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On the excitation of f modes and torsional modes by magnetar giant flares

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ABSTRACT

Magnetar giant flares may excite vibrational modes of neutron stars. Here we compute an estimate of initial post-flare amplitudes both of the torsional modes in the magnetar's crust and of the global f modes. We show that while the torsional crustal modes can be strongly excited, only a small fraction of the flare's energy is converted directly into the lowest order f modes. For a conventional model of a magnetar, with the external magnetic field of $\sim 10^{15}$ G, the gravitational wave detection of these f modes with Advanced LIGO is unlikely.

Key words: stars: neutron.

1 INTRODUCTION

The gamma- and X-ray flares from soft gamma repeaters (SGRs; Mazetz et al. 1979; Hurley et al. 1999, 2005) are believed to be powered by a sudden release of the magnetic energy stored in their host magnetars (Thompson & Duncan 1995). A SGR flare may excite vibrational modes of a magnetar (Duncan 1998). Indeed, torsional oscillations of a magnetar provide an attractive explanation for some of the quasi-periodic oscillations (QPOs) observed in the tails of giant flares (Barat et al. 1983; Israel et al. 2005; Strohmayer & Watts 2005; Colaiuda & Kokkotas 2011; Gabler et al. 2011; van Hoven & Levin 2011). Moreover, there is now some evidence of QPOs during the normal, non-giant flares in SGR 1806–20 (El-Mezeini & Ibrahim 2010).

Excitation of low-order f modes is also of considerable interest, because of the f modes' strong coupling to potentially detectable gravitational radiation. The sensitivity of the ground-based gravitational wave interferometers has improved dramatically over the last 5 years (Acernese et al. 2008; Abbott et al. 2009a), and interesting upper limits on the f-mode gravitational wave emission from the 2004 SGR 1806–20 giant flare, a possible 2009 SGR 1550–5418 giant flare, and several less energetic bursts have recently been obtained (Abbott et al. 2008; Abbott et al. 2009b; Abadie et al. 2010; see also Kalmus et al. 2009). Advanced LIGO and Advanced Virgo are expected to become operational in the next 5–7 years, and it is of interest to predict the strength of expected gravitational wave signal from future giant flares. In this paper, we compute a theoretical estimate for the amplitude of the torsional and f modes expected to be excited in a giant flare. We show that only a small fraction of the flare energy is expected to be pumped into the low-order f modes, and estimate the signal-to-noise ratio for future giant flare detection with Advanced LIGO. By contrast, the torsional modes can be strongly excited and may well be responsible for some of the observed QPOs in magnetar flares.

2 THE GENERAL FORMALISM

The giant flares release a significant fraction of the free magnetic energy stored in their host magnetars. Two distinct mechanisms for this have been proposed: (i) large-scale rearrangement of the internal field, facilitated by a major rupture of the crust (Thompson & Duncan 1995, 2001; we shall refer to it as the internal mechanism, hereinafter IM); and (ii) large-scale rearrangement of the magnetospheric field, facilitated by fast reconnection (Lyutikov 2006; Gill & Heyl 2010; we shall refer to it as the external mechanism, hereinafter EM). Both processes may well be at play: the IM would likely serve as a trigger for the EM. [However, as was argued in Lyutikov (2003), the EM may also be triggered by the slow motion of the footpoints of a magnetospheric flux tube, leading to a sudden loss of magnetostatic equilibrium.] Observationally, the extremely short, a few microseconds, rise time of the 2004 giant flare in SGR 1806–20 (Hurley et al. 2004) gives a reason to believe that the EM was at play in that source: the IM operates on a much longer Alfvén crossing time-scale of 0.05–0.1 s. The long time-scale for the IM implies that it would not be efficient in exciting the f modes which have frequencies of over a kHz; this was recently independently emphasized by Kashiyama & Ioka (2011, hereinafter KI).

