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## Antibunched Photons Emitted by a Quantum Point Contact out of Equilibrium

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Motivated by the experimental search for “GHz nonclassical light,” we identify the conditions under which current fluctuations in a narrow constriction generate sub-Poissonian radiation. Antibunched electrons generically produce bunched photons, because the same photon mode can be populated by electrons decaying independently from a range of initial energies. Photon antibunching becomes possible at frequencies close to the applied voltage  $V \times e/\hbar$ , when the initial energy range of a decaying electron is restricted. The condition for photon antibunching in a narrow frequency interval below  $eV/\hbar$  reads  $[\sum_n T_n(1 - T_n)]^2 < 2\sum_n [T_n(1 - T_n)]^2$ , with  $T_n$  an eigenvalue of the transmission matrix. This condition is satisfied in a quantum point contact, where only a single  $T_n$  differs from 0 or 1. The photon statistics is then a superposition of binomial distributions.

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In a recent experiment [1], Gabelli *et al.* have measured the deviation from Poisson statistics of photons emitted by a resistor in equilibrium at mK temperatures. By cross correlating the power fluctuations they detected photon bunching, meaning that the variance  $\text{Var}n = \langle n^2 \rangle - \langle n \rangle^2$  in the number of detected photons exceeds the mean photon count  $\langle n \rangle$ . Their experiment is a variation on the quantum optics experiment of Hanbury Brown and Twiss [2], but now at GHz frequencies.

In the discussion of the implications of their novel experimental technique, Gabelli *et al.* noticed that a general theory [3] for the radiation produced by a conductor out of equilibrium implies that the deviation from Poisson statistics can go either way: Super-Poissonian fluctuations ( $\text{Var}n > \langle n \rangle$ , signaling bunching) are the rule in conductors with a large number of scattering channels, while sub-Poissonian fluctuations ( $\text{Var}n < \langle n \rangle$ , signaling antibunching) become possible in few-channel conductors. They concluded that a quantum point contact could therefore produce GHz nonclassical light [4].

It is the purpose of this work to identify the conditions under which electronic shot noise in a quantum point contact can generate antibunched photons. The physical picture that emerges differs in one essential aspect from electron-hole recombination in a quantum dot or quantum well, which is a familiar source of sub-Poissonian radiation [5–7]. In those systems the radiation is produced by transitions between a few discrete levels. In a quantum point contact the transitions cover a continuous range of energies in the Fermi sea. As we will see, this continuous spectrum generically prevents antibunching, except at frequencies close to the applied voltage.

Before presenting a quantitative analysis, we first discuss the mechanism in physical terms. As depicted in Fig. 1, electrons are injected through a constriction in an energy range  $eV$  above the Fermi energy  $E_F$ , leaving behind holes at the same energy. The statistics of the

charge  $Q$  transferred in a time  $\tau \gg \hbar/eV$  is binomial [8], with  $\text{Var}Q/e < \langle Q/e \rangle$ . This electron antibunching is a result of the Pauli principle. Each scattering channel  $n = 1, 2, \dots, N$  in the constriction and each energy interval  $\delta E = \hbar/\tau$  contributes independently to the charge statistics. The photons excited by the electrons would inherit the antibunching if there would be a one-to-one correspondence between the transfer of an electron and the population of a photon mode. Generically, this is not what happens: A photon of frequency  $\omega$  can be excited by each scattering channel and by a range  $eV - \hbar\omega$  of initial energies. The resulting statistics of photocounts is negative-binomial [3], with  $\text{Var}n > \langle n \rangle$ . This is the same photon bunching as in black-body radiation [9].

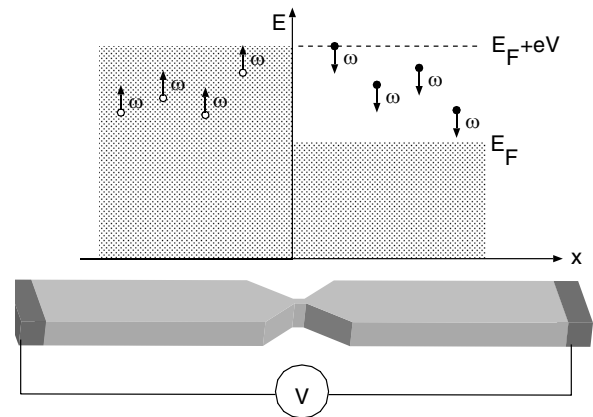


FIG. 1. Schematic diagram of a constriction in a conductor (bottom) and the energy range of electronic states (top), showing excitations of electrons (black dots) and holes (white dots) in the Fermi sea. A voltage  $V$  drops over the constriction. Electrons (holes) in an energy range  $eV - \hbar\omega$  can populate a photon mode of frequency  $\omega$ , by decaying to an empty (filled) state closer to the Fermi level.

