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Arefeva, I.; Bagrov, A.; Koshelev, A.S.

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Holographic Thermalization from Kerr-AdS

Irina Aref'eva,[a] **Andrey Bagrov,**[b] **Alexey S. Koshelev**[c]

[a] *Steklov Mathematical Institute, RAS, Gubkin str. 8, 119991 Moscow, Russia*

[b] *Lorentz Institute for Theoretical Physics, Leiden University, Niels Bohrweg 2, 2333CA Leiden, The Netherlands*

[c] *Theoretische Natuurkunde, Vrije Universiteit Brussel and The International Solvay Institutes, Pleinlaan 2, B-1050 Brussels, Belgium*

E-mail: arefeva@mi.ras.ru, bagrov@lorentz.leidenuniv.nl, alexey.koshelev@vub.ac.be

Abstract: We study thermalization of a strongly coupled theory holographically dual to a thin shell of null dust with non-zero angular momentum collapsing to Kerr-AdS. We calculate thermalization time for two point correlation functions. It happens that in the 3-dimensional case the thermalization time is just proportional to the distance between points where the correlator is evaluated. This is a very surprising and rather unexpected generalization of the same relation in the case of zero momentum.

Keywords: Holography and quark-gluon plasmas, gauge-gravity correspondence

Contents

1 Introduction

Among many questions one of the most intriguing problem in the physics of heavy ion collisions (HIC) is understanding of the short thermalization time, less than 1 fm/c. At later times a local thermodynamical equilibrium settles, and a hydrodynamic description of the quark-gluon plasma (QGP) is possible. The main difficulty is that describing the thermalization processes one has to deal with time evolution of strongly coupled systems.

A powerful approach to such problems was developed on the basis of the holographic duality between a strongly coupled quantum field theory in d-dimensional Minkowski space and the classical gravity in $d+1$-dimensional anti-de Sitter space (AdS) [1–3] during the last few years. In particular, there is a considerable progress in the holographic description of the equilibrium QGP [4]. This technique can also be applied to non-equilibrium quantum systems. Within this framework the thermalization is described as a process of black hole (BH) formation in AdS.

The gravitational collapse of matter injected into Minkowski space-time and the formation of event horizon are old problems in general relativity. The same question might be asked in AdS, and one can analyze what deformations of AdS metric end up with the BH formation. In the context of the application of AdS/CFT to HIC it is interesting to consider deformations which can be interpreted as a dual to the initial state of relativistic heavy ions.

Various models have been proposed in the literature:

- colliding gravitational shock waves [5]-[18],

- colliding shell of matter with vanishing rest mass ("null dust"), the so called AdS-Vaidya models [19]-[30],
- sudden near-boundary perturbations of the metric propagating into the bulk [31–33],
- coupled quenches [34, 35].

As stated above, an explanation of the short formation time of QGP (short thermalization time) in HIC using AdS/CFT techniques is an important and an ambitious goal. For a review we refer the reader to [36, 37], and refs. therein.

Currently the AdS-Vaidya metric [19]-[30] seems to be the most tractable model to answer this question, and we focus on it in this paper. This is a very instructive holographic toy model that captures many important features of quantum fields thermalization. In this model a thin collapsing null dust shell in AdS is considered. The final black hole state is dual to the thermalized equilibrium boundary QFT.

There is a possibility to write analytical, but implicit formula for the thermalization time of two-point correlators for a wide class of backgrounds, like Reissner-Nordström-AdS and Lifshitz-AdS [38]-[41]. In $2+1$ dimensional case the corresponding formula can be written explicitly. Numerical analysis of the thermalization time in higher dimensions indicates that it is nothing more but the causal bound on the possible space of an after quench evolution.

Results in [42–44] also show very rapid BH formation, close to the causal bound for a very wide range of BH masses. In the BTZ-Vaidya metric in AdS_3/CFT_2 case, we can write an explicit formula for the thermalization time [23–25]. Note that in those papers only the BTZ-Vaidya metric at zero angular momentum has been considered.

In the present work we extend the analysis of [23–25] to the case of rotating null dust shell collapsing into AdS-Kerr-BTZ BH and show that the thermalization time of a non-equal time two point correlators in a field theory dual to this gravitational background can be calculated exactly, and, rather unexpectedly, a presence of the second event horizon and a non-zero angular momentum does not affect it at least in the (2+1)-dimensional case. To study the same question in higher dimensions one is required to perform a numerical analysis.

Another branch of the contemporary research on AdS/CFT that our considerations might be relevant for is the holographic information theory. One of the cornerstones of modern applications of quantum field theory to solid state physics is the concept of the entanglement entropy [45]. It seems to be intimately related with ground states of strongly interacting field theories, and is widely used in theoretical many-body physics as a classifier of both quantum critical [46] and topological phases [47].

This quantity is very hard to calculate using standard field theoretical techniques. Holographic solution to this problem has been proposed in [48] in a form of conjecture, recently proven in [49]. It says that the entanglement entropy of a region $\mathcal{A}$ on the boundary is proportional to area of minimal surface in the bulk that has a common boundary with $\mathcal{A}$.

