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Distribution of Voltage Fluctuations in a Current-Biased Conductor

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We calculate the fluctuating voltage $V(t)$ over a conductor driven out of equilibrium by a current source. This is the dual of the shot noise problem of current fluctuations $I(t)$ in a voltage-biased circuit. In the single-channel case the distribution of the accumulated phase $\Phi = (e/\hbar) \int V dt$ is the Pascal (or binomial waiting-time) distribution—distinct from the binomial distribution of transferred charge $Q = \int I dt$. The weak-coupling limit of a Poissonian $P(\Phi)$ is reached in the limit of a ballistic conductor, while in the tunneling limit $P(\Phi)$ has the chi-square form.

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The current-voltage or charge-phase duality plays a central role in the theory of single-electron tunneling through tunnel junctions of small capacitance [1]. At the two extremes one has a voltage-biased junction (in which the voltage is kept fixed by a source with zero internal resistance, while the current fluctuates) and a current-biased junction (fixed current from a source with infinite internal resistance, fluctuating voltage). The two current-voltage characteristics are entirely different. In the current-biased case the Coulomb blockade introduces a jump in the voltage at low current [2], while in the voltage-biased case the Coulomb blockade is inoperative.

Quantum mechanically, the duality appears because current I and voltage V are noncommuting operators [3]. This is conveniently expressed by the canonical commutator $[\Phi, Q] = ie$ of the transferred charge $Q = \int_0^\tau I(t) dt$ and accumulated phase $\Phi = (e/\hbar) \int_0^\tau V(t) dt$ (in a given detection time τ). Moments of charge and phase determine the measured correlators of current and voltage, respectively [4].

While all moments of Q in a voltage-biased conductor are known ([5]), the dual problem (moments of Φ under current bias) has been studied only for the first two moments [6,7]. In the absence of Coulomb-blockade effects, the first two moments in the dual problems are simply related by rescaling $I(t) \rightarrow V(t) \times G$ (with G the conductance). One might surmise that this linear rescaling carries over to higher moments, so that the dual problems are trivially related in the absence of the Coulomb blockade. However, the rescaling (as derived, for example, in Ref. [8]) follows from a Langevin approach that is suspect for moments higher than the second [9,10]—so that one might expect a more complex duality relation.

The resolution of this issue is particularly urgent in view of recent proposals to measure the third moment of shot noise in a mesoscopic conductor [9–11]. Does it matter if the circuit is voltage biased or current biased, or can one relate one circuit to the other by a linear rescaling? That is the question addressed in this Letter.

We demonstrate that, quite generally, the rescaling breaks down beyond the second moment. We calculate

all moments of the phase (hence all correlators of the voltage) for the simplest case of a single-channel conductor (transmission probability Γ) in the zero-temperature limit. In this case the charge $Q \equiv qe$ for voltage bias $V_0 \equiv h\phi_0/e\tau$ is known to have the binomial distribution [5]

$$P_{\phi_0}(q) = \binom{\phi_0}{q} \Gamma^q (1 - \Gamma)^{\phi_0 - q}. \quad (1)$$

We find that the dual distribution of phase $\Phi \equiv 2\pi\phi$ for current bias $I_0 \equiv eq_0/\tau$ is the Pascal distribution [12]

$$P_{q_0}(\phi) = \binom{\phi - 1}{q_0 - 1} \Gamma^{q_0} (1 - \Gamma)^{\phi - q_0}. \quad (2)$$

(Both q and ϕ are integers for integer ϕ_0 and q_0 .)

In the more general case we have found that the distributions of charge and phase are related in a remarkably simple fashion for $q, \phi \rightarrow \infty$:

$$\ln P_q(\phi) = \ln P_\phi(q) + \mathcal{O}(1). \quad (3)$$

[The remainder $\mathcal{O}(1)$ equals $\ln(q/\phi)$ in the zero-temperature limit.] This manifestation of charge-phase duality, valid with logarithmic accuracy, holds for any number of channels and any model of the conductor. Before presenting the derivation we give an intuitive physical interpretation.

The binomial distribution (1) for voltage bias has the interpretation [5] that electrons hit the barrier with frequency eV_0/h and are transmitted independently with probability Γ . For current bias the transmission rate is fixed at I_0/e . Deviations due to the probabilistic nature of the transmission process are compensated for by an adjustment of the voltage drop over the barrier. If the transmission rate is too low, the voltage $V(t)$ rises so that electrons hit the barrier with higher frequency. The number of transmission attempts (“trials”) in a time τ is given by $(e/h) \int_0^\tau V(t) dt \equiv \phi$. The statistics of the accumulated phase ϕ is therefore given by the statistics of the number of trials needed for $I_0\tau/e$ successful transmission events. This stochastic process has the Pascal distribution (2).