In order to convert antibunched electrons into antibunched photons, it is sufficient to ensure a one-to-one correspondence between electron modes and photon modes. This can be realized by concentrating the current fluctuations in a single scattering channel and by restricting the energy range  $eV - \hbar\omega$ . Indeed, in a single-channel conductor and in a narrow frequency range  $\omega \lesssim eV/\hbar$  we obtain sub-Poissonian photon statistics regardless of the value of the transmission probability. In the more general multichannel case, photon antibunching is found if  $[\sum_n T_n(1 - T_n)]^2 < 2\sum_n [T_n(1 - T_n)]^2$  (with  $T_n$  an eigenvalue of the transmission matrix product  $tt^\dagger$ ).

The starting point of our quantitative analysis is the general relationship of Ref. [3] between the photocount distribution  $P(n)$  and the expectation value of an ordered exponential of the electrical current operator:

$$P(n) = \frac{1}{n!} \lim_{\xi \rightarrow -1} \frac{d^n}{d\xi^n} F(\xi), \quad (1)$$

$$F(\xi) = \left\langle \mathcal{O} \exp \left[ \xi \int_0^\infty d\omega \gamma(\omega) I^\dagger(\omega) I(\omega) \right] \right\rangle. \quad (2)$$

We summarize the notation. The function  $F(\xi) = \sum_{k=0}^\infty (\xi^k/k!) \langle n^k \rangle_f$  is the generating function of the factorial moments  $\langle n^k \rangle_f \equiv \langle n(n-1)(n-2)\cdots(n-k+1) \rangle$ . The current operator  $I = I_{\text{out}} - I_{\text{in}}$  is the difference of the outgoing current  $I_{\text{out}}$  (away from the constriction) and the incoming current  $I_{\text{in}}$  (toward the constriction). The symbol  $\mathcal{O}$  indicates ordering of the current operators from left to right in the order  $I_{\text{in}}^\dagger, I_{\text{out}}^\dagger, I_{\text{out}}, I_{\text{in}}$ . The real frequency-dependent response function  $\gamma(\omega)$  is proportional to the coupling strength of conductor and photodetector and proportional to the detector efficiency. Positive (negative)  $\omega$  corresponds to absorption (emission) of a photon by the detector. We consider photodetection by absorption, hence  $\gamma(\omega) \equiv 0$  for  $\omega \leq 0$ . Integrals over frequency should be interpreted as sums over discrete modes  $\omega_p = p \times 2\pi/\tau$ ,  $p = 1, 2, 3, \dots$ . The detection time  $\tau$  is sent to infinity at the end of the calculation. We denote  $\gamma_p = \gamma(\omega_p) \times 2\pi/\tau$ , so that  $\int d\omega \gamma(\omega) \rightarrow \sum_p \gamma_p$ . For ease of notation we set  $\hbar = 1$ ,  $e = 1$ .

The exponent in Eq. (2) is quadratic in the current operators, which complicates the calculation of the expectation value. We remove this complication by introducing a Gaussian field  $z(\omega)$  and performing a Hubbard-Stratonovich transformation,

$$F(\xi) = \left\langle \mathcal{O} \exp \left[ \sqrt{\xi} \int_0^\infty d\omega \gamma(\omega) (z(\omega) I^\dagger(\omega) + z^*(\omega) I(\omega)) \right] \right\rangle. \quad (3)$$

The angular brackets now indicate both a quantum mechanical expectation value of the current operators and a classical average over independent complex Gaussian

variables  $z_p = z(\omega_p)$  with zero mean and variance  $\langle |z_p|^2 \rangle = 1/\gamma_p$ .

