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**Heavy fields, reduced speeds of sound, and decoupling during inflation**Ana Achúcarro,<sup>1,2</sup> Vicente Atal,<sup>1</sup> Sebastián Céspedes,<sup>3</sup> Jinn-Ouk Gong,<sup>4</sup> Gonzalo A. Palma,<sup>3</sup> and Subodh P. Patil<sup>5</sup><sup>1</sup>*Instituut-Lorentz for Theoretical Physics, Universiteit Leiden, 2333 CA Leiden, Netherlands*<sup>2</sup>*Department of Theoretical Physics, University of the Basque Country, 48080 Bilbao, Spain*<sup>3</sup>*Physics Department, FCFM, Universidad de Chile, Blanco Encalada 2008, Santiago, Chile*<sup>4</sup>*Theory Division, CERN, CH-1211 Genève 23, Switzerland*<sup>5</sup>*Centre de Physique Théorique, Ecole Polytechnique and CNRS, Palaiseau cedex 91128, France*

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We discuss and clarify the validity of effective single-field theories of inflation obtained by integrating out heavy degrees of freedom in the regime where adiabatic perturbations propagate with a suppressed speed of sound. We show by construction that it is indeed possible to have inflationary backgrounds where the speed of sound remains suppressed and uninterrupted slow-roll persists for long enough. In this class of models, heavy fields influence the evolution of adiabatic modes in a manner that is consistent with decoupling of physical low- and high-energy degrees of freedom. We emphasize the distinction between the effective masses of the isocurvature modes and the eigenfrequencies of the propagating high-energy modes. Crucially, we find that the mass gap that defines the high-frequency modes increases with the strength of the turn, even as the naively heavy (isocurvature) and light (curvature) modes become more strongly coupled. Adiabaticity is preserved throughout, and the derived effective field theory remains in the weakly coupled regime, satisfying all current observational constraints on the resulting primordial power spectrum. In addition, these models allow for an observably large equilateral non-Gaussianity.

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The recent observation that heavy fields can influence the evolution of adiabatic modes during inflation [1] has far-reaching phenomenological implications [2–5] that, *a posteriori*, require a refinement of our understanding of how high- and low-energy degrees of freedom decouple [6] and how one splits “heavy” and “light” modes on a time-dependent background. Provided that there is only one flat direction in the inflaton potential, heavy fields (in the present context, field excitations orthogonal to the background trajectory) can be integrated out. This results in a low-energy effective field theory (EFT) for adiabatic modes exhibiting a reduced speed of sound  $c_s$ , given by

$$c_s^{-2} = 1 + 4\dot{\theta}^2/M_{\text{eff}}^2, \quad (1)$$

where  $\dot{\theta}$  is the turning rate of the background trajectory in multifield space, and  $M_{\text{eff}}$  is the effective mass of heavy fields, assumed to be much larger than the expansion rate  $H$ . Depending on the nature of the trajectory, (1) can render features in the power spectrum [3,4] and/or observably large non-Gaussianity [1,5].

Given that  $M_{\text{eff}}$  is the mass of the fields we integrate, one might doubt the validity of the EFT in the regime where the speed of sound is suppressed [7], as this requires  $\dot{\theta}^2 \gg M_{\text{eff}}^2$ . In this article we elaborate on this issue by studying the dynamics of light and heavy degrees of freedom when  $c_s^2 \ll 1$ . What emerges is a crucial distinction, in time-dependent backgrounds, between isocurvature and curvature field excitations, and the true heavy and light excitations. We show that the light (curvature) mode  $\mathcal{R}$  indeed stays coupled to the heavy (isocurvature) modes when strong turns take place ( $\dot{\theta}^2 \gg M_{\text{eff}}^2$ ); however,

decoupling between the physical low- and high-energy degrees of freedom persists in such a way that the deduced EFT remains valid. This is confirmed by a simple setup in which  $H$  decreases adiabatically, allowing for a sufficiently long period of inflation. In this construction, *high-energy degrees of freedom* are never excited, and yet *heavy fields* do play a role in lowering the speed of sound of adiabatic modes.

Although this is completely consistent with the principles of EFT, it seems to have escaped previous analyses due to some subtleties that we summarize in points i–iv below. Furthermore, inflationary scenarios with sustained turns and uninterrupted slow roll appear to be consistent with all the observational constraints on the primordial power spectrum of primordial perturbations, while predicting enhanced equilateral non-Gaussianity, and we give explicit examples at the end.

