V_d}$ in (4.2) to the correspondence sides of $\overrightarrow{I_d}$ approaches a quadratic equation of I_s with complex coefficients for whose roots, in general case, we could not find a presentable algebraic expression. For the special case of $C = L = 1/R_1 = 0$, we refer the reader to [294, 295].

During the learning process, in each (discrete) time step t every learner $l_{ij} \in \mathcal{L}$ individually and distributively updates its temporary power value P^t_{ij} in a myopic manner, i.e., supposing that all other learners $\mathcal{L}\setminus\{l_{ij}\}$ are inactive and their power values are fixed. In the following, we propose a method to distributively update the power values P^t_{ij} in order to lead it to the best value and we will show that the updating process has a fixed point, i.e., for a given t large enough $P^{t+1}_{ij} = P^t_{ij}$ $\forall\, l_{ij} \in \mathcal{L}$, and then we set $P^*_{ij} = P^{t+1}_{ij}$.

The power value of learners l_{sd} appear in both conditions (4.3), whereas that of l_{s0} only in (4.3a) and that of l_{0d} only in (4.3b). From conditions (4.3), the following equations are derived:

$$\begin{cases} P_{ij} = C_{ij}\,; \qquad \forall\, l_{ij} \in \mathcal{L} \\ C_{s0} = Q_s - \sum\limits_{d\in\mathcal{K}^-} P_{sd} \qquad \forall\, l_{s0} \in \mathcal{L}\,; \qquad //\forall\, s \in \mathcal{K}^+ \\ C_{0d} = -Q_d - \sum\limits_{s\in\mathcal{K}^+} P_{sd} + D_{0d} + \sum\limits_{s\in\mathcal{K}^+} D_{sd} \qquad \forall\, l_{0d} \in \mathcal{L}\,; \qquad //\forall\, d \in \mathcal{K}^- \\ C_{sd} = 0.5\left(Q_s - Q_d - \sum\limits_{d\neq d'\in\mathcal{K}^-} P_{sd'} - \sum\limits_{s\neq s'\in\mathcal{K}^+} P_{s'd} - P_{s0} - P_{0d} + D_{0d} + \sum\limits_{s'\in\mathcal{K}^+} D_{s'd}\right) \\ \qquad\qquad \forall\, l_{sd} \in \mathcal{L}. \qquad //\forall\, s \in \mathcal{K}^+,\ \forall\, d \in \mathcal{K}^- \end{cases} \tag{4.7}$$

According to (4.7), the rationality of each learner $l_{ij} \in \mathcal{L}$ is focusing on updating the proper value P_{ij} in order to achieve the value of C_{ij} *exactly*, i.e., $P_{sd} = C_{sd}$, $P_{s0} = C_{s0}$, and $P_{0d} = C_{0d}$. At each time step t, the learner l_{ij} will update the proper power value P_{ij}^t in a myopic manner, i.e., supposing that C_{ij}^t is a constant. To this end, we define the following learning utility function for each learner $l_{ij} \in \mathcal{L}$:

$$\begin{cases} u_{ij} = -\sqrt{|P_{ij} - C_{ij}|} + \alpha \cdot \mathrm{u}\,(P_{ij} - C_{ij})\,; & 0 \le C_{ij} \le \bar{P}_{ij} \\ u_{ij} = -\alpha & \text{otherwise.} \end{cases} \tag{4.8}$$

where $\mathrm{u}\,(\cdot)$ is the step function, with $\mathrm{u}\,(y) = 1$ if $y \ge 0$ and $\mathrm{u}\,(y) = 0$ otherwise (see Fig. 4.4), and α is a constant. When the value C_{ij} is out of the range of $[0, \bar{P}_{ij}]$, the proper learner will gain the minimum possible payoff, $-\alpha$. For a $C_{ij} \in [0, \bar{P}_{ij}]$, if $P_{ij} = C_{ij}$, the learner l_{ij} earns the highest possible payoff $u_{ij} = \alpha$. Whereas when $P_{ij} \neq C_{ij}$, the learner l_{ij} gets a payoff lower than α. The factor α is a sufficiently large positive constant that ensures u_{ij} to be positive when $P_{ij} > C_{ij}$. This is an expedient to let the learners distinguish the value of P_{ij} that is either smaller or larger than C_{ij} only by knowing their own payoffs. In practice, it is obvious that the payoff of each learner is bounded from below by the finite negative number $-\alpha$. Setting an appropriate value to α in order to guarantee a positive u_{ij} when $P_{ij} \ge C_{ij}$ is strictly dependent upon the values of Q_k. Note that the proposed framework could also consider any bounded utility function which increases as its argument moves from $\pm\infty$ to 0. This means that, for any $P_{ij} \neq C_{ij}$, each learner l_{ij} has an incentive to move towards the point zero where $P_{ij} = C_{ij}$ and the learner gains the highest possible payoff. When all learners gain α, then all conditions (4.3) hold, and obviously $P_{ij} = P_{ij}^*$.

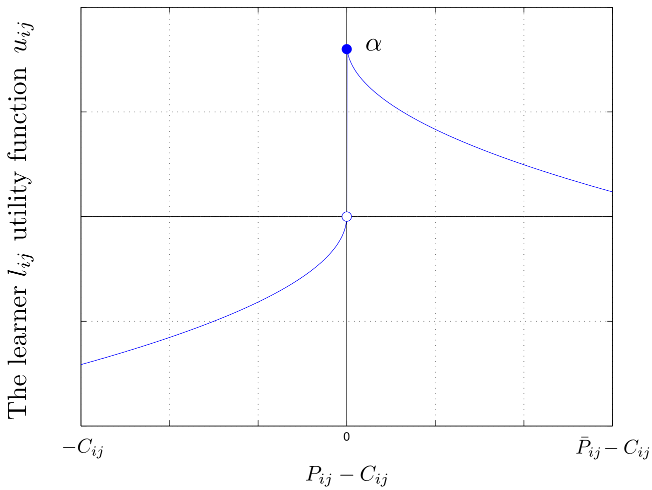


Fig. 4.4: *Learner utility as a function of* $P_{ij} - C_{ij}$ *with* $0 \leq C_{ij} \leq \bar{P}_{ij}$ *and* $\alpha \gg 0$.

The pseudocode in Tab. 4.1 shows how each learner l_{ij} takes its myopic decision during time step t. In this algorithm, $\text{sign}(\cdot)$ is our sign function, with $\text{sign}(y) = 1$ if $y \geq 0$ and $\text{sign}(y) = -1$ otherwise. The parameter $\tilde{u}_{ij}$ is the "trial" value of the current payoff of the learner l_{ij} when the tentative power $\tilde{P}_{ij}$ is adopted. As each learning time step, the power step $\Delta\tilde{P}_{ij}$ is the particular outcome (value) of a random variable uniformly distributed between 0 and $\overline{\Delta P}_{ij}$, with $\overline{\Delta P}_{ij} \ll \bar{P}_{ij}$. Optimal values for $\overline{\Delta P}_{ij}$ can be found in order to minimize the computational load of the algorithm, based on experimental results. If $u^t_{ij} < 0$, then $P^t_{ij} < C_{ij}$, and the best strategy for the learner l_{ij} is to increase its power so as to increase its payoff. Consequently, the tentative power is a random number in the interval $[P^t_{ij}, \bar{P}_{ij}]$. The learner l_{ij} accepts this value if and only if the utility u^t_{ij} increases, otherwise it ends up to keep its previous value. If $0 \leq u^t_{ij} < \alpha$, the learner l_{ij}'s best strategy is on the contrary to decrease P^t_{ij}, and thus the tentative (random) power level belongs to the interval $[0, P^t_{ij}]$. As is apparent, the convergence speed of the algorithm depends also on the choice of the maximum update step $\overline{\Delta P}_{ij}$.

The process starts at time step $t = 0$ with $P^{t=0}_{ij} = 0$ for all learners in $\mathcal{L}$. Thus, at $t = 0$, we have $C^{t=0}_{s0} = Q_s = \bar{P}_{s0} \;\; \forall s \in \mathcal{K}^+$ and $C^{t=0}_{0d} = -Q_d < \bar{P}_{0d} \; \forall d \in \mathcal{K}^-$, i.e.,

```
function  P_ij^{t+1} = Powerupdate(l_ij, t)
   if (u_ij^t = α) or (u_ij^t = −α), then P_ij^{t+1} = P_ij^t, exit;
   P̃_ij = P_ij^t; //saving the current power
   repeat
      P̂_ij = P̃_ij ; //saving tentative power
      compute ũ_ij; //tentative payoff
      ΔP̃_ij = unif [0, \overline{ΔP}_ij] ; //random power step
      P̃_ij = P̃_ij − sign(u_ij^t) · ΔP̃_ij ; //tentative power
   until (ũ_ij > u_ij^t) or (P̃_ij > P̄_ij) or (P̃_ij < 0)
   if (ũ_ij > u_ij^t), then P_ij^{t+1} = P̂_ij ; //accept
                      else P_ij^{t+1} = P_ij^t ; //discard
end function.
```

Tab. 4.1: *Myopic decision of the learner l_{ij} at time step t.*

$u_{s0}^{t=0}$ and $u_{0d}^{t=0}$ are in the range of $(-\alpha, 0)$. Depending on Q_s, $-Q_d$, and β, the values of $C_{sd}^{t=0}$ can be either in or out of the range of $[0, \bar{P}_{sd}]$. At $t = 1$, each learner l_{s0} and l_{0d} may increase the proper power amount which results that some of $C_{sd}^{t=1} < C_{sd}^{t=0}$. After some time steps, some of the learners l_{sd} will have $0 \leq C_{sd}^t \leq \bar{P}_{sd}$ and they can update the proper powers.

Each learner l_{ij} individually decides to adjust its power value P_{ij}^t in a myopic manner while supposing that the value of C_{ij}^t is fixed and all other learners are inactive. The value of C_{ij}^t depends upon the power values of other learners and therefore the decision of adjusting the power value by each learner influences other C values in conflicting and incompatible ways which prevent learners from gaining the expected payoff. At each time step t, adjusting P_{ij}^t changes all C_{s0}^t and C_{0d}^t $\forall s \in \mathcal{K}^+$ and $\forall d \in \mathcal{K}^-$. To reduce the number of occurrences of this event, we modify our algorithm by requesting each learner l_{ij} *not* to update its power value at every time step with a probability $\lambda_{ij} \in [0, 1]$. At each time step t, every learner l_{ij} picks a random number ξ_{ij}^t uniformly distributed in $[0, 1]$. If $\xi_{ij}^t > \lambda_{ij}$, then the learner applies the algorithm and possibly update P_{ij}^{t+1}, otherwise $P_{ij}^{t+1} = P_{ij}^t$. Note that each learner l_{ij} is aware of the value of λ_{ij}. The algorithm is executed in a central computing unit [9], e.g. the CDU, and it can transmit the best amount of power load over each transmission line to MGs.

We show now that our proposed algorithm reaches a fixed point at which $P_{ij}^t = C_{ij}^t$ $\forall\, l_{ij} \in \mathcal{L}$. We model the evolution of the algorithm as the output of a Markov chain with state space $\Omega = \{\omega = (\boldsymbol{u}) | \, \boldsymbol{u} \in [-\alpha, \alpha]^L\}$. At time step t, our system is in the state $\omega^t = (\boldsymbol{u}^t)$, where $\boldsymbol{u}^t$ is the set $\{u_{ij}^t\}$ $\forall \, l_{ij} \in \mathcal{L}$. For all time steps t, since u_{ij} is continuous and bounded in $[-\alpha, \alpha]$, $\boldsymbol{u}^t$ is bounded in the L-dimensional space $[-\alpha, \alpha]^L$. So, $\boldsymbol{u}^t$ is compact and Lebesgue measurable. The evolution of the Markov chain is then dictated by the strategy of the learning process. The strategy of each learner l_{ij} is to find the best power amount P_{ij}^t that leads to an increase its own payoff u_{ij}^t. In practice, each learner l_{ij} autonomously decides whether to change its power value P_{ij}^t, making its payoff better off, or to keep the power at the same power level (when its payoff is equal to α or $-\alpha$). The transition from the state $\omega = (\boldsymbol{u})$ to a new state $\breve{\omega} = (\breve{\boldsymbol{u}})$ occurs if and only if the new state $\breve{\omega}$ "dominates" the state ω, i.e., compared to $\boldsymbol{u}$, no learner gets worse off in $\breve{\boldsymbol{u}}$. The CDU computes C_{ij}^{t+1} $\forall \, l_{ij}$ using the values of P_{ij}^{t+1} and then computes all utility payoffs $\boldsymbol{u}^{t+1}$. If the transition from the state ω^t to the state ω^{t+1} occurs, then the CDU communicates to all learners sending them the proper C_{ij}^{t+1}, otherwise announce them to keep the previous power amount, i.e., $P_{ij}^{t+1} = P_{ij}^t$. The Markov process asymptotically tends towards a stable power distribution state at which no learner has any incentive to change its power value. In other words, all learners get their maximum payoffs, $\boldsymbol{u} = \{\alpha\}^L$, and consequently $P_{ij}^t = C_{ij}^t$ $\forall \, l_{ij} \in \mathcal{L}$, and then, obviously, no learner has any incentive to update its power value. Our algorithm guarantees tending the Markov chain towards a fixed point state with probability one when $t \to \infty$. Obviously, the stable state is not unique and according to the way the learners generate random numbers, the algorithm leads them to one of the possible solutions and then the power values are no longer updated.