The starting point of our derivation is a generalization to time-dependent bias voltage $V(t) = (\hbar/e)\dot{\Phi}(t)$ of an expression in the literature [5,13] for the generating functional $Z[\Phi(t), \chi(t)]$ of current fluctuations:

$$Z[\Phi, \chi] = \left\langle \overleftarrow{T} \exp \left\{ \frac{i}{e} \int dt \left[\Phi(t) + \frac{1}{2} \chi(t) \right] \hat{I}(t) \right\} \overrightarrow{T} \exp \left\{ \frac{i}{e} \int dt \left[-\Phi(t) + \frac{1}{2} \chi(t) \right] \hat{I}(t) \right\} \right\rangle. \quad (4)$$

[The notation \overleftarrow{T} (\overrightarrow{T}) denotes time ordering of the exponentials in ascending (descending) order.] Functional derivatives of the Keldysh action $\ln Z$ with respect to $\chi(t)/e$ produce cumulant correlators of the current operator $\hat{I}(t)$ to any order desired. To make the transition from voltage to current bias we introduce a second conductor B in series with the mesoscopic conductor A (see Fig. 1). The generating functional Z_{A+B} of current fluctuations in the circuit is a (path integral) convolution of Z_A and Z_B ,

$$Z_{A+B}[\Phi, \chi] = \int \mathcal{D}\Phi_1 \mathcal{D}\chi_1 Z_A[\Phi_1, \chi_1] Z_B[\Phi - \Phi_1, \chi - \chi_1]. \quad (5)$$

One can understand this expression as the average over fluctuating phases Φ_1, χ_1 at the node of the circuit shared by both conductors.

In general the functional dependence of Z_A, Z_B is rather complicated and nonlocal in time, but we have found an interesting and tractable low-frequency regime: The nonlocality may be disregarded for sufficiently slow realizations of the fluctuating phases. In this regime the functional Z can be expressed in terms of a function S ,

$$\ln Z[\Phi(t), \chi(t)] = \int dt S[\dot{\Phi}(t), \chi(t)]. \quad (6)$$

The path integral (5) can be taken in saddle-point approximation, with the result

$$S_{A+B}(\dot{\Phi}, \chi) = S_A(\dot{\Phi}_s, \chi_s) + S_B(\dot{\Phi} - \dot{\Phi}_s, \chi - \chi_s). \quad (7)$$

Here $\dot{\Phi}_s$ and χ_s stand for the (generally complex) values of $\dot{\Phi}_1$ and χ_1 at the saddle point (where the derivatives with respect to these phases vanish).

The validity of the low-frequency and saddle-point approximations depends on two time scales. The first time scale $\tau_1 = \min(\hbar/eV, \hbar/kT)$ (with T the temperature) sets the width of current pulses associated with the transfer of individual electrons. The second time scale $\tau_2 = e/I$ sets the spacing of the pulses. Let ω be the characteristic frequency of a particular realization of the fluctuating phase. For the low-frequency approximation we require $\omega\tau_1 \ll 1$ and for the saddle-point approximation $\omega\tau_2 \ll 1$. Both conditions are satisfied if frequencies greater than $\Omega_c = \min(1/\tau_1, 1/\tau_2)$ do not contribute to the path integral. To provide this cutoff we assume that $|Z(\omega)| \ll \hbar/e^2$ at frequencies $\omega \gtrsim \Omega_c$. The small high-frequency impedance acts as a ‘‘mass term’’ in the Keldysh action, suppressing high-frequency fluctuations. The low-frequency impedance can have any value. Since the frequency dependence of $Z(\omega)$ is typically on scales much below Ω_c , it can be readily accounted for within the range of validity of our approximations.

Equations (6) and (7) are quite general and now we apply them to the specific circuit of Fig. 1. We assume that the mesoscopic conductor A (conductance G) is in series with a macroscopic conductor B with frequency dependent impedance $Z(\omega)$. We denote the zero-frequency limit

by $Z(0) \equiv Z_0 \equiv z_0 \hbar/e^2$. The circuit is driven by a voltage source with voltage V_0 . Both the voltage drop V at the mesoscopic conductor and the current I through the conductor fluctuate in time for finite Z_0 , with averages $\bar{I} = V_0 G(1 + Z_0 G)^{-1}$, $\bar{V} = V_0(1 + Z_0 G)^{-1}$. Voltage bias corresponds to $Z_0 G \ll 1$ and current bias to $Z_0 G \gg 1$, with $I_0 = V_0/Z_0$ the imposed current.