The simplest setup that allows a quantitative analysis (see Refs. [3–5] for details) is a two-scalar system with an action

$$S = \int \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \gamma_{ab} \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi) \right], \quad (2)$$

(in units  $8\pi G = 1$ ) where  $R$  is the Ricci scalar,  $V$  is the scalar potential and  $\gamma_{ab}$  is the possibly noncanonical sigma-model metric of the space spanned by  $\phi^a$ , with  $a = 1, 2$ . The background solution to the equations of motion is an inflationary trajectory  $\phi_0^a(t)$  and a Friedmann-Robertson-Walker metric  $ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j$ , where  $a(t)$  is the scale factor and  $H = \dot{a}/a$  the Hubble parameter. As usual, we take unit vectors  $T^a$  and  $N^a$  tangent

and normal to the trajectory [8] given by  $T^a = \dot{\phi}_0^a/\dot{\phi}_0$  and  $D_t T^a = -\dot{\theta} N^a$ , which also defines the turning rate  $\dot{\theta}$ , the angular velocity described by the bends of the trajectory. Here  $D_t = \dot{\phi}_0^a \nabla_a$  is the covariant time derivative along the background trajectory, and  $\dot{\phi}_0^2 \equiv \gamma_{ab} \dot{\phi}_0^a \dot{\phi}_0^b$  [9]. Finally, we define slow-roll parameters  $\epsilon \equiv -\dot{H}/H^2 = \dot{\phi}_0^2/2$  and  $\eta_{\parallel} \equiv -\ddot{\phi}_0/(H\dot{\phi}_0)$ , whose smallness ensures that  $H$  evolves adiabatically for a sufficiently long time.

We are interested in the dynamics of scalar perturbations  $\delta\phi^a(t, \mathbf{x}) = \phi^a(t, \mathbf{x}) - \phi_0^a(t)$ . We work in the flat gauge and define the comoving curvature and heavy isocurvature perturbations as  $\mathcal{R} \equiv -(H/\dot{\phi})T_a \delta\phi^a$  and  $\mathcal{F} \equiv N_a \delta\phi^a$ , respectively. (A definition of  $\mathcal{R}$  and  $\mathcal{F}$  valid to all orders in perturbation theory is given in Ref. [5]). The quadratic order action for these perturbations is

$$S_2 = \frac{1}{2} \int a^3 \left[ \frac{\dot{\phi}_0^2}{H^2} \dot{\mathcal{R}}^2 - \frac{\dot{\phi}_0^2}{H^2} \frac{(\nabla \mathcal{R})^2}{a^2} + \dot{\mathcal{F}}^2 - \frac{(\nabla \mathcal{F})^2}{a^2} - M_{\text{eff}}^2 \mathcal{F}^2 - 4\dot{\theta} \frac{\dot{\phi}_0}{H} \dot{\mathcal{R}} \mathcal{F} \right]. \quad (3)$$

Here  $M_{\text{eff}}$  is the effective mass of  $\mathcal{F}$  given by

$$M_{\text{eff}}^2 = m^2 - \dot{\theta}^2, \quad (4)$$

where  $m^2 \equiv V_{NN} + \epsilon H^2 \mathbb{R}$  and  $V_{NN} \equiv N^a N^a \nabla_a \nabla_b V$ .  $\mathbb{R}$  is the Ricci scalar of the sigma-model metric  $\gamma_{ab}$ . Notice that  $\dot{\theta}$  couples both fields and reduces the effective mass, suggesting a breakdown of the hierarchy that permits a single-field effective description as  $\dot{\theta}^2 \sim m^2$ . As we are about to see, this expectation is somewhat premature. The linear equations of motion in Fourier space are

$$\ddot{\mathcal{R}} + (3 + 2\epsilon - 2\eta_{\parallel})H\dot{\mathcal{R}} + \frac{k^2}{a^2} \mathcal{R} = 2\dot{\theta} \frac{H}{\dot{\phi}_0} \left[ \dot{\mathcal{F}} + \left( 3 - \eta_{\parallel} - \epsilon + \frac{\ddot{\theta}}{H\dot{\theta}} \right) H\mathcal{F} \right], \quad (5)$$

$$\ddot{\mathcal{F}} + 3H\dot{\mathcal{F}} + \frac{k^2}{a^2} \mathcal{F} + M_{\text{eff}}^2 \mathcal{F} = -2\dot{\theta} \frac{\dot{\phi}_0}{H} \dot{\mathcal{R}}. \quad (6)$$