The advantage of a low dimensional holography is that in contrary to a higher dimensional case both two-point correlators and entropy are encoded in the same objects in the bulk, - in space-like geodesics. Performing the geodesic analysis we are testing the thermalization properties of observables of both kinds at the same time. In this regard our results on thermalization time are universal (even though we do not discuss here subtle aspects of the entanglement entropy phenomenology).

The paper is organized as follows. In section 2 we briefly recall the BTZ solution [50]. Sections 3 and 4 form the central part of the paper. In 3.1 and 3.2 we provide a detailed derivation of space-like geodesics in this space-time (in different parametrizations). In 3.3 we analyze properties of geodesics of a particular kind, which start and end up at the conformal boundary and therefore related to two-point correlation functions in the boundary field theory. In Section 4 we consider evolution of these geodesics in the background of a collapsing null shell and calculate holographically the thermalization time of related boundary correlation functions.

2 $D = 3$ Kerr-BTZ

The original BTZ formula is

$$ds^2 = -(-M + \frac{r^2}{l^2})dt^2 + \frac{dr^2}{-M + \frac{r^2}{l^2} + \frac{a^2}{r^2}} - 2adtd\phi + r^2 d\phi^2. \tag{2.1}$$

We introduce for the sequel

$$\mathcal{K} = r^2 M l^2 - r^4 - a^2 l^2 = -(r^2 - \beta_1)(r^2 - \beta_2), \tag{2.2}$$

where

$$\beta_1 = r_{H+}^2,\ \beta_2 = r_{H-}^2,\ r_{H\pm} = \sqrt{\frac{l^2 M}{2}\left(1 \pm \sqrt{1 - \frac{4a^2}{l^2 M^2}}\right)} \tag{2.3}$$

are the horizons. In the following we assume both $\beta_{1,2}$ are real meaning $\frac{4a^2}{l^2 M^2} \leq 1$. Obviously

$$\beta_1 > \beta_2 > 0. \tag{2.4}$$

We then transform the metric to

$$ds^2 = -\left(-M + \frac{r^2}{l^2}\right) dv^2 + 2dvdr - 2advd\hat{\phi} + r^2 d\hat{\phi}^2 \tag{2.5}$$

by virtue of the change of variables

$$dv = dt - \frac{r^2 l^2}{\mathcal{K}} dr, \tag{2.6}$$

$$d\hat{\phi} = d\phi - \frac{al^2}{\mathcal{K}} dr. \tag{2.7}$$

Naturally M, l, a are real and $M \geq 0,\ l > 0$. a can be taken positive as well since we always can account the change in the sign of a by replacing $\phi \to -\phi$ (or $\hat{\phi} \to -\hat{\phi}$) The relations between the old and new coordinates are

$$v = v_0 + t + \frac{l^2}{2(\beta_1 - \beta_2)} \left(\sqrt{\beta_1} \log \frac{r - \sqrt{\beta_1}}{r + \sqrt{\beta_1}} - \sqrt{\beta_2} \log \frac{r - \sqrt{\beta_2}}{r + \sqrt{\beta_2}} \right), \tag{2.8}$$

$$\hat{\phi} = \hat{\phi}_0 + \phi + \frac{a l^2}{2(\beta_1 - \beta_2)} \left(\frac{1}{\sqrt{\beta_1}} \log \frac{r - \sqrt{\beta_1}}{r + \sqrt{\beta_1}} - \frac{1}{\sqrt{\beta_2}} \log \frac{r - \sqrt{\beta_2}}{r + \sqrt{\beta_2}} \right), \tag{2.9}$$

and we put $v_0 = \hat{\phi}_0 = 0$. Note that g_{00} component of either metric vanishes only for

$$r_H = l\sqrt{M}. \tag{2.10}$$

Clearly zero mass M describes just an empty AdS space. Thus the picture we have in mind is that the mass in fact can be parametrized by step function $M\theta(v)$ where the parameter M is constant, representing the presence of the infalling null-shell at $v = 0$, pure AdS space-time for $v < 0$, and BTZ BH solution for $v > 0$. This is the simplest form of the Vaidya solution which assumes a general distribution of the mass $M(v)$.

3 Geodesics

We are going to use the two point boundary correlation functions as the thermalization probe. In AdS/CFT they can be easily approximated just by the renormalized length of a geodesic connecting these two points on the boundary [51]:

$$\langle \mathcal{O}(t, \vec{x})) \mathcal{O}(t', \vec{x}') \rangle_{ren} \sim e^{-\Delta L_{ren}}, \tag{3.1}$$

where Δ is the conformal dimension of the operator $\mathcal{O}$ under consideration, and L_{ren} is the renormalized geodesic length.