Theorem 7 *The coalition formation learning process converges a stable state.*

Proof Denote by $\mathcal{P}$ the set of all possible $\boldsymbol{u}^t$'s. Obviously $\mathcal{P} = [-\alpha, \alpha]^L$ is not empty and it is Lebesgue measurable, compact and convex set. We construct a correspondence $\Gamma : \mathcal{P} \to \mathcal{P}$ with $\Gamma(\boldsymbol{u}) = \{\breve{\boldsymbol{u}}\}$ where $\{\breve{\boldsymbol{u}}\}$ is the set of all possible next states of the (current) state $\boldsymbol{u}$ in the proposed best-response algorithm, i.e., the states $\breve{\boldsymbol{u}}$s dominates $\boldsymbol{u}$. According to the best-response algorithm, the transition probability from $\boldsymbol{u}$ to one member of $\{\breve{\boldsymbol{u}}\}$ is one and zero to the others members. Show that a fixed point exists. Now we claim that there is upper hemi-continuity (uhc)[2] for a given $\boldsymbol{u}^t$. To this end, let $\boldsymbol{u}^{-t}$ (resp. $\breve{\boldsymbol{u}}^{-t}$), for t large enough, be some sequences in $\mathcal{P}$

[2]There is uhc for $f : X \to X$ when the graph of f is close i.e., for all sequences $\{x_n\}$ and $\{y_n\}$ such that $y_n \in f(x_n)$ for all n, $x_n \to x$, and $y_n \to y$, we have $y \in f(x)$ [10].

converging to the current $\boldsymbol{u}^t$ (resp. $\check{\boldsymbol{u}}^t$). Best response transition rule confirms that : $\check{\boldsymbol{u}}^{t-1} = \Gamma(\boldsymbol{u}^{t-1}) = \boldsymbol{u}^t$. Study such correspondence sequence in best-response process confirms that: $\check{\boldsymbol{u}}^{t-1} = \boldsymbol{u}^t = \Gamma(\dots\Gamma(\Gamma(\boldsymbol{u}^{t=0})))$ after t times of Γ operation. This is somehow obvious to claim that $\check{\boldsymbol{u}}^t \in \Gamma(\boldsymbol{u})$. Then, by the arguments above, all the conditions for the Kakutani fixed point theorem [303] are satisfied, and there exists $\boldsymbol{u}^* \in \mathcal{P}$ such that $\boldsymbol{u}^* \in \Gamma(\boldsymbol{u}^*)$ which corresponds to the best value of P^*_{ij} $\forall\, l_{ij} \in \mathcal{L}$.■

Note that, during the learning process for achieving stable state, the payoffs of all coalitions are equal to $-\infty$ since the conditions (4.3) are not still satisfied. Once all learners achieve the coalition formation stable state, the conditions hold a possible coalition structure $\boldsymbol{\mathcal{M}}$ is formed, but the power dissipated is not necessarily the minimum one. In the next subsection, we propose another "complementary" Markov model which, in each of its time steps τ, executes coalition formation dynamic process and calls the achieved stable state $\omega = (\boldsymbol{u})$. The evolution during time τ leads to a network with the minimum amount of power dissipated.

4.4.2 Power loss minimization dynamic learning process

At the stable state of the coalition formation process, conditions (4.3) hold, a possible coalition structure $\boldsymbol{\mathcal{M}}$ is formed, and each coalition earns a limited payoff. The payoff of each coalition is the inverse of total power dissipated and so higher payoffs for coalitions means lower amount of power dissipated. Obviously, the coalitions' payoffs strictly depend upon the distribution of power over transmission lines, i.e., the achieved coalition formation stable state $\omega = (\boldsymbol{u})$. In general, the payoffs of coalitions and MGs is neither a convex nor a concave function of the loaded power over transmission lines. Hence, it is not possible to find an analytical solution for finding the absolute best payoffs of coalitions. With aim at minimization of the overall power dissipated, we propose a complementary dynamic learning process. In a similar fashion as the coalition formation dynamic learning, we model the evolution of these learning processes as a Markov model $\Pi = \left\{\pi = (\boldsymbol{u}, \rho) \mid \boldsymbol{u} = \{\alpha\}^L, \rho \subset \mathbb{R}^K\right\}$ wherein $\omega = (\boldsymbol{u})$ is the achieved coalition formation stable state. For the second input ρ, we will consider one of the characteristics of the network which is limited and Lebesgue measurable. In the simulation results, we will consider the following two values for ρ and we will compare the performance of them:

(*i*) We will set $\rho = \nu(\mathcal{K}) \in \mathbb{R}$. In fact, in this case we disregard the coalition structure and MGs payoffs, but we aim at maximization of $\nu(\mathcal{K})$ which is, according to (4.5), limited and Lebesgue measurable.

(ii) We will set $\rho = \boldsymbol{\phi} \in \mathbb{R}^K$ where $\boldsymbol{\phi} = \{\phi_k\}$ denotes the MGs Shapley values. In this case, the structure of the coalitions and their payoffs are important.

In the simulation results we will show that maximizing Shapley values outperforms maximizing $\nu(\mathcal{K})$ in terms of power saving. At each time step τ, the central computing unit, first, executes the coalition formation dynamic learning. Once the coalition formation learning achieves a stable state, the complementary Markov model evolution is in the state $\pi^\tau = (\boldsymbol{u}^\tau, \rho^\tau)$, i.e., either $\pi^\tau = (\boldsymbol{u}^\tau, \nu(\mathcal{K})^\tau)$ if we choose (i), or $\pi^\tau = (\boldsymbol{u}^\tau, \boldsymbol{\phi}^\tau)$ if (ii), where $\boldsymbol{u}^\tau$ is the stable state of the coalition formation dynamic learning. At the next time step $\tau + 1$, the CDU re-executes the coalition formation dynamic learning picking different random numbers and achieves a new stable state $\boldsymbol{u}^{\tau+1}$ with a different power distribution and coalition structure. The transition from π^τ to $\pi^{\tau+1}$ occurs if and only if $\rho^{\tau+1}$ dominates ρ^τ, otherwise the Markov model Π stays at the same state π^τ. If we set $\rho = \nu(\mathcal{K})$ the transition occurs when the grand coalition earns a higher payoff, whereas with $\rho = \boldsymbol{\phi}$ it occurs when no MG will get worse off. At time step $\tau + 1$, if the coalition formation learning process happens to achieve the same $\boldsymbol{u}^\tau$, again the complementary Markov model stays at the same state π^τ. Since the values of ρ are limited, with a same discussions in Theorem 7, it is easy to show that the evolution of Π achieves a fixed point for ρ, i.e., for a given time step τ large enough, $\rho^{\tau+1} = \rho^\tau$ for a $\boldsymbol{u}^{\tau+1} \neq \boldsymbol{u}^\tau$. At the fixed point of the complementary dynamic learning process, in case (i) the grand coalition earns the highest possible payoff, and in case (ii) all MGs in $\mathcal{K}$ earn the highest possible Shapley value ϕ_k. This means that the amount of power dissipated is the lowest possible. Obviously, this algorithm can not guarantee the absolute minimum power loss, since the shape of the power loss function strictly depends upon the parameters of the transmission line. For this, the power loss minimization stable point is not necessarily unique and the proposed learning process leads micro-grids to one of the stable points. For the reader's convenience, the power loss minimization algorithm is summarized in Tab. 4.2 and for better readability, the coalition formation dynamic process part is colored blue.

Initialization:

//*Initialization for* $\Pi = \{\pi = (\boldsymbol{u}, \rho)\}$ *Markov model:*

set $\tau = 0$, $\rho^{\tau} = -\infty$, and a tolerance ε;

Power loss minimization process:

while 1

  //*Coalition formation dynamic process:*

  set $t = 0$, $P^{t}_{ij} = 0$; $\forall\, l_{ij} \in \mathcal{L}$ //*Initialization for* $\Omega = \{\omega = (\boldsymbol{u})\}$ *Markov model*

  compute C^{t}_{ij}; $\forall\, l_{ij} \in \mathcal{L}$

  compute $\boldsymbol{u}^{t}$;

  repeat

    execute $P^{t+1}_{ij} = \mathsf{Powerupdate}(l_{ij}, t)$; $\forall\, l_{ij} \in \mathcal{L}$ //*parallel decisions*

    $t = t + 1$;

    compute C^{t}_{ij}; $\forall\, l_{ij} \in \mathcal{L}$

    compute $\boldsymbol{u}^{t}$;

    if $\left(\boldsymbol{u}^{t}\ \text{dominates}\ \boldsymbol{u}^{t-1}\right)$ then //$\omega^{t} = (\boldsymbol{u}^{t})$

      $\omega^{t-1} \longrightarrow \omega^{t}$; //*the transition occurs*

    else

      $\omega^{t} = \omega^{t-1}$; //*discard:* $P^{t}_{ij} = P^{t-1}_{ij}$ *and then* $C^{t}_{ij} = C^{t-1}_{ij}$, $\boldsymbol{u}^{t} = \boldsymbol{u}^{t-1}$

    end

  until $\min\limits_{l_{ij} \in \mathcal{L}} \left(\left(-\varepsilon \le u^{t}_{ij} < 0\right)\ \texttt{or}\ \left(u^{t}_{ij} \ge \alpha - \varepsilon\right)\right)$;

  // $\omega^{t} = (\boldsymbol{u}^{t})$ *is the stable state of the coalition formation dynamic learning process*

  $\tau = \tau + 1$;

  $\boldsymbol{u}^{\tau} = \boldsymbol{u}^{t}$;

  compute ρ^{τ}; //*either* $\rho = \nu(\mathcal{K})$ *or* $\rho = \phi$

  if $\left(\rho^{\tau}\ \text{dominates}\ \rho^{\tau-1}\right)$ then

    $\pi^{\tau-1} \longrightarrow \pi^{\tau}$; //*the transition to* $\pi^{\tau} = (\boldsymbol{u}^{\tau}, \rho^{\tau})$ *occurs*

    if $\left(\mid \rho^{\tau} - \rho^{\tau-1} \mid \le \varepsilon\right)$ then

      break; //*stable state*

    end

  else

    $\pi^{\tau} = \pi^{\tau-1}$; //*discard: transition does not occur*

  end

end

//π^{τ} *is the stable state of the power loss minimization dynamic learning process.*

Tab. 4.2: *Power loss minimization algorithm.*

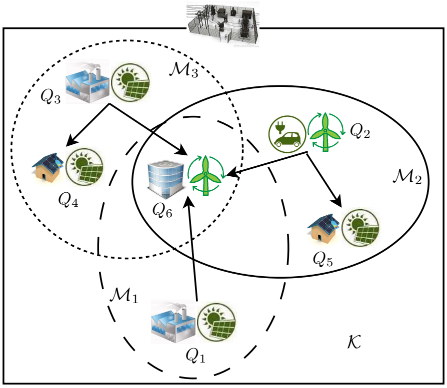


Fig. 4.5: *Snapshot of a formed coalition structure.*

4.5 Numerical results

In this section, we show the performance of the proposed algorithm. Throughout the simulations, we make use of the following transmission lines parameters: $\omega C = 4.518\ \mu\Omega^{-1}/\text{km}$, $\omega L = 367\ \text{m}\Omega/\text{km}$, $R = 37\ \text{m}\Omega/\text{km}$, and $R_1 = 1\ \Omega$ [290, p. 68]. Power transfer among MGs themselves is done at a medium voltage 100 kV, while between the central transmission unit and the MGs that is done at 345 kV [290, p. 68]. The MGs are uniform randomly distributed within an area with radius 100 km and the CDU located at the center. For each MG $k \in \mathcal{K}$, the amount of residual power Q_k is supposed to be a normal random distribution with zero mean and a standard deviation uniformly distributed in the range [100, 500] MW [291]. In the following set of evaluations, based on numerical optimizations, we consider the following parameters in the function Powerupdate: the power update step $\overline{\Delta P}_{sd} = \bar{P}_{sd}/5$, $\overline{\Delta P}_{s0} = \bar{P}_{s0}/10$, and $\overline{\Delta P}_{0d} = \bar{P}_{0d}/10$; the parameter that reduces the probability of conflicting decisions among learners $\lambda_{sd} = 0.7$, $\lambda_{s0} = 0.85$, and $\lambda_{0d} = 0.85$ for $\forall\ s \in \mathcal{K}^+$ and $\forall\ d \in \mathcal{K}^-$. All results are obtained by averaging over 2,000 random realizations of a network with different positions and different values of Q_ks.

Fig. 4.5 reports a snapshot of the achieved coalition formation dynamic learning process stable state in a network consisting of $K = 6$ MGs randomly located in an area with residual quantities Q_ks equal to $\{+150, +350, +200, \ -150, -200, -450\}$ MW, respectively. After one execution of coalition formation dynamic learning process, the power distribution at the coalition formation stable state is as follows: $P_{16} =$

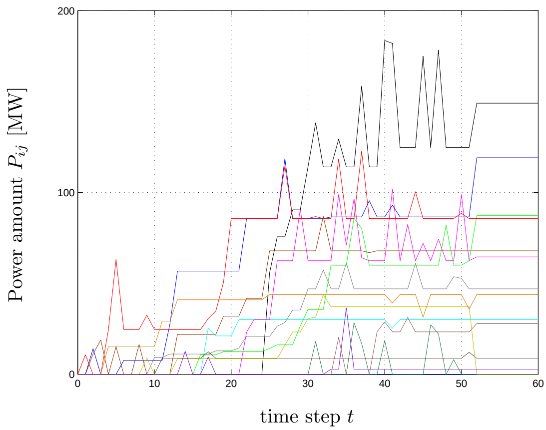


Fig. 4.6: *Learners behaviour during coalition formation process.*

$0.43Q_1$, $P_{25} = 0.38Q_2$, $P_{26} = 0.42Q_2$, $P_{34} = 0.53Q_3$, $P_{36} = 0.47Q_3$ and consequently the coalition structure is $\mathcal{M}_1 = \{1, 6\}$, $\mathcal{M}_2 = \{2, 5, 6\}$, $\mathcal{M}_3 = \{3, 4, 6\}$, and $\mathcal{K}$. The achieved coalitions payoffs are $\{107, 14.9, 29, 9.5\}$ nW^{-1}, respectively, and the Shapley values of the MGs are $\{4.426, -0.430, 0.038, 0.038, -0.430, 5.888\}$ nW^{-1}, respectively. The total amount of power dissipated, $1/\nu(\mathcal{K})$, is 59% of that of the traditional non-cooperative model which is calculated as a network wherein each MG trades only with CDU as in Fig. 4.1(a). Note that here we report the results at the stable point of one execution of coalition formation dynamic learning process only.