Note that  $\mathcal{R} = \text{constant}$  and  $\mathcal{F} = 0$  are nontrivial solutions to these equations for arbitrary  $\dot{\theta}$ . Since  $\mathcal{F}$  is heavy,  $\mathcal{F} \rightarrow 0$  shortly after horizon exit, and  $\mathcal{R} \rightarrow \text{constant}$ , as in single-field inflation.

We are interested in (5) and (6) in the limit where  $\dot{\theta}$  is constant and much greater than  $M_{\text{eff}}$ . We first consider the short-wavelength limit where we can disregard Hubble friction terms and take  $\dot{\phi}_0/H$  as a constant. In this regime, the physical wave number  $p \equiv k/a$  may be taken to be constant, and (5) and (6) simplify to

$$\begin{aligned} \ddot{\mathcal{R}}_c + p^2 \mathcal{R}_c &= +2\dot{\theta} \dot{\mathcal{F}}, \\ \ddot{\mathcal{F}} + p^2 \mathcal{F} + M_{\text{eff}}^2 \mathcal{F} &= -2\dot{\theta} \dot{\mathcal{R}}_c, \end{aligned} \quad (7)$$

in terms of the canonically normalized  $\mathcal{R}_c = (\dot{\phi}_0/H)\mathcal{R}$ . The solutions are found to be [2]

$$\begin{aligned} \mathcal{R}_c &= \mathcal{R}_+ e^{i\omega_+ t} + \mathcal{R}_- e^{i\omega_- t}, \\ \mathcal{F} &= \mathcal{F}_+ e^{i\omega_+ t} + \mathcal{F}_- e^{i\omega_- t}, \end{aligned} \quad (8)$$

where the two frequencies  $\omega_-$  and  $\omega_+$  are given by

$$\omega_{\pm}^2 = \frac{M_{\text{eff}}^2}{2c_s^2} + p^2 \pm \frac{M_{\text{eff}}^2}{2c_s^2} \sqrt{1 + \frac{4p^2(1-c_s^2)}{M_{\text{eff}}^2 c_s^{-2}}}, \quad (9)$$

with  $c_s$  given by (1). The pairs  $(\mathcal{R}_-, \mathcal{F}_-)$  and  $(\mathcal{R}_+, \mathcal{F}_+)$  represent the amplitudes of both low- and high-frequency modes, respectively, and satisfy

$$\mathcal{F}_- = \frac{-2i\dot{\theta}\omega_-}{M_{\text{eff}}^2 + p^2 - \omega_-^2} \mathcal{R}_-, \quad \mathcal{R}_+ = \frac{-2i\dot{\theta}\omega_+}{\omega_+^2 - p^2} \mathcal{F}_+. \quad (10)$$

Thus the fields in each pair oscillate coherently. Integrating out the heavy mode consists in ensuring that the high-frequency degrees of freedom do not participate in the dynamics of the adiabatic modes. This requires a hierarchy of the form  $\omega_-^2 \ll \omega_+^2$ , which from (9) requires

$$p^2 \ll M_{\text{eff}}^2 c_s^{-2}. \quad (11)$$

This defines the regime of validity of the EFT, for which one has  $\omega_+^2 \simeq M_{\text{eff}}^2 c_s^{-2} = m^2 + 3\dot{\theta}^2$  and the low-energy dispersion relation

$$\omega_-^2(p) \simeq p^2 c_s^2 + (1 - c_s^2)^2 \frac{p^4}{M_{\text{eff}}^2 c_s^{-2}}. \quad (12)$$

These expressions for  $\omega_+$  and  $\omega_-$  imply a clear distinction between low- and high-energy degrees of freedom. In terms of the low-energy frequency, condition (11) may be rewritten as  $\omega_-^2 \ll M_{\text{eff}}^2 c_s^{-2}$ , in light of which the scale  $\omega_+^2 \simeq M_{\text{eff}}^2 c_s^{-2}$  evidently cuts off the low-energy regime. Thus, we can safely consider only low-frequency modes, in which case  $\mathcal{F}$  is completely determined by  $\mathcal{R}_c$  as  $\mathcal{F} = -2\dot{\theta} \dot{\mathcal{R}}_c / (M_{\text{eff}}^2 + p^2 - \omega_-^2)$ . Notice that  $\omega_-^2 \ll M_{\text{eff}}^2 + p^2$ , so  $\omega_-^2$  may be disregarded here.