So, once we are interested in the thermalization processes, we can simply identify concepts of correlation functions and spacelike geodesics ending up on the AdS boundary. Since outside of the thin shell the metric is indistinguishable from the one of a black hole provided such a geodesic does not cross the shell, it has the same length as it would have in the background of a black hole of mass M, and therefore we may consider the corresponding correlator as a thermalized one. If the geodesic crosses the shell, its length evolves in time, and the corresponding correlator is out of the equilibrium.

3.1 Proper time τ parametrization

Equations for the geodesics in metric (2.1) are as follows

$$\ddot{r} - \frac{\mathcal{K}}{r\, l^4} \dot{t}^2 + \frac{\mathcal{K}}{r\, l^2} \dot{\phi}^2 - \frac{-r^4 + a^2 l^2}{r\mathcal{K}} \dot{r}^2 = 0, \tag{3.2}$$

$$\ddot{t} - 2\frac{r^3\, \dot{t}\dot{r}}{\mathcal{K}} + 2\frac{a\, l^2 r \dot{\phi}\dot{r}}{\mathcal{K}} = 0, \tag{3.3}$$

$$\ddot{\phi} - 2\frac{a\, r\, \dot{t}\dot{r}}{\mathcal{K}} + 2\frac{(Ml^2 - r^2) r \dot{\phi}\dot{r}}{\mathcal{K}} = 0. \tag{3.4}$$

Hereafter dot denotes a derivative with respect to the proper time τ. They can be obtained from the following corresponding Lagrangian

$$
\begin{aligned}
L &= -\frac{1}{2}\left((-M+\frac{r^2}{l^2})\dot{t}^2 - \frac{\dot{r}^2}{-M+\frac{r^2}{l^2}+\frac{a^2}{r^2}} + 2a\dot{t}\dot{\phi} - r^2\dot{\phi}^2\right) \\
&= -\frac{1}{2}\left((-M+\frac{r^2}{l^2})\dot{t}^2 + \frac{r^2l^2\dot{r}^2}{\mathcal{K}} + 2a\dot{t}\dot{\phi} - r^2\dot{\phi}^2\right),
\end{aligned} \tag{3.5}
$$

which is $+1/2$ of the metric. The conjugated momenta are

$$
p_t = -(-M+\frac{r^2}{l^2})\dot{t} - a\,\dot{\phi},\;\; p_\phi = r^2\dot{\phi} - a\dot{t},\;\; p_r = -\frac{r^2l^2\dot{r}}{\mathcal{K}}. \tag{3.6}
$$

$\dot{p}_t = \dot{p}_\phi = 0$ thanks to the fact that the initial metric does not depend on t and ϕ explicitly. Then we have

$$
\mathcal{E} \equiv p_t, \quad \mathcal{J} \equiv p_\phi, \tag{3.7}
$$

$$
\dot{t} = -\frac{\mathcal{E}r^2 + \mathcal{J}a}{(-M+\frac{r^2}{l^2})r^2 + a^2} = \left(\mathcal{E}r^2 + \mathcal{J}a\right)\frac{l^2}{\mathcal{K}}, \tag{3.8}
$$

$$
\dot{\phi} = -\frac{\mathcal{E}a + \mathcal{J}(M-\frac{r^2}{l^2})}{(-M+\frac{r^2}{l^2})r^2 + a^2} = \left(\mathcal{E}a + \mathcal{J}(M-\frac{r^2}{l^2})\right)\frac{l^2}{\mathcal{K}} = \frac{1}{r^2}(a\dot{t} + \mathcal{J}). \tag{3.9}
$$

Substituting these results into equation (3.2) one obtains

$$
\ddot{r} - \frac{\mathcal{K}}{r\,l^4}\dot{t}^2 + \frac{\mathcal{K}}{r\,l^2}\dot{\phi}^2 - \frac{-r^4 + a^2l^2}{r\mathcal{K}}\dot{r}^2 = \ddot{r} - A(r)\dot{r}^2 + B(r) = 0, \tag{3.10}
$$

where

$$
B(r) = \frac{1}{r\mathcal{K}}\left[l^2\left(\mathcal{E}a + \mathcal{J}(M-\frac{r^2}{l^2})\right)^2 - \left(\mathcal{E}r^2 + \mathcal{J}a\right)^2\right],
$$

and $A(r)$ is obvious. Than the latter equation can be recasted into a first order linear differential equation for $X(r) = \dot{r}^2$ as follows

$$
X' - 2A(r)X + 2B(r) = 0
$$

with the general solution

$$
X(r) = \left(-2\int\left(Be^{-2\int Adr}\right)dr + X_0/l^2\right)e^{2\int Adr}.
$$