Fig. 4.6 exhibits the behaviour of the learners during coalition formation dynamic process. At $t = 0$, all $P_{ij} = 0$ and only the learners of P_{s0} and P_{0d} $\forall\ s \in \mathcal{K}^+$ and $\forall\ s \in \mathcal{K}^+$ may increase the proper power, since they have a negative payoff not equal to $-\alpha$. As can be seen, after some time steps, some other learners earn a utility not equal to $-\alpha$, and so may update their power value. Despite conflicts between simultaneous and myopic decisions, Fig. 4.6 shows the convergence of P_{ij} the stable point after 52 time steps.

Fig. 4.7 reports the average number of time steps τ for achieving the stable point of the power loss minimization process in the network scenario with the above mentioned (fixed) residual quantities. The dashed line reports the grand coalition's payoff and other curves report the Shapley values of the MGs during the dynamic process as a function of τ. As can be seen, when we choose $\rho = \nu(\mathcal{K})$ as the input of the Markov

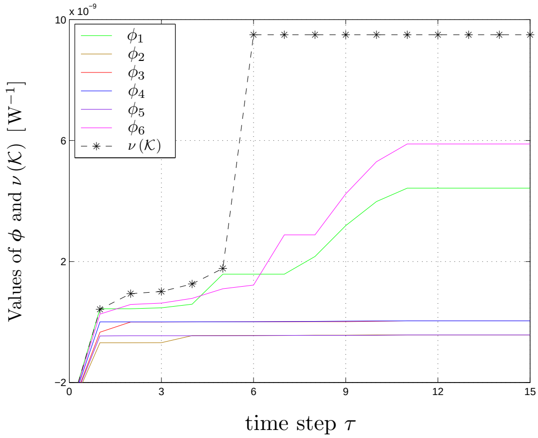


Fig. 4.7: *Shapley values and grand coalition's payoff during power loss minimization dynamic learning.*

state $\pi = (\boldsymbol{u}, \rho)$, the algorithm achieves stable state after 7 time steps, while this happens in $\tau = 12$ with $\rho = \boldsymbol{\phi}$. It is worthwhile to emphasize that, at each time step τ, first coalition formation dynamic learning process is executed (Fig. 4.6), and then the inputs of the state π^τ are updated.

To measure the power dissipated improvement, now we compare the performance of our proposed algorithm to the traditional non-cooperative power transmission. Fig. 4.9 reports the percentage of improvement in terms of average dissipated power amount expressed as the ratio of the whole power dissipated amount D^{NC} achieved by the traditional non-cooperative power transferring to the whole power dissipated amount D^{C} achieved by the proposed coalitional game theory based algorithm. The parameter $D^{NC} = \sum_{s\in\mathcal{K}^+} D_{s0} + \sum_{d\in\mathcal{K}^-} D_{0d}$ with $P_{s0} = Q_s$ and $P_{0d} = -Q_d$, and the parameter $D^{C} = 1/\nu(\mathcal{K})$. As can be seen, the growing rate of the curves slightly decreases after $K = 25$. The average of power loss improvement is 1.2% per MG with setting $\rho = \nu(\mathcal{K})$, while that is 1.5% per MG with $\rho = \boldsymbol{\phi}$. As a result, considering the Shapley values in which the coalition structure and all coalitions' payoffs are involved outperforms setting $\rho = \nu(\mathcal{K})$ in which only the grand coalition payoff is considered, at the cost of more computational complexity. The proposed algorithm in this paper outperforms the proposed coalition formation algorithm in [294].

Fig. 4.8 depicts the average of power loss fraction that is defined as the proportion of

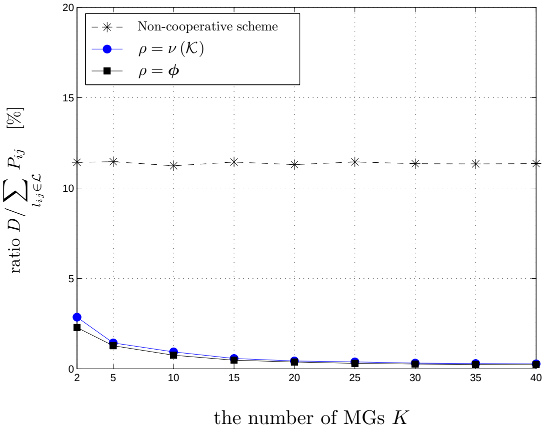


Fig. 4.8: *Average of loss fraction (power dissipated as a proportion of power loaded) as a function of K.*

the overall amount of dissipated power to the overall loaded power. The dashed line reports the traditional non-cooperative scheme with $D = \sum_{s\in\mathcal{K}^+} D_{s0} + \sum_{d\in\mathcal{K}^-} D_{0d}$ by setting $P_{s0} = Q_s$ and $P_{0d} = -Q_d$, and the solid line reports the proposed cooperative scheme with $D = 1/\nu(\mathcal{K})$. In other words, the total power dissipated in the traditional non-cooperative scheme is calculated as a network with K singleton coalitions. The average of the loss fraction in the non-cooperative scheme is 11% and that is not very sensitive to the number of MGs. However, in our cooperative scheme the average of the ratio decreases with the increasing number of MGs and its average is 0.83% with $\rho = \nu(\mathcal{K})$, and 0.69% with $\rho = \boldsymbol{\phi}$. Considering the range of the quantities (in $\pm[100,\ 500]$ MW), the power dissipated in non-cooperative scheme is significant and that reduces significantly in the cooperative scheme. As can be seen, with applying the proposed cooperative algorithm, the power dissipated will become 10% of the traditional distribution networks. As a result, setting $\rho = \boldsymbol{\phi}$ in which the coalition structure and all coalitions' payoffs are involved outperforms setting $\rho = \nu(\mathcal{K})$ in which only the grand coalition payoff is considered, at the cost of more computational complexity. Fig. 4.8 shows also that any value greater than 1.2 is a sufficient value for β in the power constraints $\bar{P}_{sd}$ and $\bar{P}_{0d}$, as we set $\beta = 2$.

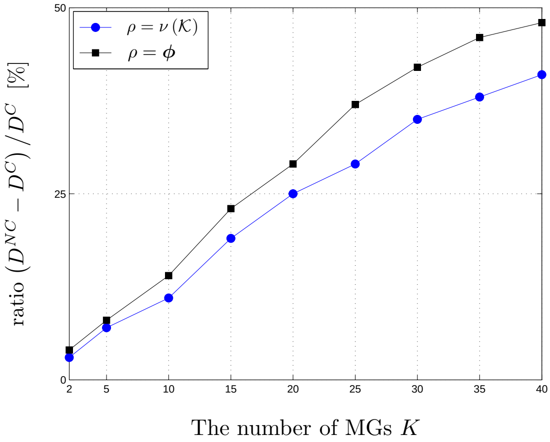


Fig. 4.9: *Percentage of power dissipated improvement as a function of K compared to the traditional power distribution.*

4.6 Conclusion

In this chapter, we have investigated the problem of power trading coordination in a smart power grid consisting of a set of micro-grids (MGs) each of which has a quantity of either surplus or deficit amount of power to sell or to acquire, respectively. The proposed coalitional game theory based approach allows micro-grids (MGs) to form a set of coalitions and trade power between themselves in order to achieve the minimum power dissipated during power generation and transfer. In this context, we propose a dynamic learning process to form a set of non necessarily disjoint and possibly singleton coalitions. In the proposed dynamic learning process each transmission line, as a learner, adjusts the best power load such that: *i*) all suppliers to be completely loaded, and *ii*) all demanders to be completely served. A complementary dynamic learning process is proposed to lead the coalition structure to the best performance in terms of power saving. Numerical results show power dissipated in the proposed cooperative smart grid is 10% of that in the traditional non-cooperative networks.

Chapter 5

Summary and perspective

In this thesis, we have used coalitional game theoretic solutions for resource management in OFDMA-based networks and smart power grids. In OFDMA networks, we studied radio resource allocation techniques in the uplink direction scenario, focusing in particular on the issue of fairness. In smart grids, we studied the problem of power trading coordination among micro-grids with the purpose of minimizing of the amount of dissipated power over transmission lines.

In OFDMA-based networks, the main concern in the identification of the game has been the best utilization of the network resources applying a low complexity algorithm. This has led us to introduce a utility function with which each active wireless terminal achieves its request data rate exactly. This fairness criterion satisfies the expectation of both the wireless service provider and each user's terminal. To cope with the non-convexity and non-concavity of the utility function we proposed a dynamic learning algorithm. The algorithm enforces every terminal to adapt its transmit power to approach the maximum value of the utility function. The proposed algorithm approaches a distribution of transmit powers over different assigned subcarriers for each user from which no terminal wishes to unilaterally deviate (the core set). We showed that the core set coincides with a sort of Nash equilibrium point for cooperative games. The convergence and stability of the algorithm was proved by using Markov modeling, and the complexity of the algorithm favorably compared to the existing literature.

In smart grids, a micro-grid acts as a seller or a buyer depending on its current generation and demand state. To model the cooperative behaviour of micro-grids in smart power grids, we proposed an algorithm based on coalitional game theory. The proposed algorithm aims at minimizing the amount of dissipated power during generation and transfer using an approach that combines coalitional game theory and dynamic learning. In particular, the coalitional game theory-based algorithm finds a set of coalitions of micro-grids that achieves a feasible power distribution condition where all the surplus power from micro-grids is absorbed by the network and, dually, all the deficit power of micro-grids is provided by the network. The dynamic learning algorithm, instead, modifies the set of coalitions in order to minimize the power loss. Members of each coalition will locally trade power among themselves. They can also trade power with the (traditional) central distribution unit, if it is necessary. We modeled the evolution of the algorithm using Markov chain and the convergence of the algorithm is proved using Kakutani fixed point theorem.

Let us draw some future works. In OFDMA-based networks, we plan a cooperative resource allocation game in a multiple-access OFDMA wireless network consisting of multiple terminals and multiple relays. The transmitted symbols by the source nodes reach the base-station and all relays. Each relay decodes/estimates the received signals on poor channels, encodes them, and forwards toward the base-station. The communication is full-duplex mode. The algorithm will consist of subcarrier assignment and power allocation at each node for maximizing the frequency spectral efficiency at the minimum cost of power consumption. At the relays and the base-station each subcarrier is allowed be shared by more terminals. There are different subcarrier assignments at each transmitter, and the relays are not constrained to transmit the same subcarriers over which receive the symbols. There are also separate power constraints on each subcarrier at every nodes. Taking into account the relaying strategy, we will introduce a condition under which source nodes discern the needed subcarriers to be assisted.

In smart grids, we plan to develop an interaction mechanism in which micro-grids can bargain with each other to achieve an agreement on both price and the amount of power. The strategies of each micro-grid correspond to determining the price at which it is willing to buy/sell energy and the quantity that it wishes to sell/buy. The objective of each micro-grid is to determine the optimal quantity and price over each transmission line at which it wants to trade so as to optimize its objective function. Our goal is to achieve a bargaining mechanism with a good tradeoff between the price at which the trade takes place and the amount of overall power loss in the whole network.

Curriculum Vitae

1975 Born on 6 September in Tehran, IRAN

1993-1998 BSc in Computer Hardware Engineering in Tehran

1998-2004 Network programmer, multimedia communications programmer, database analyser and programmer, mini-hardware designer and programmer, and network engineer in different R&D centers in Iran.