We assume zero temperature, so that the incoming current is noiseless. We may then replace  $I$  by  $I_{\text{out}}$  and restrict ourselves to energies  $\varepsilon$  in the range  $(0, V)$  above  $E_F$ . Let  $b_n^\dagger(\varepsilon)$  be the operator that creates an outgoing electron in scattering channel  $n$  at energy  $\varepsilon$ . The outgoing current is given in terms of the electron operators by

$$I_{\text{out}}(\omega) = \int_0^V d\varepsilon \sum_n b_n^\dagger(\varepsilon) b_n(\varepsilon + \omega). \quad (4)$$

Energy  $\varepsilon_p = p \times 2\pi/\tau$  is discretized in the same way as frequency. The energy and channel indices  $p, n$  are collected in a vector  $b$  with elements  $b_{pn} = (2\pi/\tau)^{1/2} b_n(\varepsilon_p)$ . Substitution of Eq. (4) into Eq. (3) gives

$$F(\xi) = \langle e^{b^\dagger Z b} e^{b^\dagger Z^\dagger b} \rangle. \quad (5)$$

The exponents contain the product of the vectors  $b, b^\dagger$  and a matrix  $Z$  with elements  $Z_{pn, p'n'} = \xi^{1/2} \delta_{nn'} z_{p-p'} \gamma_{p-p'}$ . Notice that  $Z$  is diagonal in the channel indices  $n, n'$  and lower-triangular in the energy indices  $p, p'$ .

Because of the ordering  $\mathcal{O}$  of the current operators, the single exponential of Eq. (3) factorizes into the two non-commuting exponentials of Eq. (5). In order to evaluate the expectation value efficiently, we would like to bring this back to a single exponential—but now with normal ordering  $\mathcal{N}$  of the fermion creation and annihilation operators. (Normal ordering means  $b^\dagger$  to the left of  $b$ , with a minus sign for each permutation.) This is accomplished by means of the operator identity [10]

$$\prod_i e^{b^\dagger A_i b} = \mathcal{N} \exp \left[ b^\dagger \left( \prod_i e^{A_i} - 1 \right) b \right], \quad (6)$$

valid for any set of matrices  $A_i$ . The quantum mechanical expectation value of a normally ordered exponential is a determinant [11],

$$\langle \mathcal{N} e^{b^\dagger A b} \rangle = \text{Det}(1 + AB), \quad B_{ij} = \langle b_j^\dagger b_i \rangle. \quad (7)$$

In our case  $A = e^Z e^{Z^\dagger} - 1$  and  $B = tt^\dagger$ , with  $t$  the  $N \times N$  transmission matrix of the constriction.

In the experimentally relevant case [1,12] the response function  $\gamma(\omega)$  is sharply peaked at a frequency  $\Omega \lesssim V$ , with a width  $\Delta \ll \Omega$ . We assume that the energy dependence of the transmission matrix may be disregarded on the scale of  $\Delta$ , so that we may choose an  $\varepsilon$ -independent basis in which  $tt^\dagger$  is diagonal. The diagonal elements are the transmission eigenvalues  $T_1, T_2, \dots, T_N \in (0, 1)$ . Combining Eqs. (5)–(7) we arrive at

$$F(\xi) = \left\langle \prod_{n=1}^N \text{Det}[1 + T_n(e^Z e^{Z^\dagger} - 1)] \right\rangle \\ = \left\langle \prod_{n=1}^N \text{Det}[(1 - T_n)e^{-Z^\dagger} + T_n e^Z] \right\rangle. \quad (8)$$

(In the second equality we used that  $\text{Det}e^{Z^\dagger} = 1$ , since  $Z$  is a lower-triangular matrix.) The remaining average is over the Gaussian variables  $z_p$  contained in the matrix  $Z$ .

Since the interesting new physics occurs when  $\Omega$  is close to  $V$ , we simplify the analysis by assuming that  $\gamma(\omega) \equiv 0$  for  $\omega < V/2$ . For such a response function one has  $Z^2 = 0$ . (This amounts to the statement that no electron with excitation energy  $\varepsilon < V$  can produce more than a single photon of frequency  $\omega > V/2$ .) We may therefore replace  $e^Z \rightarrow 1 + Z$  and  $e^{-Z^\dagger} \rightarrow 1 - Z^\dagger$  in Eq. (8). We then apply the matrix identity

$$\text{Det}(1 + A + B) = \text{Det}(1 - AB), \quad \text{if } A^2 = 0 = B^2, \quad (9)$$

and obtain

$$F(\xi) = \prod_p \frac{\gamma_p}{\pi} \int d^2 z_p e^{-\gamma_p |z_p|^2} \times \prod_{n=1}^N \text{Det}[1 + T_n(1 - T_n)\xi X]. \quad (10)$$

We have defined  $\xi X \equiv ZZ^\dagger$  and written out the Gaussian average. The Hermitian matrix  $X$  has elements

$$X_{pp'} = \sum_q z_{p-q} z_{p'-q}^* \gamma_{p-q} \gamma_{p'-q}. \quad (11)$$

The integers  $p, p', q$  range from 1 to  $V\tau/2\pi$ .