2.1 Excitation by the EM

During the large-scale EM event, the magnetic stresses at the stellar surface change rapidly by, at most,¹ an order of 1. The magnetosphere comes to a new equilibrium, on the very short time-scale of several Alfvén (light) crossing times, and the stresses change to new

¹ There is some observational evidence for the substantial magnetic field reconfiguration in the magnetosphere, as seen from the difference between the persistent pre-flare and post-flare pulse profiles (Palmer et al. 2005). The global change in the magnetospheric twist would result in a comparable change in the tangential magnetic field at the surface, as is evident from, for example, the twisted-magnetosphere solution by Thompson, Lyutikov & Kulkarni (2002).

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constant values. We shall characterize the change in the magnetic stress by the three components.

$$\begin{aligned}\Delta T_{rr} &= \frac{B^2}{4\pi} f_r(\theta, \phi), \\ \Delta T_{r\theta} &= \frac{B^2}{4\pi} f_\theta(\theta, \phi), \\ \Delta T_{r\phi} &= \frac{B^2}{4\pi} f_\phi(\theta, \phi),\end{aligned}\quad (1)$$

where B is some characteristic value of the surface magnetic field, and f_r , f_θ and f_ϕ are functions of the order of 1 in the strongest possible flares and are smaller for the weaker flares. Consider now a normal mode of the star with an eigenfrequency ω_n and a displacement wavefunction $\xi_n(r, \theta, \phi)$. We treat the changing surface magnetic stress as an external perturbation acting on the mode. We derive the mode excitation using the Lagrangian formalism; in the appendix, we sketch the derivation directly from the equations of motion. The Lagrangian of the free (pre-perturbation) mode is given by

$$L_{\text{free}}(a_n, \dot{a}_n) = \frac{1}{2} m_n \dot{a}_n^2 - \frac{1}{2} m_n \omega_n^2 a_n^2, \quad (2)$$

where a_n is the generalized coordinate corresponding to the normal mode, m_n is the effective mass given by

$$m_n = \int d^3r \rho(\mathbf{r}) \xi_n^2(\mathbf{r}) \quad (3)$$

and $\rho(\mathbf{r})$ is the density. The Lagrangian term characterizing the mode's interaction with external stress is given by (cf. Section 2 of Levin 1998)

$$L_{\text{int}} = a_n \int R^2 \xi_n \cdot \mathbf{F} \sin \theta \, d\theta \, d\phi, \quad (4)$$

where

$$\mathbf{F} = \Delta T_{rr} \mathbf{e}_r + \Delta T_{r\theta} \mathbf{e}_\theta + \Delta T_{r\phi} \mathbf{e}_\phi, \quad (5)$$

and the displacement ξ is evaluated at the radius of the star R . The full Lagrangian for the n th mode is given by²

$$L(a_n, \dot{a}_n) = L_{\text{free}} + E_{\text{mag}} \alpha_n \frac{a_n}{R}, \quad (6)$$

where

$$E_{\text{mag}} = \frac{B^2 R^3}{4\pi} \quad (7)$$

is the characteristic energy stored in the star's magnetic field and α_n is the coupling coefficient given by

$$\alpha_n = \int \xi_n(R, \theta, \phi) \cdot \mathbf{f}(\theta, \phi) \sin \theta \, d\theta \, d\phi, \quad (8)$$

where

$$\mathbf{f} = f_r(\theta, \phi) \mathbf{e}_r + f_\theta(\theta, \phi) \mathbf{e}_\theta + f_\phi(\theta, \phi) \mathbf{e}_\phi. \quad (9)$$

It is now trivial to find the motion resulting from the sudden introduction of the external stress at moment $t = 0$. The coordinate a_n oscillates as follows:

$$a_n(t) = \bar{a}_n [1 - \cos(\omega_n t)], \quad (10)$$

² We work in the linear regime and do not take into account the non-linear coupling between the modes. The mode amplitudes $\ll 1$ found at the end of our calculation indicate that this is a good approximation.