2005-2006 MSc in Information & Automation Engineering at the University of Ferrara, Ferrara, ITALY

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Bibliography

[1] A. R. Radwan, P. Marques, and J. Rodriguez, "C2POWER Architecture: a first approach," in *Intern. Conf. on Telecommunications and Multimedia (Temu)*, vol. 1, (Grete, Greece), pp. 1–11, July 2010. 1

[2] M. Pickavet and T. R., "Network solutions to reduce the energy footprint of ICT," in *The European Conf. and Exhibition on Optical Communications (ECOC)*, (Turin, Italy), Sep. 2010. 1

[3] "Global Action Plan - Green ICT - An Inefficient Truth," http://www.greenict.org.uk/reports-library/an-inefficient-truth/. 1

[4] Gómez-Barquero, D., *Next Generation Mobile Broadcasting*. Taylor & Francis, 2013. 1, 2

[5] A. R. Bahai, B. R. Saltzberg, and M. Ergen, *Multi-Carrier Digital Communications Theory and Applications of OFDM*. Boston, MA: Springer, 2nd ed., 2004. 1, 2

[6] A. J. Goldsmith, *Wireless Communications*. Cambridge, UK: Cambridge University Press, 2005. 2

[7] A. Carvallo and J. Cooper, *The Advanced Smart Grid: Edge Power Driving Sustainability*. Artech House, 2nd ed., 2015. 2, 3

[8] S. Bu, F. Yu, Y. Cai, and X. Liu, "When the Smart Grid Meets Energy-Efficient Communications: Green Wireless Cellular Networks Powered by the Smart Grid," *IEEE Trans. on Wireless Communications*, vol. 11, pp. 3014–3024, Aug. 2012. 2, 3

[9] S. Galli, A. Scaglione, and Z. Wang, "For the grid and through the grid: The role of power line communications in the smart grid," *Proc. of the IEEE*, vol. 99, June 2011. 3, 126

[10] M. J. Osborne and A. Rubinstein, *A Course in Game Theory.* Cambridge, MA: MIT Press, 1994. 4, 10, 11, 12, 16, 19, 24, 33, 72, 73, 74, 75, 95, 101, 115, 127

[11] R. Aumann and S. Hart, eds., *Handbook of Game Theory with Economic Applications*, vol. 2. Elsevier, 2 ed., 1995. 4, 80

[12] M. Maschler, "The bargaining set, kernel, and nucleolus," in *Handbook of Game Theory with Economic Applications* (R. Aumann and S. Hart, eds.), vol. 1, ch. 18, pp. 591–667, Elsevier, 1 ed., 1992. 4, 25

[13] H. Raiffa, *Contributions to the Theory of Game II.* Princeton, NJ: Princeton University Press, 1953. 4, 76

[14] M. Félegyházi and J. P. Hubaux, "Game theory in wireless networks: A tutorial," *ACM computing surveys*, 2006. Technical Report: LCA-REPORT-2006-002, EPFL. 4, 26, 34

[15] F. Edalat, *Real-time sub-carrier Adaptive Modulation and Coding in wideband Orthogonal Frequency Division Multiplexing wireless systems.* PhD thesis, Dept. of Electrical Engineering and Computer Science, Massachusetts Institute of Technology (MIT), Boston, USA, 2008. 6

[16] Z. Han, Z. Ji, and K. J. R. Liu, "Non-cooperative resource competition game by virtual referee in multi-cell OFDMA networks," *IEEE Journal on Selected Areas in Commun.*, vol. 25, Aug. 2007. 6, 58, 74, 105

[17] G. Dainelli, M. Moretti, A. Zalonis, and N. Dimitriou, "Distributed subcarrier allocation schemes for OFDMA multiple access systems," in *Proc. Newcom++/Cost2100 Workshop*, (Vienna, Austria), Sept. 2009. 6, 105

[18] T. K. Chee, C.-C. Lim, and J. Choii, "A cooperative game theoretic framework for resource allocation in OFDMA systems," in *Proc. IEEE Intern. Conf. Commun. Systems (ICCS)*, (Singapore, Singapore), Oct. 2006. 6, 11, 77, 106

[19] W. Noh, "A distributed resource control for fairness in OFDMA systems: English-auction game with imperfect information," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, (New Orleans, LA), pp. 1–6, Dec. 2008. 6, 77, 106

[20] M. A. McHenry and D. McCloskeyk, "Spectrum occupancy measurements Chicago, Illinois," Nov. 2005. 10

[21] C. Hyun-Ho, L. Jong Bu, H. Hyosun, and J. Kyunghun, "Optimal handover decision algorithm for throughput enhancement in cooperative cellular networks," in *IEEE Vehicular Technology Conf. (VTC)*, (Ottawa, Canada), pp. 1–5, Sep. 2010. 10

[22] B. Peleg and P. Sudhölter, *Introduction to the Theory of Cooperative Games*. Berlin, Germany: Springer-Verlag, 2nd ed., 2007. 10, 11, 12, 14, 17, 25, 73, 74, 95

[23] W. Saad, Z. Han, T. Basar, and A. Hjorungnes, "Coalitional games for distributed eavesdroppers cooperation in wireless networks," in *Intern. Workshop Game Theory in Communication Networks*, (Pisa, Italy), pp. 3365–3371, Oct. 2009. 11

[24] E. Hossain, D. I. Kim, and V. K. Bhargava, *Cooperative Cellular Wireless Networks*. New York, NY, USA: Cambridge University Press, 2011. 11

[25] S. Hew and L. White, "Cooperative resource allocation games in shared networks: symmetric and asymmetric fair bargaining models," *IEEE Trans. on Wireless Commun.*, vol. 7, pp. 4166–4175, Nov. 2008. 11

[26] F. Shams, G. Bacci, and M. Luise, "An OFDMA resource allocation algorithm based on coalitional games," *EURASIP Journal on Wireless Communications and Networking*, vol. 2011, no. 1, p. 46, 2011. 11, 15, 33

[27] H. Kwon and G. B. Lee, "Cooperative power allocation for broadcast/multicast services in cellular OFDM systems," *IEEE Trans. on Commun.*, vol. 57, pp. 3092–3102, Oct. 2009. 11

[28] E. Zeydan, D. Kivanc, U. Tureli, and C. Comaniciu, "Joint iterative beamforming and power adaptation for MIMO ad hoc networks," *EURASIP Journal on Wireless Communications and Networking*, vol. 2011, no. 1, p. 79, 2011. 11

[29] D. Li, Y. Xu, X. Wang, and M. Guizani, "Coalitional game theoretic approach for secondary spectrum access in cooperative cognitive radio networks," *IEEE Trans. on Wireless Commun.*, vol. 10, pp. 844–856, Mar. 2011. 11

[30] Z. Khan, S. Glisic, L. DaSilva, and J. Lehtomändki, "Modeling the dynamics of coalition formation games for cooperative spectrum sharing in an interference channel," *IEEE Trans. on Computational Intelligence and AI in Games*, vol. 3, pp. 17–30, Mar. 2011. 11

[31] I. Stanojev, O. Simeone, U. Spagnolini, Y. Bar-Ness, and R. Pickholtz, "Cooperative ARQ via auction-based spectrum leasing," *IEEE Trans. on Commun.*, vol. 58, pp. 1843–1856, June 2010. 11

[32] F. Javadi, M. Kibria, and A. Jamalipour, "Bilateral shapley value based cooperative gateway selection in congested wireless mesh networks," in *IEEE Global Telecommun. Conf. (GLOBECOM)*, (New Orleans, LA), pp. 1–5, Dec. 2008. 11

[33] J. Huang, Z. Han, M. Chiang, and H. Poor, "Auction-based resource allocation for multi-relay asynchronous cooperative networks," in *IEEE Intern. Conf. on Acoustics, Speech, and Signal Processing (ICASSP)*, (Las Vegas, Nev.), pp. 5356–5359, Apr. 2008. 11

[34] H. Deng, Y. Wang, and J. Lu, "Auction based resource allocation for balancing efficiency and fairness in OFDMA relay networks with service differentiation," in *IEEE Vehicular Technology Conf. (VTC)*, (Ottawa, Canada), pp. 1–5, Sept. 2010. 11

[35] V. Rodoplu and T. Meng, "Core capacity region of energy-limited, delay-tolerant wireless networks," *IEEE Trans. on Wireless Commun.*, vol. 6, no. 5, pp. 1844–1853, 2007. 13

[36] L. S. Shapley, "Cores of convex games," *International Journal of Game Theory*, vol. 1, pp. 11–26, 1971. 13, 17, 22

[37] A. ParandehGheibi, A. Eryilmaz, A. Ozdaglar, and M. Medard, "Resource allocation in multiple access channels," in *Conf. on Signals, Systems and Computers (ACSSC)*, (Pacific Grove, CA), pp. 878–882, Nov. 2007. 13

[38] D. B. Gillies, *Some theorems on n-person games.* PhD thesis: Department of Mathematics, Princeton University, N.J., 1953. 14

[39] M. M. Madiman, "Cores of cooperative games in information theory," *EURASIP Journal on Wireless Commun. and Networking*, vol. abs/0901.0062, 2009. 15

[40] D. Li, Y. Xu, J. Liu, and X. Wang, "Relay assignment and cooperation maintenance in wireless networks," in *IEEE Wireless Commun. and Networking Conf. (WCNC)*, (Sydney, Australia), pp. 1–5, Apr. 2010. 15

[41] D. Niyato and E. Hossain, "A cooperative game framework for bandwidth allocation in 4G heterogeneous wireless networks," in *IEEE Intern. Conf. on Commun. (ICC)*, vol. 9, (Istanbul, Turkey), pp. 4357–4362, June 2006. 15

[42] T. W. Sandholm and V. R. Lesser, "Coalitions among computationally bounded agents," *Artificial Intelligence*, vol. 94, pp. 99–137, 1997. 15

[43] O. Bondareva, "Some applications of the methods of linear programming to the theory of cooperative games," *Problemy Kibernetiki*, vol. 10, pp. 119–139, 1963, (Russian). 16

[44] L. S. Shapley, "On balanced sets and cores," *Naval Research Logistics Quarterly*, vol. 14, no. 4, pp. 453–560, 1967. 16

[45] A. E. Roth, *The Shapley Value. Essays in Honor of Lloyd S. Shapley.* Cambridge University Press, 1988. 17

[46] J. C. Harsanyi, "An equilibrium-point interpretation of stable sets and a proposed alternative definition," *Management Science*, vol. 20, no. 11, pp. 1472–1495, 1974. 20

[47] C. Rafels and S. Tijs, "On the cores of cooperative games and the stability of the Weber set," *International Journal of Game Theory*, vol. 26, pp. 491–499, 1997. 20

[48] L. S. Shapley, "A value for n-person games. Contribution to the theory of games," *Annals of Mathematics Studies*, vol. 2, p. 28, 1953. 21, 115, 121

[49] S. Hart, "A comparison of non-transferable utility values," discussion paper series, Center for Rationality and Interactive Decision Theory, Hebrew University, Jerusalem, 2003. 22

[50] G.-J. Otten and H. J. Peters, "The Shapley transfer procedure for NTU-games," open access publications from maastricht university, Maastricht University, 2002. 22

[51] H. P. Young, "Monotonic solutions of cooperative games," *International Journal of Game Theory*, vol. 14, pp. 65–72, 1985. 23, 25

[52] S. Kim, "Cooperative game theoretic online routing scheme for wireless network managements," *Commun., IET*, vol. 4, pp. 2074–2083, June 2010. 23

[53] M. Khouzani and S. Sarkar, "Economy of spectrum access in time varying multichannel networks," *IEEE Trans. on Mobile Computing*, vol. 9, pp. 1361–1376, Oct. 2010. 23

[54] H. Park and M. Van-Der Schaar, "Coalition-based resource negotiation for multimedia applications in informationally decentralized networks," *IEEE Trans. on Multimedia*, vol. 11, pp. 765–779, June 2009. 23

[55] S. Pechersky, "On proportional excess for NTU games," Tech. Rep. Ec-02/01, European University at St. Petersburg, Department of Economics, 2001. 24

[56] T. S. H. Driessen, "A note on the inclusion of the kernel in the core of the bilateral assignment game," *International Journal of Game Theory*, vol. 27, no. 2, pp. 301–303, 1998. 24

[57] Z. Han and H. V. Poor, "Coalition games with cooperative transmission: a cure for the curse of boundary nodes in selfish packet-forwarding wireless networks," *IEEE Trans. on Commun.*, vol. 57, pp. 203–213, Jan. 2009. 25

[58] G. Demange and M. Wooders, *Group Formation in Economics: Networks, Clubs, and Coalitions.* UK: Cambridge University Press, 2005. 26

[59] R. J. Aumann, "Acceptable points in general cooperative n-person games," in *Annals of Mathematics Studies, 40, in Contributions to the Theory of Games*, (Princeton, NJ), pp. 287–324, Princeton University Press, 1959. 26, 28

[60] B. D. Bernheim, B. Peleg, and M. D. Whinston, "Coalition-proof Nash equilibria I. concepts," *Journal of Economic Theory*, vol. 42, no. 1, pp. 1–12, 1987. 26, 28, 29, 30

[61] M. Jackson and V. D. Nouweland, "Strongly stable networks," *Games and Economic Behavior*, vol. 51, no. 2, pp. 420–444, 2005. Special Issue in Honor of Richard D. McKelvey. 28

[62] S. Zhong and F. Wu, "A collusion-resistant routing scheme for noncooperative wireless ad hoc networks," *IEEE/ACM Trans. on Networking*, vol. 18, pp. 582–595, Apr. 2010. 29

[63] E. Altman, T. Basar, I. Menache, and H. Tembine, "A dynamic random access game with energy constraints," in *Intern. Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt)*, (Seoul, Korea), pp. 1–7, June 2009. 29

[64] B. Moldovanu, "Coalition-proof Nash equilibria and the core in three-player games," *Games and Economic Behavior*, vol. 4, pp. 565–581, Oct. 1992. 30

[65] H. Konishi, M. Le Breton, and S. Weber, "Equivalence of strong and coalition-proof Nash equilibria in games without spillovers," *Economic Theory*, vol. 9, pp. 97–113, 1997. 31

[66] M. Félegyházi, M. Cagalj, S. Bidokhti, and J.-P. Hubaux, "Non-cooperative multi-radio channel allocation in wireless networks," in *Proc. IEEE Computer and Commun. Societies Conf. (INFOCOM)*, pp. 1442–1450, May 2007. 31

[67] L. Gao, X. Wang, and Y. Xu, "Multiradio channel allocation in multihop wireless networks," *IEEE Trans. on Mobile Computing*, vol. 8, pp. 1454–1468, Nov. 2009. 31

[68] IEEE 802.16 Broadband Wireless Access Working Group, "IEEE 802.16m Evaluation Methodology Document (EMD)," *Tech. Rep. IEEE 802.16m-08/004r5*, Jan. 2009. 32, 102

[69] R. J. Aumann, "Subjectivity and correlation in randomized strategies," *Journal of Mathematical Economics*, vol. 1, no. 1, pp. 67–96, 1974. 33, 35

[70] Y. Heller, *Correlated Equilibrium and Seemingly-Iterational Behavior.* PhD thesis, Faculty of exact sciences, Tel-Aviv University, Israel, 2011. 35, 39

[71] G. Bacci, M. Luise, and H. V. Poor, "Game theory and power control in ultrawideband networks," *Physical Commun.*, vol. 1, no. 1, pp. 21–39, 2008. 35

[72] N. Bonneau, E. Altman, and M. Debbah, "Correlated equilibrium in access control for wireless communications," in *Intern. Conf. on Networking (IFIP)*, (Coimbra, Portugal), pp. 15–19, May 2006. 38

[73] N. Bonneau, M. Debbah, E. Altman, and A. Hjorungnes, "Non-Atomic games for multi-user systems," *IEEE Journal on Selected Areas in Commun.*, vol. 26, pp. 1047–1058, Sept. 2008. 38