We assume that the temperature of the entire circuit is sufficiently low ($kT \ll e\bar{V}$) to neglect thermal noise relative to shot noise. (See Ref. [14] for the effects of a finite temperature of mesoscopic conductor and/or series impedance.) We also restrict ourselves to frequencies below the inverse RC time of the circuit, where $Z(\omega) \approx Z_0$. The low-temperature, low-frequency Keldysh action of the external impedance is simply $S_B(\dot{\Phi}, \chi) = i\chi\dot{\Phi}/2\pi z_0$, while the action S_A of the mesoscopic conductor is given by [5]

$$S_A(\dot{\Phi}, \chi) = \frac{\dot{\Phi}}{2\pi} S(i\chi), \quad (8)$$

$$S(\xi) = \sum_{n=1}^N \ln[1 + (e^\xi - 1)T_n].$$

The T_n 's are the transmission eigenvalues, with $\sum_n T_n = Gh/e^2 \equiv g$ the dimensionless conductance.

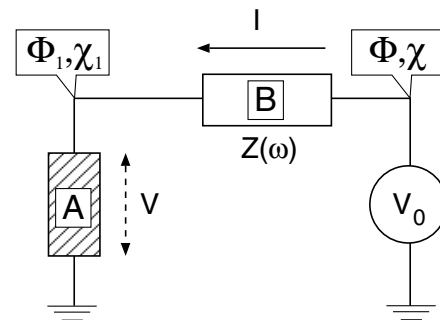


FIG. 1. Mesoscopic conductor (shaded) in a circuit containing a voltage source V_0 and series impedance $Z(\omega)$. Both the current I through the circuit and the voltage drop V over the conductor may fluctuate in time. The dual problems contrasted here are voltage bias ($Z \rightarrow 0$, fixed $V = V_0$, fluctuating I) and current bias ($Z \rightarrow \infty$, fixed $I = V_0/Z$, fluctuating V). The phases Φ, χ appearing in Eq. (5) are indicated.

We seek the cumulant generating function of charge

$$\mathcal{F}(\xi) = \ln\left(\sum_{q=0}^{\infty} e^{q\xi} P(q)\right) = \sum_{p=1}^{\infty} \frac{\langle\langle q^p \rangle\rangle \xi^p}{p!}, \quad (9)$$

where $\langle\langle q^p \rangle\rangle$ is the p th cumulant of the charge transferred during the time interval τ . It is related to the Keldysh action (7) by

$$\mathcal{F}(\xi) = \tau S_{A+B}(eV_0/\hbar, -i\xi). \quad (10)$$

We also require the cumulant generating function of phase, $\mathcal{G}(\xi)$. Since $V = V_0 - Z_0 I$ (in the absence of thermal noise from the external impedance), it is related to $\mathcal{F}(\xi)$ by a change of variables (from q to $\phi = \phi_0 - qz_0$). The relation is

$$\mathcal{G}(\xi) = \sum_{p=1}^{\infty} \frac{\langle\langle \phi^p \rangle\rangle \xi^p}{p!} = \phi_0 \xi + \mathcal{F}(-z_0 \xi). \quad (11)$$

In the limit $Z_0 \rightarrow 0$ of voltage bias the saddle point of the Keldysh action is at $\Phi_s = \Phi$, $\chi_s = \chi$, and from Eqs. (7), (9), and (11), one recovers the results of Ref. [5]: The cumulant generating function $\mathcal{F}_0(\xi) = \tau S_A(eV_0/\hbar, -i\xi) = \phi_0 S(\xi)$ and the corresponding probability distribution

$$P_{\phi_0}(q) = \lim_{x \rightarrow 0} \frac{1}{q!} \frac{d^q}{dx^q} \prod_{n=1}^N [1 + (x-1)T_n]^{\phi_0}. \quad (12)$$

The parameter $\phi_0 = eV_0\tau/\hbar$ is the number of attempted transmissions per channel, assumed to be an integer $\gg 1$. The first few cumulants are $\langle q \rangle_0 = \phi_0 g$, $\langle\langle q^2 \rangle\rangle_0 = \phi_0 \sum_n T_n (1 - T_n)$, $\langle\langle q^3 \rangle\rangle_0 = \phi_0 \sum_n T_n (1 - T_n)(1 - 2T_n)$. In the single-channel case ($N = 1$, $T_1 \equiv \Gamma$) the distribution (12) has the binomial form (1).