As linear perturbations evolve, their physical wave number  $p \equiv k/a$  decreases and the modes enter the long-wavelength regime  $p^2 c_s^2 \lesssim H^2$ , where they become strongly influenced by the background and no longer have a simple oscillatory behavior. Now the low-energy contributions to  $\mathcal{F}$  satisfy  $\dot{\mathcal{F}} \sim H\mathcal{F}$ , and because  $H^2 \ll M_{\text{eff}}^2$ , we can simply neglect time derivatives in (6). On the other hand, high-energy modes continue to evolve independently of the low-energy modes, diluting rapidly as they redshift. Thus for the entire low-energy regime (11), time derivatives of  $\mathcal{F}$  can be ignored in (6) and  $\mathcal{F}$  may be solved in terms of  $\dot{\mathcal{R}}$  as

$$\mathcal{F} = -\frac{\dot{\phi}_0}{H} \frac{2\dot{\theta}\dot{\mathcal{R}}}{k^2/a^2 + M_{\text{eff}}^2}. \quad (13)$$

Inserting (13) into (3) gives the tree-level effective action for the curvature perturbation. To quadratic order [5],

$$S_{\text{eff}} = \frac{1}{2} \int a^3 \frac{\dot{\phi}_0^2}{H^2} \left[ \frac{\dot{\mathcal{R}}^2}{c_s^2(k)} - \frac{k^2 \mathcal{R}^2}{a^2} \right], \quad (14)$$

where  $c_s^{-2}(k) = 1 + 4\dot{\theta}^2/(k^2/a^2 + M_{\text{eff}}^2)$ . This  $k$ -dependent speed of sound is consistent with the modified dispersion relation (12), where  $c_s$  is given by (1). Reference [4] studied the validity of (14) in the case where turns appear suddenly. Consistent with the present analysis, it was found that this EFT is valid even when  $\dot{\theta}^2 \gg M_{\text{eff}}^2$ , *provided the adiabaticity condition*

$$|\ddot{\theta}/\dot{\theta}| \ll M_{\text{eff}} \quad (15)$$

is satisfied. This condition states that the turn's angular acceleration must remain small in comparison to the masses of heavy modes, which otherwise would be excited. The above straightforwardly implies the more colloquial adiabaticity condition  $|\dot{\omega}_+/\omega_+^2| \ll 1$ . If (15) is violated by the background, high-energy modes can be produced and the EFT does indeed break down, as confirmed by [10].

We now outline four crucial points that underpin our conclusions:

- (i) The mixing between fields  $\mathcal{R}$  and  $\mathcal{F}$ , and modes with frequencies  $\omega_-$  and  $\omega_+$  is *inevitable* when the background trajectory bends. If one attempts a rotation in field space in order to uniquely associate fields with frequency modes, the rotation matrix would depend on the scale  $p$ , implying a nonlocal redefinition of the fields.
- (ii) Even in the absence of excited high-frequency modes, the heavy field  $\mathcal{F}$  is forced to oscillate in pace with the light field  $\mathcal{R}$  at a frequency  $\omega_-$ , so  $\mathcal{F}$  continues to participate in the low-energy dynamics of the curvature perturbations.
- (iii) When  $\dot{\theta}^2 \gg M_{\text{eff}}^2$ , the high- and low-energy frequencies become  $\omega_+^2 \simeq M_{\text{eff}}^2 c_s^{-2} \sim 4\dot{\theta}^2$  and  $\omega_-^2 \simeq p^2(M_{\text{eff}}^2 + p^2)/(4\dot{\theta}^2)$ . Thus the gap between low- and high-energy degrees of freedom is amplified, and one can consistently ignore high-energy degrees of freedom in the low-energy EFT.
- (iv) In the low-energy regime, the field  $\mathcal{F}$  exchanges kinetic energy with  $\mathcal{R}$  resulting in a reduction in the speed of sound  $c_s$  of  $\mathcal{R}$ , the magnitude of which depends on the strength of the kinetic coupling  $\dot{\theta}$ . This process is adiabatic and consistent with the usual notion of decoupling in the low-energy regime (11), as implied by (15).