[74] M. Charafeddine, Z. Han, A. Paulraj, and J. Cioffi, "Crystallized rates region of the interference channel via correlated equilibrium with interference as noise," in *IEEE Intern. Conf. on Commun. (ICC)*, (Dresden, Germany), pp. 1–6, June 2009. 39

[75] R. Dilts and T. A. Epstein, *Dynamic Learning.* Capitola, CA: MeTa Publications, 1995. 39

[76] M. Van-Der Schaar and F. Fu, "Spectrum access games and strategic learning in cognitive radio networks for delay-critical applications," *Proceedings of the IEEE*, vol. 97, pp. 720–740, Apr. 2009. 39

[77] H.-P. Shiang and M. Van-Der Schaar, "Online learning in autonomic multi-hop wireless networks for transmitting mission-critical applications," *IEEE Journal on Selected Areas in Commun.*, vol. 28, pp. 728–741, June 2010. 39

[78] Z. Lin and M. van der Schaar, "Autonomic and distributed joint routing and power control for delay-sensitive applications in multi-hop wireless networks," *IEEE Trans. on Wireless Commun.*, vol. 10, pp. 102–113, Jan. 2011. 40

[79] C. J. C. H. Watkins and P. Dayan, "Q-Learning," *Machine Learning*, vol. 8, no. 3, pp. 279–292, 1992. 40, 85

[80] C. Claus and C. Boutilier, "The dynamics of reinforcement learning in cooperative multiagent systems," in *National Conf. on Artificial Intelligence (AAAI-98)*, (Madison, Wisconsin), pp. 746–752, July 1998. 40, 41

[81] L. P. Kaelbling, M. L. Littman, and A. W. Moore, "Reinforcement learning: A survey," *Journal of Artificial Intelligence Research*, vol. 4, pp. 237–285, May 1996. 41

[82] E. Kalai and E. Lehrer, "Rational learning leads to Nash equilibrium," *Econometrica*, vol. 61, no. 5, pp. 1019–1045, 1993. 42

[83] D. Fudenberg and D. K. Levine, "Learning and equilibrium," *Annual Review of Economics*, vol. 1, no. 1, pp. 385–420, 2009. 42

[84] P. Milgrom and J. Roberts, "Rationalizability, learning, and equilibrium in games with strategic complementarities," *Econometrica*, vol. 58, pp. 1255–1277, Nov. 1990. 42

[85] C. Daskalakis, R. Frongillo, C. H. Papadimitriou, G. Pierrakos, and G. Valiant, "On learning algorithms for Nash equilibria," in *3rd International Symposium on Algorithmic Game Theory (SAGT)*, (Athens, Greece), pp. 114–125, 2010. 42

[86] A. F. Molisch, *Wireless Communications.* Hoboken, NJ: Wiley-IEEE Press, 2nd ed., 2011. 48, 52, 56

[87] M. Doelz, E. Heald, and D. Martin, "Binary data transmission techniques for linear systems," *Proceedings of the IRE*, vol. 45, pp. 656–661, May 1957. 49

[88] B. Saltzberg, "Performance of an efficient parallel data transmission system," *IEEE Trans. on Commun. Technology*, vol. 15, pp. 805–811, Dec. 1967. 49

[89] R. W. Chang, "Synthesis of band-limited orthogonal signals for multichannel data transmission," *Bell Systems Technical Journal*, vol. 45, pp. 177–1796, Dec. 1966. 49

[90] M. Zimmerman and A. Kirsch, "The AN/GSC-10 (KATHRYN) variable rate data modem for HF radio," *IEEE Trans. on Commun. Technology*, vol. 15, pp. 197–204, Apr. 1967. 49

[91] G. Porter, "Error distribution and diversity performance of a frequency-differential PSK HF modem," *IEEE Trans. on Commun. Technology*, vol. 16, pp. 567–575, Aug. 1968. 49

[92] "Orthogonal Frequency Division Multiplexing." U.S. Patent No. 3,488,4555, field Nov. 14, 1996, issued Jan. 6, 1970. 49

[93] S. Weinstein and P. Ebert, "Data transmission by frequency-division multiplexing using the discrete Fourier transform," *IEEE Trans. on Commun. Technology*, vol. 19, pp. 628–634, Oct. 1971. 49

[94] A. Peled and A. Ruiz, "Frequency domain data transmission using reduced computational complexity algorithms," in *IEEE Intern. Conf. on Acoustics, Speech, and Signal Processing (ICASSP)*, vol. 5, pp. 964–967, Apr. 1980. 50

[95] ANSI T1.413-1998, "Network and customer installation interfaces - asymmetric digital subscriber line (ADSL) metallic interface." (American National Standards Institute 1998). 50, 53

[96] J. A. Bingham, *ADSL, VDSL, and Multicarrier Modulation.* New York, NY, USA: John Wiley & Sons, Inc., 1st ed., 2000. 50

[97] H. Sari, G. Karam, and I. Jeanclaude, "Transmission techniques for digital terrestrial TV broadcasting," *IEEE Commun. Mag.*, vol. 33, pp. 100–109, Feb. 1995. 50

[98] P. Shelswell, "The COFDM modulation system: the heart of digital audio broadcasting," *Electronics Commun. Eng. J.*, vol. 7, pp. 127–136, June 1995. 50

[99] A. Jamalipour, T. Wada, and T. Yamazato, "A tutorial on multiple access technologies for beyond 3G mobile networks," *IEEE Commun. Mag.*, vol. 43, pp. 110–117, Feb. 2005. 50

[100] C. Arasu, P. Hyanki, H. Sharma, G. Lakshminarayanan, M. H. Lee, and S.-B. Ko, "PAPR reduction for improving performance of OFDM system," in *IEEE International Conf. on Communication Control and Computing Technologies (ICCCCT)*, (Tamilnadu, India), pp. 77–82, Oct. 2010. 51

[101] A.-B. Salberg and A. Swami, "Doppler and frequency-offset synchronization in wideband OFDM," *IEEE Transactions on Wireless Commun.*, vol. 4, pp. 2870–2881, Nov. 2005. 51

[102] A. Bo, Y. Zhi-xing, P. Chang-yong, G. Jian-hua, W. Yong, and L. Zhen, "On the synchronization techniques for wireless OFDM systems," *IEEE Trans. on Broadcasting*, vol. 52, pp. 236–244, June 2006. 51

[103] H. Sari and G. Karam, "Orthogonal frequency-division multiple access and its application to CATV networks," *European Trans. on Telecommun.*, vol. 9, no. 6, pp. 507–516, 1998. 51

[104] D. Shen and V.-K. Li, "Stabilized multi-channel ALOHA for wireless OFDM networks," in *IEEE Global Telecommun. Conf. (GLOBECOM)*, vol. 1, (Taipei, Taiwan), pp. 701–705, Nov. 2002. 52

[105] Y.-J. Choi, S. Park, and S. Bahk, "Multichannel random access in OFDMA wireless networks," *IEEE Journal on Selected Areas in Commun.*, vol. 24, no. 3, pp. 603–613, 2006. 52

[106] E. Yaacoub, M. Al-Alaoui, and Z. Dawy, "Novel time-frequency reservation aloha scheme for ofdma systems," in *IEEE Wireless Commun. and Networking Conf. (WCNC)*, (Quintana-Roo, Mexico), pp. 7–12, Mar. 2011. 52

[107] X. Qin and R. Berry, "Distributed power allocation and scheduling for parallel channel wireless networks," in *Intern. Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WIOPT)*, (Trentino, Italy), pp. 77–85, Apr. 2005. 52

[108] X. Qin and R. Berry, "Distributed approaches for exploiting multiuser diversity in wireless networks," *IEEE Trans. on Information Theory*, vol. 52, no. 2, pp. 392–413, 2006. 52

[109] A. Ghosh, R. Ratasuk, B. Mondal, N. Mangalvedhe, and T. Thomas, "LTE-advanced: Next-generation wireless broadband technology," *IEEE Wireless Commun.*, vol. 17, pp. 10–22, June 2010. 52

[110] T. M. Cover and J. A. Thomas, *Elements of information theory.* New York, NY, USA: Wiley-Interscience, second ed., 2006. 53, 54

[111] P. Chow, J. Cioffi, and J. Bingham, "A practical discrete multitone transceiver loading algorithm for data transmission over spectrally shaped channels," *IEEE Trans. on Commun.*, vol. 43, pp. 773–775, Feb./Mar./Apr. 1995. 53, 54, 55, 62

[112] J. Forney, G.D. and M. Eyuboğlu, "Combined equalization and coding using precoding," *IEEE Commun. Magazine*, vol. 29, pp. 25–34, Dec. 1991. 54

[113] D. Hughes-Hartogs, "Ensemble modem structure for imperfect transmission media." U.S Patents Nos. 4,679,227 (July 1988) 4,731,816 (March 1988) and 4,833,796 (May 1989). 54, 55

[114] J. Bingham, "Multicarrier modulation for data transmission: an idea whose time has come," *IEEE Commun. Magazine*, vol. 28, pp. 5–14, May 1990. 54

[115] I. Kalet, "The multitone channel," *IEEE Trans. on Commun.*, vol. 37, pp. 119–124, Feb. 1989. 54, 55

[116] L. J. Cimini, Jr., "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Trans. on Commun.*, vol. 33, pp. 665–675, July 1985. 54, 55

[117] A. Czylwik, "Adaptive OFDM for wideband radio channels," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, vol. 1, (San Francisco, CA), pp. 713–718, Nov. 1996. 55

[118] R. Fischer and J. Huber, "A new loading algorithm for discrete multitone transmission," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, vol. 1, (London, England), pp. 724–728, Nov. 1996. 55

[119] L. Van-der Perre, S. Thoen, P. Vandenameele, B. Gyselinckx, and M. Engels, "Adaptive loading strategy for a high speed OFDM-based WLAN," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, vol. 4, (Sydney, Australia), pp. 1936–1940, Nov. 1998. 55

[120] W. Zou and Y. Wu, "COFDM: an overview," *IEEE Trans. on Broadcasting*, vol. 41, pp. 1–8, Mar. 1995. 56

[121] D. MacKay and R. Neal, "Near shannon limit performance of low density parity check codes," *Electronics Letters*, vol. 32, p. 1645, Aug. 1996. 56

[122] Y. Sun, M. Karkooti, and J. R. Cavallaro, "High throughput, parallel, scalable LDPC encoder/decoder architecture for OFDM systems," in *Proc. IEEE Works. Design, Applications, Integration and Software*, (Dallas, TX), pp. 39–42, Oct. 2006. 56

[123] V. Mannoni, D. Declercq, and G. Gelle, "Optimized irregular low-density parity-check codes for multicarrier modulations over frequency-selective channels," *EURASIP Journal Applied Signal Processing*, vol. 2004, pp. 1546–1556, Jan. 2004. 56

[124] B. Hassibi and B. Hochwald, "Linear dispersion codes," in *Intern. Sympo. on Information Theory (ISIT)*, (Washington, DC), p. 325, June 2001. 56

[125] Z. Liu, Y. Xin, and G. Giannakis, "Linear constellation precoding for OFDM with maximum multipath diversity and coding gains," *IEEE Trans. on Commun.*, vol. 51, no. 3, pp. 416–427, 2003. 56

[126] Z. Wang and G. B. Giannakis, "Wireless multicarrier communications: where Fourier meets Shannon," *IEEE Signal Processing Mag.*, vol. 17, pp. 29–48, May 2000. 56

[127] D. Goeckel and G. Ananthaswamy, "On the design of multidimensional signal sets for OFDM systems," *Communications, IEEE Transactions on*, vol. 50, no. 3, pp. 442–452, 2002. 56

[128] M. McCloud, "Analysis and design of short block OFDM spreading matrices for use on multipath fading channels," *IEEE Trans. on Commun.*, vol. 53, no. 4, pp. 656–665, 2005. 56

[129] N. Prasad, L. Venturino, and X. Wang, "Diversity-multiplexing tradeoff analysis for OFDM systems with subcarrier grouping, linear precoding, and linear detection," *IEEE Trans. on Information Theory*, vol. 56, no. 12, 2010. 56

[130] E. Yaacoub and Z. Dawy, “A survey on uplink resource allocation in OFDMA wireless networks,” *IEEE Commun. Surveys Tutorials*, vol. 14, no. 22, 2012. 57

[131] A. Hamza, S. Khalifa, H. Hamza, and K. Elsayed, “A survey on inter-cell interference coordination techniques in OFDMA-based cellular networks,” *IEEE Communications Surveys Tutorials*, vol. PP, no. 99, pp. 1–29, 2013. 57, 84

[132] M. Salem, A. Adinoyi, M. Rahman, H. Yanikomeroglu, D. Falconer, Y.-D. Kim, E. Kim, and Y.-C. Cheong, “An overview of radio resource management in relay-enhanced OFDMA-based networks,” *IEEE Commun. Surveys Tutorials*, vol. 12, no. 3, pp. 422–438, 2010. 57, 87

[133] K. Akkarajitsakul, E. Hossain, D. Niyato, and D. I. Kim, “Game theoretic approaches for multiple access in wireless networks: A survey,” *IEEE Commun. Surveys & Tutorials*, vol. 13, pp. 372–395, 3rd Quarter 2011. 57, 72

[134] V. Ramamurthi and C. Wei-Peng, “Mobility based MIMO link adaptation in LTE-advanced cellular networks,” in *Intern. Conf. on Broadband, Wireless Computing, Communication and Applications (BWCCA)*, (Fukuoka, Japan), pp. 235–241, Nov. 2010. 57

[135] L. J. Cimini, B. Daneshrad, and N. Sollenberger, “Clustered OFDM with transmitter diversity and coding,” in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, vol. 1, (San Francisco, CA), pp. 703–707, Nov. 1996. 57