where the amplitude is given by

$$\bar{a}_n = \frac{\alpha_n E_{\text{mag}}}{m_n \omega_n^2 R}. \quad (11)$$

The energy in the excited mode is given by

$$E_n = \frac{\alpha_n^2 E_{\text{mag}}^2}{2m_n \omega_n^2 R^2}. \quad (12)$$

We now briefly revisit the mode excitation by the IM. In this case, the interaction Lagrangian of a mode with the magnetic field is described by the following volume integral:

$$L_{\text{int}} = a_n \int d^3r \mathbf{f}_L(\mathbf{r}) \cdot \xi_n(\mathbf{r}), \quad (13)$$

where $\mathbf{f}_L = (\nabla \times \mathbf{B}) \times \mathbf{B}$ is the Lorentz force per unit volume. Since $f_L \sim B^2/R$, one can see that the coupling of the internal field variation to the mode is of the same order of magnitude as that of the external field variation, provided that the external and internal fields are of the same order of magnitude. However, the IM acts on a much longer time-scale,³ $\tau_{\text{Alfvén}} \sim 0.1$ s, than the typical f-mode period of $\tau_f \sim 0.0005$ s, so the f-mode oscillator would be adiabatically displaced without the excitation of the periodic oscillations. One can show that the typical suppression factor of the IM relative to the EM excitation is *at least* of the order of $2\pi\tau_{\text{Alfvén}}/\tau_f$ in the mode amplitude.⁴ This factor is so large that even if the internal field were stronger than the external field by an order of magnitude, the IM excitation would still be suppressed relative to the EM one.

Is there a way around this suppression factor? Potentially, the IM could feature a collection of many localized magnetohydrodynamic (MHD) excitation, with the time-scale for each one being determined by the Alfvén crossing time of each of the excitation domain. If the domains were small enough, their time-scales could be more closely matched with the f-mode period (Melatos, private communication). However, in this case, the magnitude of the overlap integral in equation (13) would be reduced by a factor of $\sim (R/\Delta R)^3$, where ΔR is the characteristic size of the excited domain. The domains would contribute incoherently to the amplitude of the excited mode; thus, the contribution of an individual domain would have to be multiplied by $(R/\Delta R)^{3/2}$ in the (somewhat unlikely) limit where the active domains occupy the whole star. Thus, while the time-scale of the mini-flares could be well matched with the f-mode period, their overall contribution to the overlap integral in equation (13) would be suppressed by $\sim (R/\Delta R)^{3/2}$. In the optimal case that the mini-flares have the same time-scale as the f-mode period, $R/\Delta R \sim \tau_{\text{Alfvén}}/\tau_f$. Therefore, the collection of mini-flares would not give us any gain in the mode-excitation amplitude, as compared to the IM estimate given in the previous paragraph.

Two applications of the formalism for the mode excitation by the EM developed above are presented in the next two sections.

³ This time-scale could be shorter by a factor of $\sqrt{x_p} \sim 0.2$ (where x_p is the proton fraction) if the superfluid neutrons are decoupled from the MHD (Easson & Pethick 1979; van Hoven & Levin 2008; Andersson, Glampedakis & Samuelsson 2009). However, even in this case, the time-scale $\tau_{\text{Alfvén}}$ on which the IM acts is still a factor of ~ 40 larger than the f-mode period τ_f .

⁴ This can be formalized by the following argument: consider a harmonic oscillator of proper frequency ω_0 , initially at rest, which is externally driven by force $f(t)$. The amplitude of the induced oscillation at the proper frequency is proportional to $\tilde{f}(\omega_0)$, the Fourier transform of $f(t)$ evaluated at ω_0 . For a step function, representing the rapid transition (several light-crossing times) to the new magnetospheric equilibrium in the EM, $\tilde{f}(\omega) \propto 1/\omega$. On the other hand, for a smooth pulse of duration τ , as expected in the IM, the Fourier transform is suppressed and scales at most as $\tilde{f}(\omega) \propto (\omega\tau)^{-1}/\omega$ when $\omega\tau \gg 1$.