At the core of these four observations is the simple fact that in time-dependent backgrounds, the eigenmodes and eigenvalues of the mass matrix along the trajectory do not necessarily coincide with the curvature and isocurvature

fluctuations and their characteristic frequencies. With this in mind, it is possible to state more clearly the refined sense in which decoupling is operative: *while the fields  $\mathcal{R}$  and  $\mathcal{F}$  inevitably remain coupled, high- and low-energy degrees of freedom effectively decouple.*

We now briefly address the evolution of modes in the ultraviolet regime  $p^2 \gtrsim M_{\text{eff}}^2 c_s^{-2}$ . Here both modes have similar amplitudes and frequencies, and so in principle could interact via relevant couplings beyond linear order (which are proportional to  $\dot{\theta}$ ). Because these interactions must allow for the nontrivial solutions  $\mathcal{R} = \text{constant}$  and  $\mathcal{F} = 0$  (a consequence of the background time-reparametrization invariance), their action is very constrained [5]. Moreover, in the regime  $p^2 \gg M_{\text{eff}}^2 c_s^{-2}$  the coupling  $\dot{\theta}$  becomes negligible when compared to  $p$ , and one necessarily recovers a very weakly coupled set of modes, whose  $p \rightarrow \infty$  limit completely decouples  $\mathcal{R}$  from  $\mathcal{F}$ . This can already be seen in (13), where contributions to the effective action for the adiabatic mode at large momenta from having integrated out  $\mathcal{F}$ , are extremely suppressed for  $k^2/a^2 \gg M_{\text{eff}}^2$ , leading to high-frequency contributions to (14) with  $c_s = 1$ .

We now analyze a model of uninterrupted slow-roll inflation that executes a constant turn in field space, implying an almost constant, suppressed speed of sound for the adiabatic mode. Take fields  $\phi^1 = \theta$ ,  $\phi^2 = \rho$  with a metric  $\gamma_{\theta\theta} = \rho^2$ ,  $\gamma_{\rho\rho} = 1$ ,  $\gamma_{\rho\theta} = \gamma_{\theta\rho} = 0$  and potential

$$V(\theta, \rho) = V_0 - \alpha\theta + m^2(\rho - \rho_0)^2/2. \quad (16)$$

This model would have a shift symmetry along the  $\theta$  direction were it not broken by a nonvanishing  $\alpha$ . This model is a simplified version of one studied in Ref. [11], where the focus instead was on the regime  $M_{\text{eff}} \sim m \sim H$  (see also Ref. [12] where the limit  $M_{\text{eff}}^2 \gg H^2 \gg \dot{\theta}^2$  is analyzed). The background equations of motion are

$$\begin{aligned} \ddot{\theta} + 3H\dot{\theta} + 2\dot{\theta}\dot{\rho}/\rho &= \alpha/\rho^2, \\ \ddot{\rho} + 3H\dot{\rho} + \rho(m^2 - \dot{\theta}^2) &= m^2\rho_0. \end{aligned} \quad (17)$$

The slow-roll attractor is such that  $\dot{\rho}$ ,  $\ddot{\rho}$  and  $\ddot{\theta}$  are negligible. This means that  $H$ ,  $\rho$  and  $\dot{\theta}$  remain nearly constant and satisfy the following algebraic equations near  $\theta = 0$ :

$$\begin{aligned} 3H\dot{\theta} &= \frac{\alpha}{\rho^2}, & \dot{\theta}^2 &= m^2(1 - \rho_0/\rho), \\ 3H^2 &= \frac{1}{2}\rho^2\dot{\theta}^2 + V_0 + \frac{1}{2}m^2(\rho - \rho_0)^2. \end{aligned} \quad (18)$$

These equations describe a circular motion with a radius of curvature  $\rho$  and angular velocity  $\dot{\theta}$ . Here  $M_{\text{eff}}^2 = m^2 - \dot{\theta}^2$ , implying the strict bound  $m^2 > \dot{\theta}^2$ . Thus the only way to obtain a suppressed speed of sound is if  $\dot{\theta}^2 \simeq m^2$ . Our aim is to find the parameter ranges such that the background attractor satisfies  $\epsilon \ll 1$ ,  $c_s^2 \ll 1$  and  $H^2 \ll M_{\text{eff}}^2$  simultaneously. These are given by

$$1 \gg \frac{\rho_0}{4} \left( \frac{m\sqrt{3V_0}}{\alpha} \right)^{1/2} \gg \frac{V_0}{6m^2} \gg \frac{\alpha}{4\sqrt{3V_0}m}. \quad (19)$$