Computing everything we get

$$
\dot{r}^2 = \frac{1}{r^2l^2}\left[X_0\mathcal{K} + \mathcal{L}\right],\; \text{where } \mathcal{L} = \mathcal{J}^2Ml^2 + 2l^2\mathcal{E}\mathcal{J}a + r^2(l^2\mathcal{E}^2 - \mathcal{J}^2), \tag{3.11}
$$

computing the square root we come to two branches

$$
\dot{r} = \pm\frac{1}{rl}\sqrt{X_0\mathcal{K} + \mathcal{L}}. \tag{3.12}
$$

To simplify the succeeding analysis we introduce a few notations

$$\begin{aligned} X_0\mathcal{K}+\mathcal{L} &= -X_0(r^2-\gamma_1)(r^2-\gamma_2), \\ \gamma_1+\gamma_2 &= Ml^2+\frac{l^2\mathcal{E}^2-\mathcal{J}^2}{X_0}, \\ \gamma_1\gamma_2 &= l^2a^2-\frac{l^2\mathcal{J}}{X_0}(M\mathcal{J}+2a\mathcal{E}). \end{aligned} \tag{3.13}$$

We note by product that (3.12) can be integrated out explicitly

$$\tau-\tau_0=\frac{l}{2\sqrt{-X_0}}\log\left(r^2-\frac{1}{2}(\gamma_1+\gamma_2)+\sqrt{(r^2-\gamma_1)(r^2-\gamma_2)}\right) \tag{3.14}$$

and further inverted to give

$$r(y)=\frac{1}{\sqrt{2y}}\sqrt{\left(y+\frac{\gamma_1+\gamma_2}{2}\right)^2-\gamma_1\gamma_2}, \quad \text{where } y=e^{2\frac{\sqrt{-X_0}}{l}(\tau-\tau_0)}. \tag{3.15}$$

Using equation (2.6) one readily gets

$$\dot{v}=(r^2(\mathcal{E}-\dot{r})+a\mathcal{J})\frac{l^2}{\mathcal{K}}. \tag{3.16}$$

This expression is important since it determines whether the geodesics are thermal or not as we shall see shortly.

3.2 Radial coordinate r parametrization

Hereafter we introduce the prime for a derivative with respect to r. Computing $t(r)$ we write

$$t'=\frac{\dot{t}}{\dot{r}}=\frac{(\mathcal{E}r^2+\mathcal{J}a)\frac{l^2}{\mathcal{K}}}{\pm\frac{1}{rl}\sqrt{X_0\mathcal{K}+\mathcal{L}}}. \tag{3.17}$$

To find out $t(r)$ we have to integrate the RHS with respect to r. Therefore the integral of interest is

$$I=\int\frac{x-\alpha}{(x-\beta_1)(x-\beta_2)\sqrt{(x-\gamma_1)(x-\gamma_2)}}dx, \tag{3.18}$$

where $x=r^2$. This can be computed with the following result:

$$\begin{aligned} I=\frac{1}{(\beta_1-\beta_2)}\Bigg[&\frac{\alpha-\beta_1}{\sqrt{B_1}}\ln(X_1-\text{sign}(x-\beta_1)\sqrt{X_1^2-(\gamma_1-\gamma_2)^2})- \\ &-\frac{\alpha-\beta_2}{\sqrt{B_2}}\ln(X_2-\text{sign}(x-\beta_2)\sqrt{X_2^2-(\gamma_1-\gamma_2)^2})\Bigg]\equiv I_-, \end{aligned} \tag{3.19}$$

where

$$B_{1,2}=(\beta_{1,2}-\gamma_2)(\beta_{1,2}-\gamma_1), \quad X_{1,2}=-(2\beta_{1,2}-\gamma_1-\gamma_2)-\frac{2B_{1,2}}{x-\beta_{1,2}}.$$

An important technical note here is that accounting for two branches we must have $\pm I_-$ and we note that

$$\begin{aligned} -I_- =& \frac{1}{(\beta_1-\beta_2)}\left[\frac{\alpha-\beta_1}{\sqrt{B_1}}\ln(X_1+\mathrm{sign}(x-\beta_1)\sqrt{X_1^2-(\gamma_1-\gamma_2)^2})-\right.\\ &\left.-\frac{\alpha-\beta_2}{\sqrt{B_2}}\ln(X_2+\mathrm{sign}(x-\beta_2)\sqrt{X_2^2-(\gamma_1-\gamma_2)^2})\right]+C_\alpha=I_++C_\alpha, \end{aligned} \tag{3.20}$$

where the signs in front of $\mathrm{sign}(x-\beta_{1,2})$ inside the logs are altered. The constant C_α is given by

$$C_\alpha=-\frac{2}{(\beta_1-\beta_2)}\left[\frac{\alpha-\beta_1}{\sqrt{B_1}}-\frac{\alpha-\beta_2}{\sqrt{B_2}}\right]\ln(\gamma_1-\gamma_2).$$

In our study we have to take different branches based on the sign in front of this square root making the second form more convenient to us. The first form with an overall minus sign is still valid but would force us having different integration constants to glue both branches together. Surely, $-I_+\neq I_-$.