3 F MODES AND GRAVITATIONAL WAVES

In order to estimate an effective f-mode mass, we have computed the $l = 2$ f-mode displacement functions for a neutron star⁵ in the Cowling approximation. Convenient scalings are

$$\begin{aligned} m_n &= q_M M, \\ \omega_n^2 &= q_\omega \frac{GM}{R^3}, \\ \xi_r(R, \theta, \phi) &= a_n Y_{2m}(\theta, \phi). \end{aligned} \quad (14)$$

In our fiducial model, $q_M = 0.046$, where we have normalized the mode wavefunction so that $e_r \cdot \xi_{2m}(R, \theta, \phi) = Y_{2m}(\theta, \phi)$. Our reference number $q_\omega = 1.35$ was obtained using a fitting formula for fully relativistic f-mode frequencies⁶ from Andersson & Kokkotas (1996). The amplitude of the f mode is given by

$$\frac{\bar{a}_{2m}}{R} = \frac{\alpha_{2m}}{q_m q_\omega} \frac{E_{\text{mag}}}{E_{\text{grav}}}, \quad (15)$$

where

$$E_{\text{grav}} = \frac{GM^2}{R} \quad (16)$$

is of the same order as the gravitational binding energy of the neutron star. We get

$$\frac{\bar{a}_{2m}}{R} \sim 3 \times 10^{-6} \alpha_{2m} \left(\frac{B}{10^{15} \text{ G}} \right)^2 \left(\frac{R}{10 \text{ km}} \right)^4 \left(\frac{1.4 M_\odot}{M} \right)^2. \quad (17)$$

The energy in the f mode is

$$E_f = \frac{\alpha_{2m}^2}{2q_m q_\omega} \frac{E_{\text{mag}}^2}{E_{\text{grav}}} \sim 1.5 \times 10^{-6} \alpha_{2m}^2 E_{\text{mag}} \quad (18)$$

for our fiducial parameters. This energy is drained from the star primarily through the emission of gravitational waves. The total amount of energy carried by gravitational waves is therefore

$$E_{\text{GW}} = E_f = \frac{2\pi^2 d^2 f^2 c^3}{G} \int_{-\infty}^{\infty} \langle h^2 \rangle dt, \quad (19)$$

where $f = \omega_n/2\pi$ is the f-mode frequency in Hz, $\langle h^2 \rangle$ is the direction and polarization averaged value of the square of the gravitational wave strain h as measured by observers at distance d from the source. This expression allows us to estimate the expected signal-to-noise ratio for ground-based gravitational wave interferometers (cf. Abadie et al. 2010). One can use the fact that nearly all the gravitational wave signal is expected to arrive in a narrow band around the f-mode frequency, and that the signal form (the exponentially

decaying sinusoid) is known. The Wiener-filter expression for the signal-to-noise ratio can be written as

$$\frac{S}{N} \approx \left[\frac{1}{S_h(f)} \int_{-\infty}^{\infty} |\tilde{h}^2(f')| df' \right]^{1/2} \quad (20)$$

$$\sim \left[\frac{G}{2\pi^2 c^3} \frac{E_f}{S_h(f) f^2 d^2} \right]^{1/2}, \quad (21)$$

where $\tilde{h}(f)$ is the Fourier transform of the time-dependent gravitational wave strain $h(t)$. As is standard for narrow-band signal, we have used Parseval's theorem to convert the integral over f to the integral over t from equation (19), and, following Abadie et al. (2010), we have approximated h^2 with the average $\langle h^2 \rangle$. At frequencies of a few kHz, the spectral density, $S_h(f)$, of the ground-based detectors like Advanced LIGO and Advanced Virgo is dominated by shot-noise and is proportional to f^2 . This makes the signal-to-noise ratio for observations of magnetar f modes excited in giant flares particularly sensitive to frequency ($\propto f^{-3}$). For Advanced LIGO, we find