If these hierarchies are satisfied, the solutions to (18) are well approximated by

$$\rho^2 = \frac{\alpha}{\sqrt{3V_0}m}, \quad \dot{\theta} = m - \frac{m\rho_0}{2} \left( \frac{m\sqrt{3V_0}}{\alpha} \right)^{1/2}, \quad (20)$$

and  $H^2 = V_0/3$ , up to fractional corrections of order  $\epsilon$ ,  $c_s^2$  and  $H^2/M_{\text{eff}}^2$ . We note that the first inequality in (19) implies  $\rho \gg \rho_0$ , and so the trajectory is displaced off the adiabatic minimum at  $\rho_0$ . However, the contribution to the total potential energy implied by this displacement is negligible compared to  $V_0$ . After  $n$  cycles around  $\rho = 0$  one has  $\Delta\theta = 2\pi n$ , and the value of  $V_0$  has to be adjusted to  $V_0 \rightarrow V_0 - 2\pi n\alpha$ . This modifies the expressions in (20) accordingly, and allows us to easily compute the adiabatic variation of certain quantities, such as  $s \equiv \dot{c}_s/(c_s H) = -\epsilon/4$ , and  $\eta_{\parallel} = -\epsilon/2$ , where  $\epsilon = \sqrt{3}\alpha m^2/(2V_0^{3/2})$ . These values imply a spectral index  $n_{\mathcal{R}}$  for the power spectrum  $\mathcal{P}_{\mathcal{R}} = H^2/(8\pi^2\epsilon c_s)$  given by  $n_{\mathcal{R}} - 1 = -4\epsilon + 2\eta_{\parallel} - s = -19\epsilon/4$ .

It is possible to find reasonable values of the parameters such that observational bounds are satisfied. Using (20) we can relate the values of  $V_0$ ,  $\alpha$ ,  $m$  and  $\rho_0$  to the measured values  $\mathcal{P}_{\mathcal{R}}$  and  $n_{\mathcal{R}}$ , and to hypothetical values for  $c_s$  and  $\beta \equiv H/M_{\text{eff}}$  as

$$\begin{aligned} V_0 &= 96\pi^2(1 - n_{\mathcal{R}})\mathcal{P}_{\mathcal{R}}c_s/19, \\ m^2 &= 8\pi^2(1 - n_{\mathcal{R}})\mathcal{P}_{\mathcal{R}}/(19c_s\beta^2), \\ \alpha &= 6(16/19)^2\pi^2(1 - n_{\mathcal{R}})^2\mathcal{P}_{\mathcal{R}}c_s^2\beta, \\ \rho_0 &= 16c_s^3\beta\sqrt{2(1 - n_{\mathcal{R}})/19}. \end{aligned} \quad (21)$$

Following WMAP7, we take  $\mathcal{P}_{\mathcal{R}} = 2.42 \times 10^{-9}$  and  $n_{\mathcal{R}} = 0.98$  [13]. Then, as an application of relations (21), we look for parameters such that

$$c_s^2 \simeq 0.06, \quad M_{\text{eff}}^2 \simeq 250H^2, \quad (22)$$

(which imply  $H^2 \simeq 1.4 \times 10^{-10}$ ) according to which  $V_0 \simeq 5.9 \times 10^{-10}$ ,  $\alpha \simeq 1.5 \times 10^{-13}$ ,  $m \simeq 4.5 \times 10^{-4}$  and  $\rho_0 \simeq 6.8 \times 10^{-3}$ , from which we note that  $m$ ,  $\rho_0$  and  $\alpha^{1/4}$  are naturally all of the same order. In terms of the mass scale  $m$ , the above implies that we have excited a *field* with mass of order  $H^2/m^2 \sim 1/1450$ , completely consistent with decoupling and the validity of the EFT. We have checked numerically that the background equations of motion are indeed well approximated by (20), up to fractional corrections of order  $c_s^2$ . More importantly, we obtain the same nearly scale-invariant power spectrum  $\mathcal{P}_{\mathcal{R}}$  using both the full two-field theory described by (5) and (6), and the single-field EFT described by the action (14). The evolution of curvature perturbations in the EFT compared to the full two-field theory for the long-wavelength modes is almost indistinguishable given the effectiveness with