Thus we have

$$t(r)=t_0-\frac{\mathcal{E}l^3}{2\sqrt{-X_0}}I_\mp|_{\alpha=-\frac{\mathcal{J}a}{\mathcal{E}}}\,. \tag{3.21}$$

Calculations for ϕ are identical just with the different parameter α:

$$\phi'=\frac{\dot\phi}{\dot r}=\frac{\left(\mathcal{E}a+\mathcal{J}(M-\frac{r^2}{l^2})\right)\frac{l^2}{\mathcal{K}}}{\pm\frac{1}{rl}\sqrt{X_0\mathcal{K}+\mathcal{L}}}, \tag{3.22}$$

providing

$$\phi(r)=\phi_0+\frac{\mathcal{J}l}{2\sqrt{-X_0}}I_\mp|_{\alpha=\frac{\mathcal{E}a+\mathcal{J}M}{\mathcal{J}/l^2}}\,. \tag{3.23}$$

Some nice cancelations happen here. Namely

$$\frac{\alpha-\beta_{1,2}}{\sqrt{B_{1,2}}}|_{\alpha=-aJ/\mathcal{E}}=-s_{1,2}\frac{\sqrt{\beta_{1,2}}}{l\mathcal{E}},\quad \frac{\alpha-\beta_{1,2}}{\sqrt{B_{1,2}}}|_{\alpha=Ml^2+l^2a\mathcal{E}/\mathcal{J}}=s_{1,2}\frac{\sqrt{\beta_{2,1}}}{J}, \tag{3.24}$$

where

$$\begin{aligned} &B_1=(\sqrt{\beta_1}\mathcal{E}l+\sqrt{\beta_2}\mathcal{J})^2=S_1^2,\quad B_2=(\sqrt{\beta_2}\mathcal{E}l+\sqrt{\beta_1}\mathcal{J})^2=S_2^2,\\ &s_1=\mathrm{sign}(S_1),\ s_2=\mathrm{sign}(S_2). \end{aligned} \tag{3.25}$$

Hence finally we get

$$t(r)=t_0+\frac{l^2}{2\sqrt{-X_0}(\beta_1-\beta_2)}\left(s_1\sqrt{\beta_1}\ln F_{1\mp}-s_2\sqrt{\beta_2}\ln F_{2\mp}\right), \tag{3.26}$$

$$\phi(r)=\phi_0+\frac{l}{2\sqrt{-X_0}(\beta_1-\beta_2)}\left(s_1\sqrt{\beta_2}\ln F_{1\mp}-s_2\sqrt{\beta_1}\ln F_{2\mp}\right), \tag{3.27}$$

$$F_{1,2\mp}=X_{1,2}\mp\mathrm{sign}(x-\beta_{1,2})\sqrt{X_{1,2}^2-(\gamma_1-\gamma_2)^2},\quad x=r^2.$$

3.3 Geodesics of interest which start and finish at $r = \infty$

Here we specialize to the type of geodesics which begin and end at the boundary $r = \infty$. Returning to (3.12) we see that for geodesics reaching the boundary $r = +\infty$ we should have $X_0 = -1$. Moreover, we must guarantee that a turning point exists meaning that at least one of $\gamma_{1,2} > 0$. If both are greater than 0 we assume

$$\gamma_1 > \gamma_2. \tag{3.28}$$

Also we assume the geodesics of interest do not cross any of the horizons. Writing down some elementary school level formulae we see that assuming $\gamma_1 + \gamma_2 = 2b, \gamma_1\gamma_2 = c$ one has

$$\gamma_{1,2} = b \pm \sqrt{b^2 - c}.$$

Now there are two cases

- $c > 0$
  In this case we must worry about $b^2 > c$ so that roots exist an then b must be positive. Indeed, if it is negative then both roots are negative as well. If all the conditions are met the greater root is given by the plus sign

- $c < 0$
  In this case we must not worry about $b^2 > c$ as well as b can be any. The greater root is given by the plus sign as in the previous case.

Hence the turning point we are interested in is

$$r^2 = \gamma_1 = b + \sqrt{b^2 - c} \text{ with } \begin{cases} c < 0 \text{ or} \\ c > 0, \ b > 0 \text{ and } b^2 > c. \end{cases} \tag{3.29}$$

Relations (3.25) imply $B_{1,2} > 0$ which in turn implies $\beta_1 < \gamma_2$ and one can check that this leaves only with the second possibility in (3.29)

So in total we have

$$\beta_2 < \beta_1 < \gamma_2 < \gamma_1, \tag{3.30}$$

$$\gamma_1 + \gamma_2 > 0, \gamma_1\gamma_2 > 0, \tag{3.31}$$

$$b^2 - c = \frac{(\gamma_1 + \gamma_2)^2}{4} - \gamma_1\gamma_2 = \frac{(\gamma_1 - \gamma_2)^2}{4} > 0. \tag{3.32}$$

The last condition (3.32) is not trivial from the very beginning once we go back to the parameters of our problem. It is read

$$(Ml^2 - l^2\mathcal{E}^2 + J^2)^2 - 4(l^2a^2 + l^2J(MJ + 2a\mathcal{E})) = (J^2 + l^2(\mathcal{E}^2 - M))^2 - 4l^2(\mathcal{E}J + a)^2 > 0. \tag{3.33}$$

