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Gauge theory and nematic order : the rich landscape of orientational phase transition

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Citation

Liu, K. (2016, September 6). *Gauge theory and nematic order : the rich landscape of orientational phase transition*. *Casimir PhD Series*. Retrieved from https://hdl.handle.net/1887/42793

Version:	Not Applicable (or Unknown)
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Note: To cite this publication please use the final published version (if applicable).

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Issue Date: 2016-09-06

Summary

Crystals break both the translational and rotational symmetry of isotropic space, and are classified by space groups. On the other hand, nematic liquid crystals, which only break the rotational symmetry but are invariant under translations, should be in principle classified by point groups, i.e. subgroups of the full rotational group $O(3)$ of isotropic space. Traditionally, the study of nematic phases has mainly been focused on uniaxial nematics with $D_{\infty h}$-point group symmetry and biaxial nematics with D_{2h}-point group symmetry. However, in three spatial dimensions, point groups encompass seven infinite families of axial groups and seven polyhedral groups. Each point group can correspond to a nematic phase. The conventional approaches to study these phases, such as Landau-de Gennes theory or associated lattice models, involve order parameter tensors of high rank and complicated form, and are in general too unwieldy in practice. This thesis is devoted to a gauge theoretical description of nematic phases and the associated phase transitions.

The underlying theory is a non-Abelian discrete gauge theory which is formulated by a point-group-symmetric gauge theory coupled to $O(3)$ matter. The gauge symmetry in the theory identifies the orientations of the $O(3)$ degrees of freedom under given local point group transformations, and thus realizing the rotational symmetries associated with nematic order. The $O(3)$ matter with local gauge symmetries can be interpreted as coarse-grained order parameter fields. Similar lattice gauge theories have been extensively studied in particle physics and the phase structure and broken symmetry phases have been enumerated. Accordingly, the nematic phase and the isotropic liquid phase are realized by the so called Higgs phase and the confined phase of the gauge theory, respectively. Since this approach emphasizes the symmetries from the outset, it circumvents the global Landau-de Gennes order parameter description, but does not directly depend on the complex high-rank order parameter tensors. Moreover, this theory can fit nematics with any three-dimensional point-group symmetries into a unified framework and treat all of them on a general footing in an efficient way.

The proposed gauge theory can act as a generator of nematic order parameter tensors. This is a significant advantage in comparison with the

traditional Landau-de Gennes theory and the associated lattice models. Those methods require order parameters as a priori degrees of freedom. As a consequence, before one can explicitly formulate a Landau-de Gennes theory or an associated lattice model, one first needs to construct the highly non-trivial order parameters for each symmetry and then enumerate their couplings case-by-case. However, with the gauge theory, we are in principle able to derive the minimal set of order parameter tensors allowed by the symmetries for all point groups, and have enumerated the explicit form of those tensors for many physically interesting symmetries including all the crystallographic point groups, the icosahedral groups as well as all infinite axial groups. The derivation also allows one to elucidate their general structure and interrelations. To the best of my knowledge, this has never been done before in such a general and systematic way, although this problem has been intensively discussed in the liquid crystal community since the 1970s.

Furthermore, this gauge theory allows us to compare different nematic symmetries against a common reference. We were therefore able to quantify the orientational fluctuations of nematic orders with increasing symmetry and identified an order-by-disorder mechanism for a vestigial chiral liquid phase. We discovered that toward the top of the three-dimensional point-group hierarchy, the fluctuations associated with the nematic order increase tremendously. For the chiral polyhedral nematics, for which the left-right hand symmetry of isotropic space is broken, fluctuations in orientations may become sufficiently severe, such that the orientational order and the chiral order can be destroyed separately, with the ramification that room opens for a vestigial liquid phase breaking only chirality. In these observations we again benefit from the gauge theoretical description offering a common microscopic reference making it possible to compare nematics with different symmetries. In the framework of traditional Landau-de Gennes theory, however, the theoretical description of different nematic phases are formulated in terms of different order parameter tensors and/or couplings, rendering such comparisons to be rather ad hoc and obscure.

Finally, we also show that the gauge theory provides a convenient way to access the anisotropy between the axial and in-plane order of axial nematics. We were therefore able to predict and verify many anisotropy-induced vestigial phases. This extends the extensively studied biaxial-uniaxial transition of D_{2h} biaxial nematics to a much broader class, identifying generalized biaxial-uniaxial transitions for all nematics with a finite axial-point-group symmetry as well as additional transitions between two different but related biaxial nematics and different two uniaxial nematics.