which (11) is satisfied, with a marginal difference  $\Delta\mathcal{P}_{\mathcal{R}}/\mathcal{P}_{\mathcal{R}} \simeq 0.008$ . This is of order  $(1 - c_s^2)H^2/M_{\text{eff}}^2$ , which is consistent with the analysis of Ref. [4]. Despite the suppressed speed of sound in this model, a fairly large tensor-to-scalar ratio of  $r = 16\epsilon c_s \simeq 0.02$  is predicted.

As expected, for  $c_s^2 \ll 1$  a sizable value of  $f_{\text{NL}}^{(\text{eq})}$  is implied. The cubic interactions leading to this were deduced in Ref. [5], which for constant turns is given by Ref. [14]

$$f_{\text{NL}}^{(\text{eq})} = \frac{125}{108} \frac{\epsilon}{c_s^2} + \frac{5}{81} \frac{c_s^2}{2} \left(1 - \frac{1}{c_s^2}\right)^2 + \frac{35}{108} \left(1 - \frac{1}{c_s^2}\right). \quad (23)$$

This result is valid for any single-field system with constant  $c_s$  obtained by having integrated out a heavy field. We note that this prediction is clearly distinguishable from those of other single-field models (such as Dirac-Born-Infeld inflation) through the different sign and magnitude for the  $M_3$  coefficient generated in the EFT expansion of Ref. [15], as derived in Ref. [5].

Recalling that the spectral index  $n_T$  of tensor modes is  $n_T = -2\epsilon$ , for  $c_s \ll 1$  we find a consistency relation between three potentially observable parameters, given by  $f_{\text{NL}}^{(\text{eq})} = -20.74n_T^2/r^2$ . In the specific case of the values in (22), we have  $f_{\text{NL}}^{(\text{eq})} \simeq -4.0$ . This value is both large and negative, so future observations could constrain this type of scenario. Finally, one can ask if the EFT corresponding to (22) remains weakly coupled throughout. For this, one needs to satisfy [5]  $\omega_- < \Lambda_{\text{sc}}$ , where  $\Lambda_{\text{sc}}$  is the scale at which our low-energy EFT becomes strongly coupled. For the standard case in which  $\omega_-^2 = c_s^2 p^2$  this scale is found to be given by  $\Lambda_{\text{sc}}^4 \simeq 4\pi\epsilon H^2 c_s^5/(1 - c_s^2)$  [15]. Nevertheless, for small values of  $c_s$  the scaling properties offered by the quartic piece in the modified dispersion relation (12) necessarily pushes the value of  $\Lambda_{\text{sc}}$  to a larger value [16]. For the present case, this effect implies a strong coupling scale given by  $\Lambda_{\text{sc}} \simeq (8\pi c_s^2)^{2/5} \times [2\epsilon H^2/(M_{\text{eff}}^2 c_s^{-4})]^{2/5} M_{\text{eff}} c_s^{-1}$  [17]. For instance, for the values (22) we find that  $\Lambda_{\text{sc}}/M_{\text{eff}} c_s^{-1} \simeq 2$ , implying that the EFT obtained by integrating a heavy field remains weakly coupled all the way up to its cutoff scale  $M_{\text{eff}} c_s^{-1}$ . Furthermore, although we did not address how inflation ends, the choice (22) allows for at least 45  $e$ -folds of inflation, which is necessary to solve the horizon and flatness problems. We stress that various other values can be chosen in (22) to arrive at similar conclusions. For example, requiring 35  $e$ -folds with  $M_{\text{eff}}^2 \simeq 100H^2$ ,  $c_s^2 \simeq 0.02$ , implies  $V_0 \simeq 3.4 \times 10^{-10}$ ,  $\alpha \simeq 8.1 \times 10^{-13}$ ,  $m \simeq 3.8 \times 10^{-4}$ ,  $\rho_0 \simeq 2.1 \times 10^{-4}$ . In this case we find  $f_{\text{NL}}^{(\text{eq})} \simeq -14$ .