We rewrite it as follows

$$(\rho_- + \sqrt{l}\mu_1)(\rho_- - \sqrt{l}\mu_1)(\rho_+ + \sqrt{l}\mu_2)(\rho_+ - \sqrt{l}\mu_2) > 0, \tag{3.34}$$

where

$$\mu_1 = \sqrt{Ml + 2a} > 0, \quad \mu_2 = \sqrt{Ml - 2a} > 0, \quad \rho_+ = \mathcal{E}l + J, \quad \rho_- = \mathcal{E}l - J. \tag{3.35}$$

Now we can readily see the solution

$$|\rho_-| > \sqrt{l}\mu_1, |\rho_+| > \sqrt{l}\mu_2 \text{ or} \tag{3.36}$$

$$|\rho_-| < \sqrt{l}\mu_1, |\rho_+| < \sqrt{l}\mu_2 \tag{3.37}$$

which are five domains on (ρ_-, ρ_+) plane (Fig. 1a). Horizons can be written in terms of $\mu_{1,2}$ as follow

$$\beta_1 = \frac{l}{4}(\mu_1 + \mu_2)^2, \quad \beta_2 = \frac{l}{4}(\mu_1 - \mu_2)^2. \tag{3.38}$$

Note that due to $a > 0$ condition we have $\mu_1 > \mu_2$.

Condition (3.30) can be taken as $\beta_1 + \beta_2 < \gamma_1 + \gamma_2$ which is explicitly

$$\rho_-\rho_+ < 0. \tag{3.39}$$

This is Fig. 1b.

The less simple relation is $\beta_1 < \gamma_2$ is hold upon:

$$\rho_-\rho_+ < -l\mu_1\mu_2, \tag{3.40}$$

and

$$\frac{\mu_1^2}{\rho_-^2} + \frac{\mu_2^2}{\rho_+^2} < \frac{2}{l} \tag{3.41}$$

(3.40) is depicted on Fig. 1c, and this supersedes (3.39). The last inequality holds in the domain (3.36), see Fig. 1d.

Examining (3.31) we write

$$Ml^2 > \rho_-\rho_+ \tag{3.42}$$

which is true thanks to (3.40) (see Fig. 1e), and

$$\mu_1^2\rho_+^2 + \mu_2^2\rho_-^2 - (\mu_1^2 + \mu_2^2)\rho_+\rho_- + \frac{l}{4}(\mu_1^2 - \mu_2^2)^2 > 0, \tag{3.43}$$

and this clearly holds in the domain (3.39) which is enough for our purposes (see also Fig. 1f).

The intersection of all the conditions is depicted graphically in Fig. 1h.

4 Thermalization

Thermal geodesics are those inside the shell and we thus must find out whether geodesics cross the shell or not. This question is translated into the analysis of $\dot{v} = 0$ point using (3.16). This happens when

$$r^2(\mathcal{E} - \dot{r}) + a\mathcal{J} = r^2\mathcal{E} + a\mathcal{J} \mp \frac{r}{l}\sqrt{(r^2 - \gamma_1)(r^2 - \gamma_2)} = 0. \tag{4.1}$$

Here we have taken already $X_0 = -1$. Then we come to the algebraic condition

$$l^2(r^2\mathcal{E} + a\mathcal{J})^2 = r^2(r^2 - \gamma_1)(r^2 - \gamma_2) \tag{4.2}$$

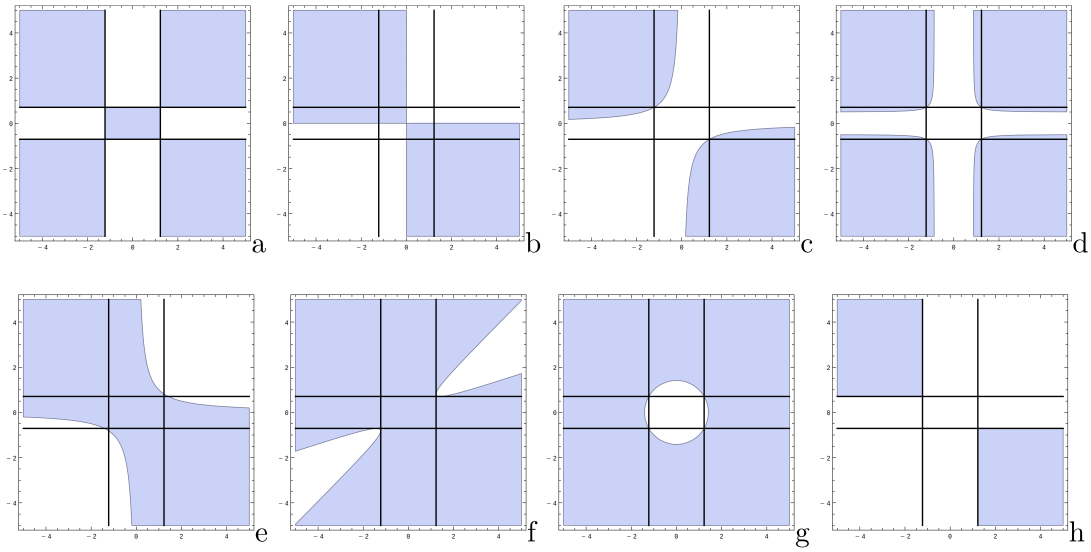

Figure 1. Domains for $M = 1,\ l = 1,\ a = 1/4$. Horizontal axis is ρ_- and vertical one is ρ_+, vertical black lines are $|\rho_-| = \sqrt{l}\mu_1 = \sqrt{3/2}$, horizontal black lines are $|\rho_+| = \sqrt{l}\mu_2 = \sqrt{1/2}$.