[136] M. Johansson, “Benefits of multiuser diversity with limited feedback,” in *IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, (Rome, Italy), pp. 155–159, June 2003. 57

[137] D. Gesbert and M. S. Alouini, “Selective multi-user diversity,” in *Proc. IEEE Intern. Symp. on Signal Processing and Information Technology (ISSPIT)*, (Darmstadt, Germany), pp. 162–165, Dec. 2003. 57

[138] P. Svedman, S. Wilson, J. Cimini, L.J., and B. Ottersten, “A simplified opportunistic feedback and scheduling scheme for OFDM,” in *IEEE Vehicular Technology Conference (VTC)*, vol. 4, (Milan, Italy), pp. 1878–1882, May 2004. 57

[139] Z. Han and K. J. R. Liu, "Fair multiuser channel allocation for OFDMA networks using Nash bargaining solutions and coalitions," *IEEE Trans. on Commun.*, vol. 53, pp. 1366–1376, Aug. 2005. 58, 75, 106

[140] D. Yu, D. Wu, Y. Cai, and W. Zhong, "Power allocation based on power efficiency in uplink OFDMA systems: A game theoretic approach," in *Proc. IEEE Intern. Conf. Commun. Systems (ICCS)*, (Singapore, Singapore), pp. 92–97, Nov. 2008. 58, 73

[141] M. Moretti, N. Dimitriou, G. Danielli, V. Corvino, R. Verdone, F. Brah, L. Vanderdorpe, and V. Tralli, "Channel and interference based cross layer OFDMA scheduling," in *Newcom++/ACORN workshop*, (Barcelona, Spain), pp. 1–5, Mar. 2009. 58, 72

[142] M. Pischella and J. Belfiore, "Distributed margin adaptive resource allocation in MIMO OFDMA networks," *IEEE Trans. on Commun.*, vol. 58, pp. 2371–2380, Aug. 2010. 58

[143] C. Tan, T. Chuah, and S. Tan, "Adaptive multicast scheme for OFDMA based multicast wireless systems," *Electronics Letters, 28th*, vol. 47, pp. 570–572, Apr. 2011. 58

[144] H. Tabassum, Z. Dawy, and M. Alouini, "Sum rate maximization in the uplink of multi-cell OFDMA networks," in *Intern. Wireless Commun. and Mobile Computing Conf. (IWCMC)*, (Istanbul, Turkey), pp. 1152–1157, July 2011. 59

[145] D. Kivanc, G. Li, and H. Liu, "Computationally efficient bandwidth allocation and power control for OFDMA," *IEEE Trans. on Wireless Commun.*, vol. 2, pp. 1150–1158, Nov. 2003. 59

[146] X. Wang, M. Tao, J. Mo, and Y. Xu, "Power and subcarrier allocation for physical-layer security in OFDMA-based broadband wireless networks," *IEEE Trans. on Information Forensics and Security*, vol. 6, pp. 693–702, Sept. 2011. 60

[147] R. Cheng and S. Verdù, "Gaussian multiaccess channels with ISI: capacity region and multiuser water-filling," *IEEE Trans. on Information Theory*, vol. 39, pp. 773–785, May 1993. 60

[148] M. Chiang, *Geometric Programming for Communication Systems.* Hanover, MA: now Publishers Inc., first ed., 2005. ISBN: 1-933019-09-3. 61, 62, 65, 70, 72

[149] A. Goldsmith and P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Trans. on Information Theory*, vol. 43, pp. 1986–1992, Nov. 1997. 61

[150] W. Rhee and J. M. Cioffi, "Increase in capacity of multiuser OFDM system using dynamic subchannel allocation," in *Proc. IEEE Veh. Technol. Conf. (VTC)-Spring*, (Tokyo, Japan), pp. 1085–1089, May 2000. 61, 62, 63, 64

[151] G. Debreu, *Theory of Value: An Axiomatic Analysis of Economic Equilibrium.* John Wiley and Sons Inc., New York, 1959. 62, 63

[152] C. Y. Wong, R. Cheng, K. Lataief, and R. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE Journal on Selected Areas in Commun.*, vol. 17, pp. 1747–1758, Oct. 1999. 62, 63

[153] D. Bertsekas and R. Gallager, *Data Networks.* Prentice Hall, second ed., 1992. ISBN: 0-13-200916-1. 62

[154] W. Yu and J. Cioffi, "FDMA capacity of the Gaussian multiple access channel with ISI," in *IEEE Intern. Conf. on Commun. (ICC)*, vol. 3, (New Orleans, Louisiana), pp. 1365–1369, 2000. 64

[155] F. Kelly, "Charging and rate control for elastic traffic," *European Trans. on Telecommun.*, vol. 8, pp. 33–37, 1997. 64

[156] P. Tsiaflakis, *Resource management and optimization in multi-user DSL systems.* PhD thesis, Department of Electrical Eng., Katholieke Universiteit Leuven, Leuven, Belgium, 2009. 64

[157] I. Toufik and R. Knopp, "Channel allocation algorithms for multi-carrier systems," in *IEEE Vehicular Technology Conf. (VTC)*, vol. 2, (Los Angeles, CA), pp. 1129–1133, Sept. 2004. 64

[158] Kelly F.P. and Maulloo A.K. and Tan D.K.H., "Rate control for communication networks: shadow prices, proportional fairness and stability," *Journal of the Operational Research Society*, vol. 49, no. 3, pp. 237–252, 1998. 65, 66

[159] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication.* Cambridge, UK: Cambridge University Press, first ed., 2005. 66

[160] Z. Shen, J. Andrews, and B. Evans, "Optimal power allocation in multiuser OFDM systems," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, vol. 1, (San Francisco, CA), pp. 337–341, Dec. 2003. 67

[161] I. C. Wong, Z. Shen, B. L. Evans, and J. G. Andrews, "A low complexity algorithm for proportional resource allocation in OFDMA systems," in *IEEE Workshop on Signal Processing Systems (SIPS)*, (Austin, Texas), pp. 1–6, Oct. 2004. 67

[162] Z. Shen, J. Andrews, and B. Evans, "Adaptive resource allocation in multiuser OFDM systems with proportional rate constraints," *IEEE Trans. on Wireless Commun.*, vol. 4, pp. 2726–2737, Nov. 2005. 67

[163] J. Mo and J. Walrand, "Fair end-to-end window-based congestion control," *IEEE/ACM Trans. on Networking*, vol. 8, pp. 556–567, Oct. 2000. 67

[164] J.-W. Lee, R. Mazumdar, and N. Shroff, "Nonconvexity issues for Internet rate control with multiclass services: stability and optimality," in *IEEE Computer and Commun. Societies Conf. (INFOCOM)*, vol. 1, (Hong Kong), p. 4, Mar. 2004. 68

[165] S. Shenker, "Fundamental design issues for the future Internet," *IEEE Journal on Selected Areas in Commun.*, vol. 13, pp. 1176–1188, Sept. 1995. 68

[166] Z. Cao and E. Zegura, "Utility max-min: an application-oriented bandwidth allocation scheme," in *Proc. IEEE Computer and Commun. Societies Conf. (INFOCOM)*, vol. 2, (New York, USA), pp. 793–801, Mar. 1999. 68, 69

[167] J.-W. Cho and S. Chong, "Utility max-min flow control using slope-restricted utility functions," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, vol. 2, (St. Louis, MO), pp. 6 pp. –824, Dec. 2005. 69

[168] A. Dewey and E. Jury, "A stability inequality for a class of nonlinear feedback systems," *IEEE Trans. on Automatic Control*, vol. 11, pp. 54–62, Jan. 1966. 69

[169] J. Huang, V. Subramanian, R. Agrawal, and R. Berry, “Downlink scheduling and resource allocation for OFDM systems,” *IEEE Trans. on Wireless Commun.*, vol. 8, pp. 288–296, Jan. 2009. 69, 72

[170] J. Huang, V. Subramanian, R. Agrawal, and R. Berry, “Joint scheduling and resource allocation in uplink OFDM systems for broadband wireless access networks,” *IEEE Journal on Selected Areas in Commun.*, vol. 27, pp. 226–234, Feb. 2009. 69, 72

[171] C. Zhou, G. Wunder, and T. Michel, “Utility maximization for OFDMA systems over discrete sets,” in *IEEE Intern. Conf. on Commun. (ICC)*, (Glasgow, Scotland), pp. 5726–5731, June 2007. 70

[172] S. Kim and J.-W. Lee, “Joint resource allocation for uplink and downlink in wireless networks: A case study with user-level utility functions,” in *IEEE Vehicular Technology Conf. (VTC)*, (Barcelona, Spain), Apr. 2009. 70

[173] M. Mehrjoo, S. Moazeni, and X. Shen, “An interior point penalty method for utility maximization problems in OFDMA networks,” in *IEEE Intern. Conf. on Commun. (ICC)*, (Dresden, Germany), pp. 1–5, June 2009. 70

[174] G. Miao, N. Himayat, and G. Li, “Energy-efficient link adaptation in frequency-selective channels,” *IEEE trans. on Commun.*, vol. 58, no. 2, pp. 545–554, 2010. 70

[175] C. Xiong, G. Li, S. Zhang, Y. Chen, and S. Xu, “Energy-efficient resource allocation in OFDMA networks,” *IEEE trans. on Commun.*, vol. 60, no. 12, pp. 3767–3778, 2012. 70

[176] C. Isheden, Z. Chong, E. Jorswieck, and G. Fettweis, “Framework for link-level energy efficiency optimization with informed transmitter,” *IEEE trans. on Wireless Commun.*, vol. 11, no. 8, pp. 2946–2957, 2012. 70

[177] A. W. Marshall and I. Olkin, *Inequalities: theory of majorization and its applications.* New York: Academic Press, 1979. 70

[178] I. Stancu-Minasian, *Fractional Programming: Theory, Methods and Applications.* Environment & Policy, Springer, 1997. 70

[179] D. Y. Zhangyu Guan and H. Zhang, "Optimal and fair resource allocation for multiuser wireless multimedia transmissions," *EURASIP Journal on Wireless Communications and Networking*, vol. 2009, no. 17, pp. 1–10, 2009. 70

[180] A. Tölli, *Resource management in cooperative MIMO-OFDM cellular systems.* PhD thesis, Department of Electrical and Information Eng., University of Oulu, Oulu, Finland, 2008. 70

[181] L. Georgiadis, M. J. Neely, and L. Tassiulas, "Resource allocation and cross-layer control in wireless networks," *Found. Trends Networking*, vol. 1, pp. 1–144, Apr. 2006. 71

[182] M. Bohge, J. Gross, A. Wolisz, and M. Meyer, "Dynamic resource allocation in OFDM systems: an overview of cross-layer optimization principles and techniques," *IEEE Network*, vol. 21, pp. 53–59, Jan.-Feb. 2007. 71

[183] E. Baccarelli, M. Biagi, N. Cordeschi, and C. Pelizzoni, "Optimal cross-layer bandwidth adaptation for maximum-throughput VBR media wireless content delivery," in *Proc. IEEE Intern. Conf. Commun. (ICC)*, (Dresden, Germany), pp. 1–6, June 2009. 71

[184] J.-M. Liang, J.-J. Chen, Y.-C. Wang, and Y.-C. Tseng, "A cross-layer framework for overhead reduction, traffic scheduling, and burst allocation in IEEE 802.16 OFDMA networks," *IEEE Trans. on Veh. Technol.*, vol. 60, pp. 1740–1755, May 2011. 71

[185] D. Hui and V. Lau, "Design and analysis of delay-sensitive decentralized cross-layer OFDMA systems with efficient feedback algorithm," *IEEE Trans. on Wireless Commun.*, vol. 8, pp. 5844–5851, Dec. 2009. 71

[186] D. Wang, P. Cosman, and L. Milstein, "Cross layer resource allocation design for uplink video OFDMA wireless systems," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM)*, (Houston, TX), pp. 1–6, Dec. 2011. 71

[187] I. Koutsopoulos and L. Tassiulas, "Cross-layer adaptive techniques for throughput enhancement in wireless OFDM-based networks," *IEEE/ACM Trans. Networking*, vol. 14, pp. 1056–1066, Oct. 2006. 72

[188] G. Song and Y. Li, "Cross-layer optimization for OFDM wireless networks-part I: theoretical framework," *IEEE Trans. on Wireless Commun.*, vol. 4, pp. 614–624, Mar. 2005. 72

[189] G. Song and Y. Li, "Cross-layer optimization for OFDM wireless networks-part II: algorithm development," *IEEE Trans. on Wireless Commun.*, vol. 4, pp. 625–634, Mar. 2005. 72

[190] T. Jiang, L. Song, and Y. Zhang, *Orthogonal Frequency Division Multiple Access Fundamentals and Applications.* Boston, MA, USA: Auerbach Publications, 2010. 72

[191] M. Morelli, M. Moretti, G. Imbarlina, and N. Dimitriou, "Low complexity SNR estimation for transmissions over time-varying flat-fading channels," in *IEEE Wireless Communications and Networking Conf. (WCNC)*, (Budapest, Hungary), pp. 1–4, Apr. 2009. 72

[192] C. M. Yen, C. J. Chang, and L. C. Wang, "A utility-based TMCR scheduling scheme for downlink multiuser MIMO-OFDMA systems," *IEEE Trans. on Vehicular Technology,*, vol. 59, pp. 4105–4115, Oct. 2010. 72

[193] D. Wu, D. Yu, and Y. Cai, "Subcarrier and power allocation in uplink OFDMA systems based on game theory," in *Intern. Conf. on Neural Networks and Signal Processing*, (Nanjing , China), pp. 522–526, June 2008. 73

[194] V. Shah, N. Mandayam, and D. Goodman, "Power control for wireless data based on utility and pricing," in *Proc. IEEE Intern. Symp. Personal, Indoor, and Mobile Radio Commun.*, vol. 3, (Boston, MA), pp. 1427–1432, Sept. 1998. 73