In summary, the active ingredients of this toy example are rather minimal and may well parametrize a generic class of inflationary models, such as axion-driven inflationary scenarios in string theory. Our results complement those of Refs. [1–5] and emphasize the refined sense in which EFT techniques are applicable during slow-roll

inflation [15,18]. In particular, contrary to the standard perspective regarding the role of ultraviolet physics during inflation, heavy fields may influence the evolution of curvature perturbations  $\mathcal{R}$  in a way consistent with decoupling between low- and high-energy degrees of freedom.

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- [1] A.J. Tolley and M. Wyman, *Phys. Rev. D* **81**, 043502 (2010).
- [2] A. Achúcarro, J.-O. Gong, S. Hardeman, G. A. Palma, and S.P. Patil, *Phys. Rev. D* **84**, 043502 (2011).
- [3] A. Achúcarro, J.-O. Gong, S. Hardeman, G. A. Palma, and S.P. Patil, *J. Cosmol. Astropart. Phys.* **01** (2011) 030.
- [4] S. Cespedes, V. Atal, and G.A. Palma, *J. Cosmol. Astropart. Phys.* **05** (2012) 008.
- [5] A. Achúcarro, J.-O. Gong, S. Hardeman, G. A. Palma, and S.P. Patil, *J. High Energy Phys.* **05** (2012) 066.
- [6] For an early discussion see C.P. Burgess, J.M. Cline, F. Lemieux, and R. Holman, *J. High Energy Phys.* **02** (2003) 048. For a general discussion of the applicability of effective theories on time-dependent backgrounds see C.P. Burgess, *Living Rev. Relativity* **7**, 5 (2004).
- [7] S. Cremonini, Z. Lalak, and K. Turzyski, *Phys. Rev. D* **82**, 047301 (2010); S. Cremonini, Z. Lalak, and K. Turzyski, *J. Cosmol. Astropart. Phys.* **03** (2011) 016; G. Shiu and J. Xu, *Phys. Rev. D* **84**, 103509 (2011); A. Avgoustidis, S. Cremonini, A.C. Davis, R.H. Ribeiro, K. Turzyski, and S. Watson, *J. Cosmol. Astropart. Phys.* **06** (2012) 025.
- [8] C. Gordon, D. Wands, B.A. Bassett, and R. Maartens, *Phys. Rev. D* **63**, 023506 (2000); S.G. Nibbelink and B.J.W. van Tent, [arXiv:hep-ph/0011325](https://arxiv.org/abs/hep-ph/0011325); S.G. Nibbelink and B.J.W. van Tent, *Classical Quantum Gravity* **19**, 613 (2002).
- [9] Projecting the equations of motion on the normal direction relates the turning rate to the potential gradient as  $\dot{\theta} \equiv V_N / \dot{\phi}_0$  (with  $V_N \equiv N^a V_a$ ). The tangential projection gives the usual single-field equations  $\ddot{\phi}_0 + 3H\dot{\phi}_0 + V_T = 0$  with  $V_T \equiv T^a V_a$ .
- [10] X. Gao, D. Langlois, and S. Mizuno, *J. Cosmol. Astropart. Phys.* **10** (2012) 040.
- [11] X. Chen and Y. Wang, *Phys. Rev. D* **81**, 063511 (2010).
- [12] X. Chen and Y. Wang, *J. Cosmol. Astropart. Phys.* **09** (2012) 021; S. Pi and M. Sasaki, *J. Cosmol. Astropart. Phys.* **10** (2012) 051.
- [13] E. Komatsu *et al.* (WMAP Collaboration), *Astrophys. J. Suppl. Ser.* **192**, 18 (2011).
- [14] A. Achúcarro, J.-O. Gong, G. A. Palma, and S.P. Patil (to be published).
- [15] C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan, and L. Senatore, *J. High Energy Phys.* **03** (2008) 014.
- [16] D. Baumann and D. Green, *J. Cosmol. Astropart. Phys.* **09** (2011) 014.
- [17] R. Gwyn, G. A. Palma, M. Sakellariadou, and S. Sypsas, [arXiv:1210.3020](https://arxiv.org/abs/1210.3020).
- [18] S. Weinberg, *Phys. Rev. D* **77**, 123541 (2008); C.P. Burgess, J.M. Cline, and R. Holman, *J. Cosmol. Astropart. Phys.* **10** (2003) 004.