which can be explicitly solved for r^2 as

$$r^2 \text{ is equal to } \mathcal{J}^2, \beta_1, \text{ or } \beta_2. \tag{4.3}$$

We assume $\dot{v} = 0$ point does not coincide with a horizon and analyze $r = |\mathcal{J}|$ then assuming moreover $\mathcal{J}^2 > \beta_1$ (to avoid the horizon crossing). Also as mentioned before the turning point is γ_1 and in order to have the thermalization we must require $\mathcal{J}^2 \geq \gamma_1$, which is always true upon

$$Ml^2 < l^2\mathcal{E}^2 + \mathcal{J}^2 \tag{4.4}$$

to be one extra condition in the game. It is however always satisfied in the domain (3.36), see also Fig. 1g.

All the quantities associated with the $\dot{v} = 0$ point hold the star subscript. Our goal now is to compute v_*. First we define

$$r_* = |\mathcal{J}| \tag{4.5}$$

and introduce

$$\chi = (x - \gamma_1)(x - \gamma_2). \tag{4.6}$$

Note that at the $\dot{v} = 0$ point $\chi = l^2(\mathcal{E}\mathcal{J} + a)^2$. Equation (4.1) evaluated at $r = r_*$ reads

$$r_*^2\mathcal{E} + a\mathcal{J} \mp r_*|\mathcal{E}\mathcal{J} + a| = 0 \tag{4.7}$$

or a bit more complicated

$$\mathcal{J}(\mathcal{E}\mathcal{J} + a)[1 \mp s_{\mathcal{J}}s] = 0, \text{ where } s_{\mathcal{J}} = \text{sign}(\mathcal{J}),\ s = \text{sign}(\mathcal{E}\mathcal{J} + a). \tag{4.8}$$

- If both signs are the same then we get the upper sign here.

- If the signs are opposite then we get the lower sign.

All the other formulae must be taken with the appropriate sign respectively.

Several quantities are important to be introduced

$$t_{\mp} = t_{\mp}(\infty), \quad \phi_{\mp} = \phi_{\mp}(\infty), \quad \Delta\phi = \phi_{+} - \phi_{-}. \tag{4.9}$$

Then with the help of intermediate calculations given in the Appendix one yields

$$v_{*} = \frac{t_{+} + t_{-}}{2} + \mathrm{sign}(\mathcal{J}) l \frac{\Delta\phi}{2}. \tag{4.10}$$

Since v_* is the minimal value of the v coordinate on the geodesics the condition $v_* > 0$ guarantees the corresponding geodesics is always under the shell. This in turn is the sufficient condition for the thermalization to happen.

Surprisingly, this formula is the same as for $a = 0$ case. No question that the space separation $\Delta\Phi$ is different but the structure is identical. Consequently the thermalization time as a function of the space separation is the same. It is just equal to the probe length $\Delta\Phi$. This makes us thinking that other results like non-thermal geodesics analysis or analysis of geodesics which cross horizons can be easily or perhaps even identically generalized to the Kerr-BTZ space-time.

5 Conclusions

In this paper we have considered thermalization of a particular kind of non-local observables, two-point non-equal time correlation functions, in (1+1)-dimensional quantum field theory dual to the AdS-BTZ-Vaidya space-time. Our results are in agreement with those of [24], moreover, we have shown that non-zero angular momentum does not affect the formula which selects thermal geodesics at all.

However many other issues deserve more detailed analysis. In particular if one is talking about bulk/boundary duality he might wonder whether bulk region bounded by a minimal surface as introduced in [48] is really the most natural dual to the boundary region $\mathcal{A}$. This question is still under discussion, and an alternative object named causal wedge (and its boundary dual — the holographic causal information) has been defined in [52]. Without getting into details we just mention that this wedge is a region in the bulk which can be causally defined by boundary conditions on $\mathcal{A}$.

Non-trivial issue is arising when we are interested in out-of-equilibrium dynamics in AdS/CFT. Suppose we consider a time dependent bulk solution and a dual non-equilibrium field theory, is the causality preserved in such a system or not? In other words, is it true that throughout the whole evolution the boundary region and its bulk counterpart are casually related? This question was addressed in [53] for a particular case of non-rotating AdS-Vaidya metric, and this test of time evolution of holographic causal information has shown that this observable is evolving in a normal causal way. The question how the conclusions might change provided we consider the Kerr-BTZ space-time, which causal structure is very different due to the presence of two event horizons is still open.