The Gaussian average is easy if the dimensionless shot noise power  $S = \sum_n T_n(1 - T_n)$  is  $\gg 1$ . We may then do the integrals of Eq. (10) in saddle-point approximation, with the result [13]

$$\ln F(\xi) = -\frac{\tau}{2\pi} \int_0^V d\omega \ln[1 - \xi S \gamma(\omega)(V - \omega)]. \quad (12)$$

The logarithm  $\ln F(\xi)$  is the generating function of the factorial cumulants  $\langle\langle n^k \rangle\rangle_f$  [14]. By expanding Eq. (12) in powers of  $\xi$  we find

$$\langle\langle n^k \rangle\rangle_f = (k-1)! \frac{\tau}{2\pi} \int_0^V d\omega [S \gamma(\omega)(V - \omega)]^k. \quad (13)$$

Equations (12) and (13) represent the multimode superposition of independent negative-binomial distributions [9]. All factorial cumulants are positive, in particular, the second, so  $\text{Varn} > \langle n \rangle$ . This is super-Poissonian radiation.

When  $S$  is not  $\gg 1$ , e.g., when only a single-channel contributes to the shot noise, the result (12) and (13) remains valid if  $V - \Omega \gg \Delta$ . This was the conclusion of Ref. [3], that narrow-band detection leads generically to a negative-binomial distribution. However, the saddle-

point approximation breaks down when the detection frequency  $\Omega$  approaches the applied voltage  $V$ . For  $V - \Omega \lesssim \Delta$  one has to calculate the integrals in Eq. (10) exactly.

We have evaluated the generating function (10) for a response function of the block form

$$\gamma(\omega) = \begin{cases} \gamma_0 & \text{if } V - \Delta < \omega < V, \\ 0 & \text{if } \omega < V - \Delta, \end{cases} \quad (14)$$

with  $\Delta < V/2$ . The frequency dependence for  $\omega > V$  is irrelevant. In the case  $N = 1$  of a single channel, with transmission probability  $T_1 \equiv T$ , we find [15]

$$\ln F(\xi) = \frac{\tau}{2\pi} \int_{V-\Delta}^V d\omega \ln[1 + \xi \gamma_0 T(1 - T)(V - \omega)] \\ = \frac{\tau \Delta}{2\pi} \frac{(1+x) \ln(1+x) - x}{x}, \quad (15)$$

with  $x \equiv \xi \gamma_0 T(1 - T)\Delta$ . This is a superposition of binomial distributions. The factorial cumulants are

$$\langle\langle n^k \rangle\rangle_f = (-1)^{k+1} \frac{(k-1)!}{k+1} \frac{\tau \Delta}{2\pi} [T(1 - T)\gamma_0 \Delta]^k. \quad (16)$$

The second factorial cumulant is negative, so  $\text{Varn} < \langle n \rangle$ . This is sub-Poissonian radiation.

We have not found such a simple closed form expression in the more general multichannel case, but it is straightforward to evaluate the low-order factorial cumulants from Eq. (10). We find

$$\langle n \rangle = \frac{\tau \Delta}{2\pi} \gamma_0 \Delta \frac{1}{2} S_1, \quad (17)$$

$$\langle\langle n^2 \rangle\rangle_f = \frac{\tau \Delta}{2\pi} (\gamma_0 \Delta)^2 \frac{1}{3} (S_1^2 - 2S_2), \quad (18)$$

$$\langle\langle n^3 \rangle\rangle_f = \frac{\tau \Delta}{2\pi} (\gamma_0 \Delta)^3 \frac{1}{6} (3S_1^3 - 15S_1 S_2 + 15S_3), \quad (19)$$

with  $S_p = \sum_n [T_n(1 - T_n)]^p$ . Antibunching therefore requires  $S_1^2 < 2S_2$ .