[195] H. Kwon and B. G. Lee, "Distributed resource allocation through noncooperative game approach in multi-cell OFDMA systems," in *IEEE Intern. Conf. on Commun. (ICC)*, vol. 9, (Istanbul, Turkey), pp. 4345–4350, June 2006. 73

[196] C. Tan, L. Sim, T. Chuah, and S. Tan, "Blotto game-based low-complexity fair multiuser subcarrier allocation for uplink OFDMA networks," *EURASIP Journal on Wireless Commun. and Networking*, vol. 2011, no. 1, pp. 1–14, 2011. 74

[197] A. Zappone, G. Alfano, S. Buzzi, and M. Meo, "Energy-efficient non-cooperative resource allocation in multi-cell OFDMA systems with multiple base station antennas," in *IEEE Online Conf. on Green Commun. (GreenCom)*, pp. 82–87, Sept. 2011. 74

[198] G. Bacci, A. Bulzomato, and M. Luise, "Uplink power control and subcarrier assignment for an OFDMA multicellular network based on game theory," in *Proc. Int. Conf. on Performance Evaluation Methodologies and Tools (ValueTools)*, (Paris, France), May 2011. 74

[199] D. Monderer and L. S. Shapley, "Potential games," *Games and Economic Behavior*, vol. 14, no. 1, pp. 124–143, 1996. 74

[200] G. Bacci, L. Sanguinetti, M. Luise, and H. V. Poor, "A game-theoretic approach for energy-efficient contention-based synchronization in OFDMA systems," *IEEE Trans. Signal Processing*, vol. 61, pp. 1258–1271, Mar. 2013. 74

[201] H. Boche and M. Schubert, "A generalization of Nash bargaining and proportional fairness to log-convex utility sets with power constraints," *IEEE Trans. on Information Theory*, vol. 57, no. 6, pp. 3390–3404, 2011. 75

[202] R. Burkard, M. Dell'Amico, and S. Martello, *Assignment Problems*. Philadelphia, PA: Society for Industrial and Applied Mathematics (SIAM), 2009. 75

[203] K.-D. Lee and V. Leung, "Fair allocation of subcarrier and power in an OFDMA wireless Mesh network," *IEEE Journal on Selected Areas in Commun.*, vol. 24, pp. 2051–2060, Nov. 2006. 75

[204] E. Kalai and M. Smorodinsky, "Other solutions to Nash's bargaining problem," *Econometrica*, vol. 4, pp. 513–518, 1975. 76

[205] A. E. Roth, "An impossibility result concerning n-person bargaining games," *International Journal of Game Theory*, vol. 8, pp. 129–132, 1979. 76

[206] J. Chen and A. Swindlehurst, "Applying bargaining solutions to resource allocation in multiuser MIMO-OFDMA broadcast systems," *IEEE Journal of Selected Topics in Signal Processing*, vol. 6, no. 2, pp. 127–139, 2012. 77

[207] S. Alavi, C. Zhou, and W. W. Gen, "Efficient and fair resource allocation scheme for OFDMA networks based on auction game," in *IEEE Veh. Technol. Conf. (VTC)*, (Québec City, Canada), pp. 1–5, Sep. 2012. 77

[208] A. El-Hajj, Z. Dawy, and W. Saad, "A stable matching game for joint uplink/downlink resource allocation in OFDMA wireless networks," in *IEEE Intern. Conf. on Commun. (ICC)*, (Ottawa, Canada), pp. 5354–5359, June 2012. 77

[209] E. G. Larsson and E. A. Jorswieck, "Competition versus cooperation on the MISO interference channel," *IEEE J. Sel. Areas Commun.*, vol. 26, pp. 1059–1069, Sept. 2008. 79

[210] A. Gjendemsjø, D. Gesbert, G. Øien, and S. Kiani, "Binary power control for sum rate maximization over multiple interfering links," *Wireless Communications, IEEE Transactions on*, vol. 7, no. 8, pp. 3164–3173, 2008. 82

[211] L. Venturino, N. Prasad, and X. Wang, "Coordinated linear beamforming in downlink multi-cell wireless networks," *IEEE Trans. on Wireless Commun.*, vol. 9, no. 4, pp. 1451–1461, 2010. 82

[212] W. Yu, T. Kwon, and C. Shin, "Joint scheduling and dynamic power spectrum optimization for wireless multicell networks," in *Conf. on Information Sciences and Systems (CISS)*, (Princeton, NJ), pp. 1–6, Mar. 2010. 82

[213] D. Bilios, C. Bouras, V. Kokkinos, A. Papazois, and G. Tseliou, "A performance study of fractional frequency reuse in OFDMA networks," in *Wireless and Mobile Networking Conf. (WMNC)*, (Paris, France), pp. 38–43, Apr. 2012. 83

[214] M. Amer, "Optimal configuration of fractional frequency reuse system for LTE cellular networks," in *IEEE Veh. Technol. Conf. (VTC)*, (Québec City, Canada), pp. 1–5, Sep. 2012. 83

[215] Huawei, "R1-050507: Soft frequency reuse scheme for UTRAN LTE," in *3GPP TSG RAN WG1 Meeting #41*, May 2005. 83

[216] K. Doppler, C. Wijting, and K. Valkealahti, "Interference aware scheduling for soft frequency reuse," in *IEEE Veh. Technol. Conf. (VTC)*, (Barcelona, Spain), pp. 1–5, Apr. 2009. 83

[217] H. Kwon, W.-I. Lee, and B. G. Lee, "Low-overhead resource allocation with load balancing in multi-cell OFDMA systems," in *IEEE Veh. Technol. Conf. (VTC)*, vol. 5, (Stockholm, Sweden), pp. 3063–3067, May-June 2005. 83

[218] D. Kimura, Y. Harada, and H. Seki, "De-centralized dynamic ICIC using X2 interfaces for downlink LTE systems," in *IEEE Veh. Technol. Conf. (VTC)*, (Budapest), pp. 1–5, May 2011. 83

[219] S.-E. Elayoubi, O. Ben Haddada, and B. Fourestiè, "Performance evaluation of frequency planning schemes in OFDMA-based networks," *IEEE Trans. on Wireless Commun.*, vol. 7, no. 5, pp. 1623–1633, 2008. 83

[220] M. Qian, W. Hardjawana, Y. Li, B. Vucetic, J. Shi, and X. Yang, "Inter-cell interference coordination through adaptive soft frequency reuse in LTE networks," in *Wireless Commun. and Networking Conf. (WCNC)*, (Paris, France), pp. 1618–1623, Apr. 2012. 83

[221] K. Yang, N. Prasad, and X. Wang, "An auction approach to resource allocation in uplink OFDMA systems," *IEEE Trans. on Signal Processing*, vol. 57, pp. 4482–4496, Nov. 2009. 84

[222] Q. D. La, Y. H. Chew, and B. H. Soong, "Performance analysis of downlink multi-cell OFDMA systems based on potential game," *IEEE Trans. on Wireless Commun.*, vol. 11, no. 9, pp. 3358–3367, 2012. 84

[223] S. Alavi, C. Zhou, and W. W. Gen, "Distributed resource allocation scheme for multicell OFDMA networks based on combinatorial auction," in *IEEE Veh. Technol. Conf. (VTC)*, (Québec City, Canada), pp. 1–5, Sep. 2012. 84

[224] S. Buzzi, G. Colavolpe, D. Saturnino, and A. Zappone, "Potential games for energy-efficient power control and subcarrier allocation in uplink multicell OFDMA systems," *IEEE Journal of Selected Topics in Signal Processing*, vol. 6, no. 2, pp. 89–103, 2012. 84

[225] J. Zheng, Y. Cai, and D. Wu, "Subcarrier allocation based on correlated equilibrium in multi-cell OFDMA systems," *EURASIP Journal on Wireless Commun. and Networking*, vol. 2012, no. 1, p. 233, 2012. 84

[226] I. Stiakogiannakis and D. Kaklamani, "A radio resource management framework for multi-user multi-cell OFDMA networks based on game theory," *Wireless Personal Commun.*, vol. 69, no. 2, pp. 745–770, 2013. 84

[227] D. López-Pérez and X. Chu, "Inter-cell interference coordination for expanded region picocells in heterogeneous networks," in *Intern. Conf. on Computer Commun. and Networks (ICCCN)*, (Maui, Hawaii), pp. 1–6, August 2011. 85

[228] T. Lee, J. Yoon, S. Lee, and J. Shin, "Interference management in OFDMA femtocell systems using fractional frequency reuse," in *Intern. Conf. on Commun., Circuits and Systems (ICCCAS)*, (Nice, France), pp. 176–180, Sep. 2010. 85

[229] H. Zhang, X. Chu, W. Ma, W. Zheng, and X. Wen, "Resource allocation with interference mitigation in OFDMA femtocells for co-channel deployment," *EURASIP Journal on Wireless Commun. and Networking*, vol. 2012, no. 1, pp. 1–9, 2012. 85

[230] S. Wang, J. Wang, J. Xu, Y. Teng, and K. Horneman, "Fairness guaranteed cooperative resource allocation in femtocell networks," *Wireless Personal Communications*, pp. 1–17, 2013. 85

[231] C.-H. Ko and H.-Y. Wei, "On-demand resource-sharing mechanism design in two-tier OFDMA femtocell networks," *IEEE Trans. on Veh. Technol.*, vol. 60, no. 3, pp. 1059–1071, 2011. 85

[232] S. Sardellitti, A. Carfagna, and S. Barbarossa, "Optimal resource allocation in fetmocell networks based on Markov modeling of interferers' activity," *EURASIP Journal on Wireless Commun. and Networking*, vol. 2012, p. 371, 2012. 85

[233] A. Galindo-Serrano, L. Giupponi, and G. Auer, "Distributed learning in multiuser OFDMA femtocell networks," in *IEEE Veh. Technol. Conf. (VTC)*, (Budapest), pp. 1–6, May 2011. 85

[234] R. Xie, F. Yu, and H. Ji, "Interference management and power allocation for energy-efficient cognitive femtocell networks," *Mobile Networks and Applications*, pp. 1–13, 2013. 85

[235] Q. Han, K. Ma, X. Wang, X. Guan, and J. Ma, "Stackelberg game based interference management for two-tier femtocell networks," *Wireless Networks*, pp. 1–13, 2013. 85

[236] S. Kim, "Femtocell network power control scheme based on the weighted voting game," *EURASIP Journal on Wireless Commun. and Networking*, vol. 2013, no. 1, p. 44, 2013. 85

[237] D. Karvounas, A. Georgakopoulos, V. Stavroulaki, N. Koutsouris, K. Tsagkaris, and P. Demestichas, "Resource allocation to femtocells for coordinated capacity expansion of wireless access infrastructures," *EURASIP Journal on Wireless Commun. and Networking*, vol. 2012, no. 1, p. 310, 2012. 85

[238] T. Kudo and T. Ohtsuki, "Cell range expansion using distributed Q-learning in heterogeneous networks," *EURASIP Journal on Wireless Commun. and Networking*, vol. 2013, no. 1, p. 61, 2013. 85

[239] L. B. Le, D. Niyato, E. Hossain, D. I. Kim, and D. T. Hoang, "QoS-aware and energy-efficient resource management in OFDMA femtocells," *IEEE Trans. on Wireless Commun.*, vol. 12, no. 1, pp. 180–194, 2013. 85

[240] G. Li and H. Liu, "On the capacity of broadband relay networks," in *Asilomar Conference on Signals, Systems and Computers*, (PACIFIC GROVE, CA, USA), pp. 7–10, Nov. 2004. 86

[241] G. Li and H. Liu, "Resource allocation for OFDMA relay networks with fairness constraints," *IEEE Journal on Selected Areas in Commun.*, vol. 24, no. 11, pp. 2061–2069, 2006. 86

[242] I. Hammerström and A. Wittneben, "On the optimal power allocation for nonregenerative OFDM relay links," in *IEEE Intern. Conf. on Commun. (ICC)*, vol. 10, (Istanbul, Turkey), pp. 4463–4468, June 2006. 86

[243] L. Vandendorpe, R. Duran, J. Louveaux, and A. Zaidi, "Power allocation for OFDM transmisson with DF relaying," in *IEEE Symp. on Commun. and Vehicular Technology in the Benelux*, (Delft, Netherlands), pp. 1–6, Nov. 2007. 86

[244] L. Vandendorpe, J. Louveaux, O. Oguz, and A. Zaidi, “Power allocation for improved DF relayed OFDM transmission: The individual power constraint case,” in *IEEE Intern. Conf. on Commun. (ICC)*, (Dresden, Germany), pp. 1–6, June 2009. 86

[245] W. Dang, M. Tao, H. Mu, and J. Huang, “Subcarrier-pair based resource allocation for cooperative multi-relay OFDM systems,” *IEEE Trans. on Wireless Commun.*, vol. 9, no. 5, pp. 1640–1649, 2010. 86

[246] T. Wang and L. Vandendorpe, “Sum rate maximized resource allocation in multiple DF relays aided OFDM transmission,” *IEEE Journal on Selected Areas in Commun.*, vol. 29, no. 8, pp. 1559–1571, 2011. 86

[247] H. Li, H. Luo, J. Guo, and C. Li, “Dynamic resource allocation in OFDMA-based DF cooperative relay networks,” *Wireless Personal Commun.*, vol. 62, no. 3, pp. 655–670, 2012. 86

[248] M. Zhou, Q. Cui, M. Valkama, and X. Tao, “Energy-efficient resource allocation for OFDMA-based two-way relay channel with physical-layer network coding,” *EURASIP Journal on Wireless Commun. and Networking*, vol. 2012, no. 1, pp. 1–11, 2012. 86

[249] IEEE 802.16j-06/026r4, “Part 16: Air interface for fixed and mobile broadband wireless access systems: Multihop relay specification,” June 2007. 86