Another problem to be analyzed is the possible relation between AdS-Vaidya space-times and the HIC phenomenology. Although here we are discussing only AdS_3/CFT_2 case, we may interpret the AdS-Kerr case as a dual to HIC with non-zero impact parameter in higher dimensions. There are two possibilities how the centrality dependence of the thermalization process quantities may appear. The first one is related with the geometry of the dual space, i.e. with the fact that we deal with Kerr-AdS, and the second one is just based on a simple collision picture of two "pancakes", since now the maximal distance between points in cross-section area is $2\sqrt{R_A^2 - b^2}$ instead of $2R_A$ where R_A is the radius of the ion, and b is the impact parameter of the collision. The second scenario does not depend on the dimension of the space-time, while the first one can.

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A Derivation of (4.10)

First we massage the logarithms in integration result (3.19) as follows

$$F_{1,2\mp} = \frac{(x-\beta_{1,2})^2 - \chi - B_{1,2} \mp 2\sqrt{B_{1,2}\chi}}{x-\beta_{1,2}},$$

where we dropped $\text{sign}(x-\beta_{1,2})$ factor as it is equal to $+1$ here. Then from (2.8) one has

$$v_* = t_* + \frac{l^2}{2(\beta_1-\beta_2)}\left(\sqrt{\beta_1}\log\frac{r_*-\sqrt{\beta_1}}{r_*+\sqrt{\beta_1}} - \sqrt{\beta_2}\log\frac{r_*-\sqrt{\beta_2}}{r_*+\sqrt{\beta_2}}\right), \tag{A.1}$$

$$\begin{aligned} t_* =& \frac{l^2}{2(\beta_1-\beta_2)}\left[s_1\sqrt{\beta_1}\ln\left(\frac{(\mathcal{J}^2-\beta_1)^2 - l^2(\mathcal{E}\mathcal{J}+a)^2 - B_1 - 2ls_1s_{\mathcal{J}}(\mathcal{E}\mathcal{J}+a)S_1}{\mathcal{J}^2-\beta_1}\right) - \right. \\ & \left. - s_2\sqrt{\beta_2}\ln\left(\frac{(\mathcal{J}^2-\beta_2)^2 - l^2(\mathcal{E}\mathcal{J}+a)^2 - B_2 - 2ls_2s_{\mathcal{J}}(\mathcal{E}\mathcal{J}+a)S_2}{\mathcal{J}^2-\beta_2}\right)\right] + t_0, \end{aligned} \tag{A.2}$$

where we took $\mp s = -s_{\mathcal{J}}$ from (4.8). Factorizing, simplifying, and regrouping terms a number of times one can get

$$\begin{aligned} v_* =& \frac{t_+ + t_-}{2} - \frac{l^2}{2(\beta_1-\beta_2)}\left[s_1\sqrt{\beta_1} - s_2\sqrt{\beta_2}\right]\ln(\gamma_1-\gamma_2) + \\ & + \frac{l^2}{2(\beta_1-\beta_2)}\left[s_1\sqrt{\beta_1}\ln[\mathcal{J}^2 - \mathcal{E}^2 l^2 + \beta_1 - \beta_2 - 2s_1 s_{\mathcal{J}} S_2] - \right. \\ & \left. - s_2\sqrt{\beta_2}\ln[\mathcal{J}^2 - \mathcal{E}^2 l^2 + \beta_2 - \beta_1 - 2s_2 s_{\mathcal{J}} S_1]\right], \end{aligned} \tag{A.3}$$

where we have used

$$t_0 = \frac{t_+ + t_-}{2} - \frac{l^2}{2(\beta_1 - \beta_2)}\left[s_1\sqrt{\beta_1} - s_2\sqrt{\beta_2}\right]\ln(\gamma_1 - \gamma_2). \tag{A.4}$$

Now we compute explicitly $\Delta\phi$. We transform it to look similar to (A.3) as follows

$$\begin{aligned}\Delta\phi = & -\frac{l}{(\beta_1 - \beta_2)}\left[s_1 s_{\mathcal{J}}\sqrt{\beta_1} - s_2 s_{\mathcal{J}}\sqrt{\beta_2}\right]\ln(\gamma_1 - \gamma_2)+ \\ & + \frac{l}{(\beta_1 - \beta_2)}\Big[s_1 s_{\mathcal{J}}\sqrt{\beta_1}\ln\left(\mathcal{J}^2 - l^2\mathcal{E}^2 + \beta_1 - \beta_2 - s_1 s_{\mathcal{J}} 2S_2\right) - \\ & - s_2 s_{\mathcal{J}}\sqrt{\beta_2}\ln\left(\mathcal{J}^2 - l^2\mathcal{E}^2 + \beta_2 - \beta_1 - s_2 s_{\mathcal{J}} 2S_1\right)\Big].\end{aligned} \tag{A.5}$$

Juxtaposing (A.3), and (A.5) it becomes clear that we come to (4.10).

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