After these preparations we are now ready to generalize all of this to finite Z_0 , and, in particular, to derive the dual distribution of phase (2) under current bias. The key equation that allows us to do that follows directly from Eqs. (7) and (10):

$$\mathcal{F}(\xi) = \frac{\phi_0}{z_0} [\xi - \sigma(\xi)], \quad \sigma + z_0 S(\sigma) = \xi. \quad (13)$$

The implicit function $\sigma(\xi)$ (which determines the saddle point of the Keldysh action) provides the cumulant generating function of charge \mathcal{F} for arbitrary series resistance $z_0 = (e^2/h)Z_0$. One readily checks that $\mathcal{F}(\xi) \rightarrow \phi_0 S(\xi)$ in the limit $z_0 \rightarrow 0$, as it should.

By expanding Eq. (13) in powers of ξ we obtain a relation between the cumulants $\langle\langle q^p \rangle\rangle$ of charge at $Z_0 \neq 0$ and the cumulants $\langle\langle q^p \rangle\rangle_0$ at $Z_0 = 0$. The Langevin approach discussed in the introduction predicts that the fluctuations are rescaled by a factor of $1 + z_0 g$ as a result of the series resistance. Indeed, to second order we find $\langle\langle q^2 \rangle\rangle = (1 + z_0 g)^{-3} \langle\langle q^2 \rangle\rangle_0$, in agreement with Ref. [8]. However, if we go to higher cumulants we find that other terms appear, which cannot be incorporated by any re-

scaling. For example, Eq. (13) gives for the third cumulant

$$\langle\langle q^3 \rangle\rangle = \frac{\langle\langle q^3 \rangle\rangle_0}{(1 + z_0 g)^4} - \frac{3z_0 g}{(1 + z_0 g)^5} \frac{(\langle\langle q^2 \rangle\rangle_0)^2}{\langle q \rangle_0}. \quad (14)$$

The first term on the the right-hand side has the expected scaling form, but the second term does not. This is generic for $p \geq 3$: $\langle\langle q^p \rangle\rangle = (1 + z_0 g)^{-p-1} \langle\langle q^p \rangle\rangle_0$ plus a nonlinear (rational) function of lower cumulants [15]. All terms are of the same order of magnitude in $z_0 g$, so one cannot neglect the nonlinear terms. The Langevin approach ignores the nonlinear feedback that causes the mixing in of lower cumulants. This deficiency can be corrected; see Ref. [14].

Turning now to the limit $z_0 g \rightarrow \infty$ of current bias, we see from Eq. (13) that $\mathcal{F} \rightarrow \mathcal{F}_\infty$ with

$$\mathcal{F}_\infty(\xi) = q_0 \xi - q_0 S^{\text{inv}}(\xi/z_0) \quad (15)$$

defined in terms of the functional inverse S^{inv} of S . The parameter $q_0 = \phi_0/z_0 = I_0\tau/e$ (assumed to be an integer $\gg 1$) is the number of charges transferred by the imposed current I_0 in the detection time τ . Transforming from charge to phase variables by means of Eq. (11), we find that $\mathcal{G} \rightarrow \mathcal{G}_\infty$ with

$$\mathcal{G}_\infty(\xi) = -q_0 S^{\text{inv}}(-\xi). \quad (16)$$

In the single-channel case Eq. (16) reduces to $\mathcal{G}_\infty(\xi) = -q_0 \ln[1 + \Gamma^{-1}(e^{-\xi} - 1)]$, corresponding to the Pascal distribution (2). The first three cumulants are $\langle \phi \rangle = q_0/\Gamma$, $\langle\langle \phi^2 \rangle\rangle = (q_0/\Gamma^2)(1 - \Gamma)$, $\langle\langle \phi^3 \rangle\rangle = (q_0/\Gamma^3)(1 - \Gamma)(2 - \Gamma)$.

For the general multichannel case a simple expression for $P_{q_0}(\phi)$ can be obtained in the ballistic limit (all T_n 's close to 1) and in the tunneling limit (all T_n 's close to 0). In the ballistic limit one has $\mathcal{G}_\infty(\xi) = q_0 \xi/N + q_0(N - g)(e^{\xi/N} - 1)$, corresponding to a Poisson distribution in the discrete variable $N\phi - q_0 = 0, 1, 2, \dots$. In the tunneling limit $\mathcal{G}_\infty(\xi) = -q_0 \ln(1 - \xi/g)$, corresponding to a chi-square distribution $P_{q_0}(\phi) \propto \phi^{q_0-1} e^{-g\phi}$ in the continuous variable $\phi > 0$. In contrast, the charge distribution $P_{\phi_0}(q)$ is Poissonian both in the tunneling limit (in the variable q) and in the ballistic limit (in the variable $N\phi_0 - q$).