$$\begin{aligned} \frac{S}{N} &\approx 0.07 \alpha_{2m} \left(\frac{2000 \text{ Hz}}{f} \right)^3 \left(\frac{B}{10^{15} \text{ G}} \right)^2 \left(\frac{1 \text{ kpc}}{d} \right) \\ &\times \left(\frac{R}{10 \text{ km}} \right)^2 \left(\frac{0.07 M_\odot}{m_n} \right)^{1/2}. \end{aligned} \quad (22)$$

Here we have used tabulated⁷ $S_h(f)$ from the LIGO document LIGO-T0900288, which gives $S_h(f) = 8.4 \times 10^{-47} \text{ Hz}^{-1} (f/2000 \text{ Hz})^2$ for the shot-noise-dominated part of the curve.

4 TORSIONAL MODES

Intuitively, one expects torsional modes to be strongly excited during the magnetar flares (Duncan 1998), since it is the free energy of the twisted magnetic field that is being released. These have much lower proper frequencies than the f modes (with the fundamental believed to be in the range 10–40 Hz, see Steiner & Watts 2009, and references therein), which can be well matched to the Alfvén frequencies inside the star. Thus, both the EM and the IM are likely to play a role in the torsional mode excitation. Here, we consider the EM explicitly but keep in mind that the IM would give a similar answer.

For the torsional modes in the crust, the displacement is given by

$$\xi_{nlm}(r, \theta, \phi) = g_n(r) \mathbf{r} \times \nabla Y_{lm}(\theta, \phi), \quad (23)$$

and it is convenient to normalize the wavefunctions so that $g_n(R) = 1$. Here $n = 0, 1, \dots$ is the number of radial nodes.

With this normalization, the effective mode mass $m_{nlm} \sim m_{\text{crust}} \sim 0.01M$, and from equation (11), one gets for the mode amplitude normalized by the star radius

$$\frac{a_{nlm}}{R} \sim 0.01 \alpha_{nlm} \left(\frac{B}{10^{15} \text{ G}} \right)^2 \frac{R}{10 \text{ km}} \frac{0.014 M_\odot}{m_{nlm}} \left(\frac{100 \text{ Hz}}{f} \right)^2. \quad (24)$$

Thus, we see that for a reasonable range of parameters, it is feasible that the crustal torsional modes would be strongly excited by a giant flare.

⁷ These sensitivity curves represent the incoherent sum of principal sources of noise as they are currently understood.

4.1 Magnetic modes

Recently, KI suggested that certain types of MHD modes that may be strongly excited during a giant flare are coupled to gravitational radiation and may therefore become an interesting source for Advanced LIGO. KI focus on the polar modes of Sotani & Kokkotas (2009); the MHD modes found by Lander & Jones (2011a,b) also satisfy some of KI's criteria.

While interesting, this idea has potential caveats that need further investigation. KI assume that the oscillations are long-lived, $\sim 10^7$ oscillation periods. However, MHD modes are notoriously capricious. While the idealized modes of Sotani & Kokkotas (2009) and Lander & Jones (2011a,b) are protected by symmetry, the global magnetic modes in more realistic configurations may couple to a variety of localized Alfvén-type modes and may thus be quickly damped via phase mixing and resonant absorption (Goedbloed & Poedts 2004; van Hoven & Levin 2011). Thus, in our view, there is currently no compelling reason to believe that the magnetic modes can be substantially longer lived than the observed magnetar QPOs.