[250] E. Weiss, S. Max, O. Klein, G. Hiertz, and B. Walke, “Relay-based vs. conventional wireless networks: Capacity and spectrum efficiency,” in *IEEE Intern. Symp. on Personal, Indoor and Mobile Radio Commun. (PIMRC)*, (Athens, Greece), pp. 1–5, Sep. 2007. 86

[251] R. Pabst, D. Schultz, and B. H. Walke, “Performance evaluation of a relay-based 4G network deployment with combined SDMA/OFDMA and resource partitioning,” in *IEEE Veh. Technol. Conf. (VTC)*, (Marina Bay, Singapore), pp. 2001–2005, May 2008. 86

[252] Y. Pan, A. Nix, and M. Beach, “Distributed resource allocation for OFDMA-based relay networks,” *IEEE Trans. on Veh. Technol.*, vol. 60, no. 3, pp. 919–931, 2011. 86

[253] G. Sidhu, F. Gao, and A. Nallanathan, "A joint resource allocation scheme for multi-relay aided uplink multi-user OFDMA system," in *Intern. conf. on Wireless Commun. and Signal Processing (WCSP)*, (Suzhou, China), pp. 1–6, Oct. 2010. 86

[254] J. A. Han and W. S. Jeon, "Game-theoretic approach to distributed scheduling for relay-aided OFDMA systems," in *IEEE Veh. Technol. Conf. (VTC)*, (Budapest), pp. 1–5, May 2011. 86

[255] L. Jiang, J. Pang, G. Shen, and D. Wang, "A game theoretic channel allocation scheme for multi-user OFDMA relay system," in *IEEE Wireless Commun. and Networking Conf. (WCNC)*, (Quintana-Roo, Mexico), pp. 298–303, Mar. 2011. 86

[256] A. Mukherjee and H. Kwon, "General auction-theoretic strategies for distributed partner selection in cooperative wireless networks," *IEEE Trans. on Commun.*, vol. 58, pp. 2903–2915, Oct. 2010. 86

[257] L. Zhai, H. Ji, X. Li, and Y. Tang, "Coalition graph game for joint relay selection and resource allocation in cooperative cognitive radio networks," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, (Anaheim, CA, USA), pp. 1818–1823, Dec. 2012. 86

[258] J. Joung and S. Sun, "Power efficient resource allocation for downlink OFDMA relay cellular networks," *IEEE Trans. on Signal Processing*, vol. 60, no. 5, pp. 2447–2459, 2012. 86

[259] H. Zhang, X. Wang, Y. Liu, L. Zheng, and T. Bohnert, "Resource allocation for relay-assisted OFDMA systems using inter-cell interference coordination," *EURASIP Journal on Wireless Commun. and Networking*, vol. 2012, no. 1, p. 156, 2012. 86

[260] P. Soldati and M. Johansson, "A low-signalling scheme for distributed resource allocation in multi-cellular OFDMA systems," in *IEEE Global Telecommun. Conf. (GLOBECOM)*, (New Orleans, LA, USA), pp. 1–6, Nov.-Dec. 2008. 87

[261] X. Yu, T. Wu, J. Huang, and Y. Wang, "A non-cooperative game approach for distributed power allocation in multi-cell OFDMA-relay networks," in *IEEE*

Veh. Technol. Conf. (VTC), (Marina Bay, Singapore), pp. 1920–1924, May 2008. 87

[262] Y. Hua, Q. Zhang, and Z. Niu, "Resource allocation in multi-cell OFDMA-based relay networks," in *IEEE Computer and Commun. Societies Conf. (INFOCOM)*, (San Diego, LA, USA), pp. 1–9, Mar. 2010. 87

[263] Z. Hou, Y. Cai, and D. Wu, "Resource allocation based on integer programming and game theory in uplink multi-cell cooperative OFDMA systems," *EURASIP Journal on Wireless Commun. and Networking*, vol. 2011, no. 1, pp. 1–10, 2011. 87

[264] Z. Jin and L. Vandendorpe, "Resource allocation in multi-cellular DF relayed OFDMA systems," in *IEEE Global Telecommun. Conf. (GLOBECOM)*, (Houston, Texas, USA), pp. 1–5, Dec. 2011. 87

[265] Z. Hou, Y. Cai, and D. Wu, "Resource allocation based on integer programming and game theory in uplink multi-cell cooperative OFDMA systems," *EURASIP Journal on Wireless Commun. and Networking*, vol. 2011, no. 1, p. 169, 2011. 87

[266] Z. Jin, T. Wang, J.-B. Wei, and L. Vandendorpe, "Resource allocation for maximizing weighted sum of per cell min-rate in multi-cell DF relay aided downlink OFDMA systems," in *IEEE Inter. Symp. on Personal Indoor and Mobile Radio Commun. (PIMRC)*, (Sydney, Australia), pp. 1845–1850, Sep. 2012. 87

[267] "IEEE 802.22," http://www.ieee802.org/22/. 87

[268] H. Mahmoud, T. Yucek, and H. Arslan, "OFDM for cognitive radio: merits and challenges," *IEEE Wireless Communications*, vol. 16, no. 2, pp. 6–15, 2009. 87

[269] T. Weiss, J. Hillenbrand, A. Krohn, and F. Jondral, "Mutual interference in OFDM-based spectrum pooling systems," in *Veh. Technol. Conf. (VTC)*, vol. 4, (Milan, Italy), pp. 1873–1877, May 2004. 87

[270] Q. Zhao and B. Sadler, "A survey of dynamic spectrum access," *IEEE Signal Processing Magazine*, vol. 24, no. 3, pp. 79–89, 2007. 88

[271] C. Y. Wang, K. T. Hong, and H. Wei, "Nash bargaining solution for cooperative shared-spectrum WLAN networks," in *Proc. IEEE Int. Symp. Personal, Indoor, and Mobile Radio Commun. (PIMRC)*, (Athens, Greece), Sept. 2007. 88

[272] D. Sun and B. Zheng, "A novel sub-carrier and power joint allocation algorithm for multi-user cognitive OFDM," in *IEEE Intern. Conf. on Signal Processing (ICSP)*, (Beijing, China), pp. 1458–1462, Oct. 2010. 88

[273] X. Zhou, B. Wu, P.-H. Ho, and X. Ling, "An efficient power allocation algorithm for OFDM based underlay cognitive radio networks," in *IEEE Global Telecommun. Conf. (GLOBECOM)*, (Houston, TX), pp. 1–5, Dec. 2011. 88

[274] J. Mao, G. Xie, J. Gao, and Y. Liu, "Energy efficiency optimization for OFDM-based cognitive radio systems: A water-filling factor aided search method," *IEEE Trans. on Wireless Commun.*, vol. PP, no. 99, pp. 1–10, 2013. 88

[275] L. P. Qian and Y. Jun, "Monotonic optimization for non-concave power control in multiuser multicarrier network systems," in *IEEE Conf. on Computer Commun. (INFOCOM)*, (Rio de Janeiro, Brazil), pp. 172–180, Apr. 2009. 88

[276] Y. Ma, D. I. Kim, and Z. Wu, "Optimization of OFDMA-based cellular cognitive radio networks," *IEEE trans. on Communications*, vol. 58, no. 8, pp. 2265–2276, 2010. 88

[277] L. Vijayandran, S.-S. Byun, G. Oien, and T. Ekman, "Increasing sum-rate in large-scale cognitive radio networks by centralized power and spectrum allocation," *EURASIP Journal on Wireless Commun. and Networking*, vol. 2012, no. 1, p. 362, 2012. 88

[278] Q. Yang, S. Wang, and M. Ge, "Cooperative resource allocation in OFDM-based multicell cognitive radio systems," in *Intern. Conf. on Computing, Networking and Communications (ICNC)*, (San Diego, USA), pp. 724–728, Jan. 2013. 88

[279] S. Wang, F. Huang, and C. Wang, "Adaptive proportional fairness resource allocation for OFDM-based cognitive radio networks," *Wireless Networks*, vol. 19, no. 3, pp. 273–284, 2013. 88

[280] X. Huang and B. Beferull-Lozano, “Joint optimization of detection and power allocation for OFDM-based cognitive radios,” in *IEEE Global Telecommun. Conf. (GLOBECOM)*, (Miami, Florida, USA), pp. 1–5, Dec. 2010. 88

[281] I. Stupia, L. Vandendorpe, R. Andreotti, and V. Lottici, “A game theoretical approach for coded cooperation in cognitive radio networks,” in *Intern. Sympo. on Commun. Control and Signal Processing (ISCCSP)*, (Rome, Italy), pp. 1–6, May 2012. 88

[282] T. Forde and L. Doyle, “A combinatorial clock auction for OFDMA-based cognitive wireless networks,” in *Intern. Sympo. on Wireless Pervasive Computing (ISWPC)*, (Santorini, Greece), pp. 329–333, May 2008. 88

[283] E. Biglieri, J. Proakis, and S. S. (Shitz), “Fading channels: Information-theoretic and communications aspects,” *IEEE Trans. on Information Theory*, vol. 44, pp. 2619–2692, Oct. 1998. 94

[284] M. J. Osborne and A. Rubinstein, *Bargaining and Markets*. San Diego, CA: Academic Press, 1990. 96

[285] M. Agastya, “Perturbed adaptive dynamics in coalition form games,” *J. Economic Theory*, vol. 89, pp. 207–233, Dec. 1999. 97

[286] J. G. Kemeny and J. L. Snell, *Finite Markov Chains*. Berlin, Germany: Springer-Verlag, 2nd ed., 1976. 97, 99, 100

[287] Y. J. Zhang and K. B. Letaief, “An efficient resource-allocation scheme for spatial multiuser access in MIMO/OFDM systems,” *IEEE Trans. on Communications*, vol. 53, pp. 107–116, Jan. 2005. 105

[288] L. Yang, Z. He, L. Wang, and W. Wu, “A new utility based subcarrier and power joint allocation scheme in uplink OFDMA systems,” in *Proc. IEEE Veh. Technol. Conf. (VTC)*, (Dublin, Ireland), pp. 2756–2760, Apr. 2007. 105

[289] T. Wang and L. Vandendorpe, “Resource allocation for maximizing weighted sum min-rate in downlink cellular OFDMA systems,” in *Proc. IEEE Int. Conf. Commun.*, (Cape Town, South Africa), May 2010. 106

[290] J. Machowski, J. W. Bialek, and J. R. Bumby, *Power System Dynamics: Stability and Control.* New York, USA: Wiley, 2 ed., 2008. 114, 116, 118, 131

[291] E. J. Coster, "Distribution grid operation including distributed generation," *Eindhoven University of Technology*, 2010. 114, 131

[292] L. Ochoa and G. Harrison, "Minimizing energy losses: Optimal accommodation and smart operation of renewable distributed generation," *IEEE Trans. on Power Systems*, vol. 26, pp. 198–205, Feb. 2011. 114

[293] P. Tenti, A. Costabeber, P. Mattavelli, and D. Trombetti, "Distribution loss minimization by token ring control of power electronic interfaces in residential micrograds," *IEEE Trans. on Industrial Electronics*, vol. 59, pp. 3817–3826, Oct. 2012. 114

[294] W. Saad, Z. Han, and H. Poor, "Coalitional game theory for cooperative micro-grid distribution networks," in *IEEE Intern. Conf. on Communications Workshops (ICC)*, (Kyoto, Japan), pp. 1–5, June 2011. 114, 115, 116, 118, 123, 133

[295] C. Wei, Z. Fadlullah, N. Kato, and A. Takeuchi, "GT-CFS: A game theoretic coalition formulation strategy for reducing power loss in micro grids," *IEEE Trans. on Parallel and Distributed Systems*, vol. 25, pp. 2307–2317, Sept. 2014. 114, 115, 116, 118, 123

[296] H. Akhavan-Hejazi and H. Mohsenian-Rad, "A stochastic programming framework for optimal storage bidding in energy and reserve markets," in *IEEE PES Conf. Innovative Smart Grid Technologies (ISGT)*, (Washington DC, USA), Feb. 2013. 115

[297] J. Lassila, J. Haakana, V. Tikka, and J. Partanen, "Methodology to analyze the economic effects of electric cars as energy storages," *IEEE Trans. on Smart Grid*, vol. 3, Mar. 2012. 115

[298] Y. Wang, W. Saad, Z. Han, H. Poor, and T. Basar, "A game-theoretic approach to energy trading in the smart grid," *IEEE Trans. on Smart Grid*, vol. 5, pp. 1439–1450, May 2014. 115

[299] A.-H. Mohsenian-Rad, V. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia, "Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid," *IEEE Trans. on Smart Grid*, vol. 1, pp. 320–331, Dec. 2010. 115, 118

[300] P. Vytelingum, S. Ramchurn, T. Voice, A. Rogers, and N. Jennings, "Agent-based modeling of smart-grid market operations," in *IEEE Power and Energy Society General Meeting*, (Detroit Michigan, USA), pp. 1–8, July 2011. 115

[301] F. Shams and M. Luise, "Basics of coalitional games with applications to communications and networking," *EURASIP Journal on Wireless Communications and Networking*, vol. 2013, no. 1, 2013. 115, 119

[302] W. Saad, Z. Han, H. Poor, and T. Basar, "Game-theoretic methods for the smart grid: An overview of microgrid systems, demand-side management, and smart grid communications," *IEEE Signal Processing Magazine*, vol. 29, pp. 86–105, Sept. 2012. 115

[303] S. Kakutani, "A generalization of Brouwer's fixed point theorem," *Duke Mathematical Journal*, vol. 8, no. 3, pp. 457–459, 1941. 116, 128

[304] N. Yahyasoltani, *Dynamic learning and resource management under uncertainties for smart grid and cognitive radio networks.* PhD thesis, Department of Computer Engineering, University of Minnesota, USA, 2014. 122