For large q_0 and ϕ , when the discreteness of these variables can be ignored, we may calculate $P_{q_0}(\phi)$ from $\mathcal{G}_\infty(\xi)$ in saddle-point approximation. If we also calculate $P_{\phi_0}(q)$ from $\mathcal{F}_0(\xi)$ in the same approximation (valid for large ϕ_0 and q), we find that the two distributions have a remarkably similar form:

$$P_{\phi_0}(q) = N_{\phi_0}(q) \exp[\tau \Sigma(2\pi\phi_0/\tau, q/\tau)], \quad (17)$$

$$P_{q_0}(\phi) = N_{q_0}(\phi) \exp[\tau \Sigma(2\pi\phi/\tau, q_0/\tau)]. \quad (18)$$

The same exponential function

$$\Sigma(x, y) = S_A(x, -i\xi_s) - y\xi_s, \quad (19)$$

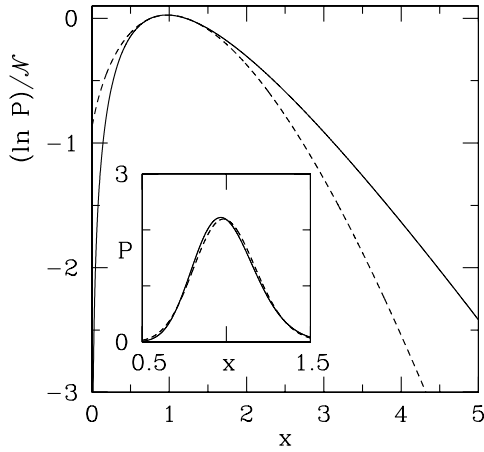


FIG. 2. Comparison of the distributions of charge (dashed curve, with $x = q/\langle q \rangle$) and of phase (solid curve, with $x = \phi/\langle \phi \rangle$), calculated from Eqs. (20) and (21) for $\mathcal{N} = q_0 = \phi_0\Gamma = 30$ transferred charges in the tunneling limit $\Gamma \ll 1$. The main plot emphasizes the non-Gaussian tails on a semi-logarithmic scale; the inset shows on a linear scale that the Gaussian body of the distributions coincides.

appears in both distributions (with ξ_s , the location of the saddle point). The preexponential functions N_{ϕ_0} and N_{q_0} are different, determined by the Gaussian integration around the saddle point. Since these two functions vary only algebraically, rather than exponentially, we conclude that Eq. (3) holds with the remainder $\mathcal{O}(1) = \ln(q/\phi)$ obtained by evaluating $\ln[2\pi(\partial^2\Sigma/\partial x^2)^{1/2}(\partial^2\Sigma/\partial y^2)^{-1/2}]$ at $x = 2\pi\phi/\tau$, $y = q/\tau$.

The distributions of charge and phase are compared graphically in Fig. 2, in the tunneling limit $\Gamma \ll 1$. We use the rescaled variable $x = q/\langle q \rangle$ for the charge and $x = \phi/\langle \phi \rangle$ for the phase and take the same mean number $\mathcal{N} = q_0 = \phi_0\Gamma$ of transferred charges in both cases. We plot the asymptotic large- \mathcal{N} form of the distributions,

$$P_{\text{charge}}(x) = (\mathcal{N}/2\pi)^{1/2} x^{-1/2} e^{\mathcal{N}(x-1-x\ln x)}, \quad (20)$$

$$P_{\text{phase}}(x) = (\mathcal{N}/2\pi)^{1/2} x^{-1} e^{\mathcal{N}(1-x+\ln x)}, \quad (21)$$

corresponding to the Poisson and chi-square distribution, respectively. Since the first two moments are the same, the difference appears in the non-Gaussian tails. The difference should be readily visible as a factor of 2 in a measurement of the third cumulant: $\langle\langle x^3 \rangle\rangle = \mathcal{N}^{-2}$ for the charge and $\langle\langle x^3 \rangle\rangle = 2\mathcal{N}^{-2}$ for the phase.

In summary, we have demonstrated theoretically that electrical noise becomes intrinsically different when the conductor is current biased rather than voltage biased. While the second moments can be related by a rescaling with the conductance, the third and higher moments cannot. From a fundamental point of view, the limit of

full current bias is of particular interest. The counterpart of the celebrated binomial distribution of transferred charge [5] turns out to be the Pascal distribution of phase increments.

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