5 DISCUSSION

In this paper, we have computed the excitation of the f modes and crustal torsional neutron star modes by a giant flare. Corsi & Owen (2011) recently computed the magnetic energy that can be released during the flare⁸ and found values comparable to E_{mag} . However, in this work, we showed that only a small fraction of the released flare energy is converted into the f modes and that the associated gravitational wave emission is correspondingly weaker than has been previously hoped (cf. Abadie et al. 2010 and Corsi & Owen 2011). From equation (22), our fiducial model does not look promising for future Advanced LIGO detection of a giant flare, even if $\alpha_{2m} \sim 1$, that is, if the flare comprises a global reconfiguration of the magnetospheric field so that the released electromagnetic energy is of the order of the total magnetic energy of the star, $\sim 10^{47}$ erg (the most energetic of the three observed giant flares released few $\times 10^{46}$ erg). However, if the surface field is significantly larger than 10^{15} G and/or the star is greater than 10 km in radius (which would reduce the f-mode frequency and increase the contact surface area), then one can become more hopeful about the potential detection.

On the other hand, we have seen that there is no difficulty in exciting the crustal torsional modes to a large amplitude. Whether or not this leads to the observed QPOs in the flare's tail (Israel et al. 2005; Strohmayer & Watts 2005; Watts & Strohmayer 2006) depends crucially on the dynamics of hydromagnetic coupling between the crustal modes and the Alfvén modes of the magnetar core (Levin 2006, 2007; Colaiuda & Kokkotas 2011; Gabler et al. 2011; van Hoven & Levin 2011).

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⁸ These analytical calculations necessarily make simplifying assumptions about the structure of an equilibrium magnetic field inside the magnetar, but they are likely to give correct order-of-magnitude values.

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APPENDIX A: DERIVATION OF THE MODE EXCITATION USING EQUATIONS OF MOTION

In this appendix, we derive, for completeness, the formalism for mode excitation directly from the equations of motion (cf. Unno et al. 1989). Let $\xi(\mathbf{r}, t)$ be the small displacement of the star from its equilibrium position. The equations of motion are given by

$$\rho \ddot{\xi} = \mathbf{F}(\xi) + f_{\text{ext}}(\mathbf{r}, t), \quad (\text{A1})$$

where ρ is the density, $\mathbf{F}(\xi)$ is the restoring force linear in ξ and f_{ext} is the external force per unit volume. For a normal-mode eigenfunction ξ_n with the angular frequency ω_n , one has $\mathbf{F}_n = -\rho \omega_n^2 \xi_n$.

We now decompose the star displacement into its eigenmodes

$$\boldsymbol{\xi}(\mathbf{r}, t) = \sum_n a_n(t) \boldsymbol{\xi}_n(\mathbf{r}) \quad (\text{A2})$$

and substitute this series into equation (A1) to obtain

$$\sum_n [\ddot{a}_n + \omega_n^2 a_n] \boldsymbol{\xi}_n(\mathbf{r}) = \mathbf{f}_{\text{ext}}(\mathbf{r}, t). \quad (\text{A3})$$

Taking a dot product of the above equation with $\boldsymbol{\xi}_k(\mathbf{r})$, integrating over the volume of the star and using the orthogonality relation

$$\int d^3r \rho \boldsymbol{\xi}_n \cdot \boldsymbol{\xi}_k \propto \delta_{nk}, \quad (\text{A4})$$

we obtain the equation of motion for a_k :

$$\ddot{a}_k + \omega_k^2 a_k = \frac{\alpha_k(t)}{m_k}, \quad (\text{A5})$$

where

$$\alpha_k = \int d^3r \mathbf{f}_{\text{ext}} \cdot \boldsymbol{\xi}_k(\mathbf{r}) \quad (\text{A6})$$

and

$$m_k = \int d^3r \rho(\mathbf{r}) \boldsymbol{\xi}_k^2(\mathbf{r}). \quad (\text{A7})$$

These equations of motion are identical to those derived from the Lagrangian in equations (2) and (13). For the case when the external force is applied at the surface, one recovers equations of motion derived from equations (2) and (4).

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