The condition on antibunching can be generalized to arbitrary frequency dependence of the response function  $\gamma(\omega)$  in the range  $V - \Delta < \omega < V$  of detected frequencies. For  $\Delta < V/2$  we find

$$\text{Varn} - \langle n \rangle = \frac{\tau}{2\pi} \int_{V-\Delta}^V d\omega' \gamma(\omega') \int_{\omega'}^V d\omega (V - \omega) \\ \times \left[ 2S_1^2 - 4S_2 - (V - \omega) S_1^2 \frac{d}{d\omega} \right] \gamma(\omega). \quad (20)$$

We see that the antibunching condition  $S_1^2 < 2S_2$  derived for the special case of the block function (14) is more generally a sufficient condition for antibunching to occur, provided that  $d\gamma/d\omega \geq 0$  in the detection range. It does not matter if the response function drops off at  $\omega > V$ ,

provided that it increases monotonically in the range  $(V - \Delta, V)$ . A steeply increasing response function in this range is more favorable, but not by much. For example, the power law  $\gamma(\omega) \propto (\omega - V + \Delta)^p$  gives the antibunching condition  $S_1^2 < 2S_2 \times [1 + p/(1 + p)]$ , which is only weakly dependent on the power  $p$ .

In conclusion, we have presented both a qualitative physical picture and a quantitative analysis for the conversion of electron to photon antibunching. A simple criterion, Eq. (18), is obtained for sub-Poissonian photon statistics, in terms of the transmission eigenvalues  $T_n$  of the conductor. Since an  $N$ -channel quantum point contact has only a single  $T_N$  different from 0 or 1, it should generate antibunched photons in a frequency band  $(V - \Delta, V)$ —regardless of the value of  $T_N$ . The statistics of these photons is the superposition (15) of binomial distributions, inherited from the electronic binomial distribution. There are no stringent conditions on the band width  $\Delta$ , as long as it is  $< V/2$  (in order to prevent multiphoton excitations by a single electron [16]). This should make it feasible to use the cross-correlation technique of Ref. [1] to detect the emission of nonclassical microwaves by a quantum point contact.

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- [9] The *negative-binomial* distribution  $P(n) \propto \binom{n+\nu-1}{n} [\nu/\langle n \rangle + 1]^{-n}$  counts the number of partitions of  $n$  bosons among  $\nu = \tau\delta\omega/2\pi$  states in a frequency interval  $\delta\omega$ . The *binomial* distribution  $P(n) \propto \nu n [\nu/\langle n \rangle - 1]^{-n}$  counts the number of partitions of  $n$  fermions among  $\nu$  states.
- [10] Equation (6) is the multimatrix generalization of the well-known identity  $\exp(b^\dagger A b) = \mathcal{N} \exp[b^\dagger (e^A - 1)b]$ .
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- [12] R. Aguado and L. P. Kouwenhoven, Phys. Rev. Lett. **84**, 1986 (2000).
- [13] The saddle point is at  $z_p = 0$ , so to integrate out the Gaussian fluctuations around the saddle point we may linearize the determinant in Eq. (10):  $\prod_n \text{Det}[1 + T_n(1 - T_n)\xi X] = \exp[\xi S \text{Tr} X + \mathcal{O}(X^2)]$ . The result is Eq. (12).
- [14] Factorial cumulants are constructed from factorial moments in the usual way. The first two are  $\langle\langle n \rangle\rangle_f = \langle n \rangle$ ,  $\langle\langle n^2 \rangle\rangle_f = \langle n^2 \rangle_f - \langle n \rangle^2 = \text{Varn} - \langle n \rangle$ .
- [15] Using computer algebra, we find that  $\ln \langle \text{Det}[1 + \xi T(1 - T)X] \rangle = \sum_{m=1}^M \ln[1 + m\xi\gamma_0 T(1 - T)(2\pi/\tau)]$ , for each matrix dimensionality  $M$  that we could check. We are confident that this closed form holds for all  $M$ , but we have not yet found an analytical proof. Equation (15) follows in the limit  $M \equiv \tau\Delta/2\pi \rightarrow \infty$  upon conversion of the summation into an integration.
- [16] Multiphoton excitations do not contribute to  $\text{Varn}$  if  $T_n \in \{0, 1/2, 1\}$  for all  $n$  [cf. Ref. [3], Eq. (19)]. For a quantum point contact, one finds that antibunching persists when  $\Delta > V/2$  provided that  $T_N(1 - T_N) > 1/6$ .