Universiteit
Leiden
The Netherlands

Gauge theory and nematic order : the rich landscape of orientational phase transition

Liu, K.

Citation
Liu, K. (2016, September 6). *Gauge theory and nematic order : the rich landscape of orientational phase transition*. *Casimir PhD Series*. Retrieved from https://hdl.handle.net/1887/42793

Version:	Not Applicable (or Unknown)
License:	Licence agreement concerning inclusion of doctoral thesis in the Institutional Repository of the University of Leiden
Downloaded from:	https://hdl.handle.net/1887/42793

Note: To cite this publication please use the final published version (if applicable).

Cover Page

Universiteit Leiden

The handle http://hdl.handle.net/1887/42793 holds various files of this Leiden University dissertation.

Author: Liu, K.
Title: Gauge theory and nematic order : the rich landscape of orientational phase transition
Issue Date: 2016-09-06

Gauge Theory and Nematic Order

The rich landscape of orientational phase transition

Proefschrift

ter verkrijging van
de graad van Doctor aan de Universiteit Leiden,
op gezag van Rector Magnificus
prof. mr. C.J.J.M. Stolker,
volgens besluit van het College voor Promoties
te verdedigen op dinsdag 6 september 2016
klokke 13:45 uur

door

Ke Liu

geboren te Jize (Hebei), China in 1987

Promotor:	Prof. Dr. J. Zaanen
Promotiecommissie:	Prof. Dr. H. T. C. Stoof (Utrecht University)
	Dr. J. van Wezel (University of Amsterdam)
	Prof. Dr. E. R. Eliel
	Dr. D. J. Kraft
	Prof. Dr. K. E. Schalm
	Prof. Dr. V. Vitelli

Casimir PhD series 2016-19
ISBN 978-90-8593-263-5
An electronic version of this thesis can be found at
https://openaccess.leidenuniv.nl

The work described in this thesis is carried out at the Lorentz Institute, Leiden University. The author was supported by a state scholarship program funded by China Scholarship Council (CSC).

The cover shows an oriental landscape excerpted from a classical Chinese drawing, Dwelling in the Fuchun Mountains. The work was created by Gong-Wang Huang in 1300s and now is in public domain. The patterns of seals show my name and signature in ancient Chinese.

To My Family and Fuzi

Contents

1 Introduction **7**

2 Preliminary: C_N Quantum nematics in $2+1$ dimensions **13**
2.1 Topological melting of a crystal 13
2.1.1 Topological defects in crystals 13
2.1.2 Defect mediated melting 15
2.2 Quantum field theory description of dislocation melting and quantum nematics 16
2.2.1 Stress-strain duality 16
2.2.2 Quantum nematic as dual stress superconductor . . 18
2.2.3 Collective modes of quantum nematics 20
2.3 Order parameter theory for $2+1$ d quantum nematics . . . 22
2.3.1 $SO(2)/Z_N$ Gauge theory and C_N quantum nematics 23
2.3.2 Phase diagram of the $SO(2)/Z_N$ model 24
2.4 Concluding remarks 30

3 Generalized $3D$ nematics and discrete non-Abelian gauge theory **31**
3.1 Gauge theory description of generalized nematic phases . . 33
3.1.1 Uniaxial nematics and Z_2 gauge theory 34
3.1.2 Generalized nematics and non-Abelian gauge theory 35
3.1.3 Realization of the generalized nematic orders 38
3.2 Nematic transitions 41
3.2.1 Giant thermal fluctuations of highly symmetric nematics 41
3.2.2 The chiral liquid as fluctuation driven vestigial phase 44
3.3 Concluding remarks 46
3.A Details of simulations and generalized nematic order parameters 49
3.A.1 Strength of the nematic ordering 49
3.B Detailed analysis of the fluctuation induced chiral phase . . 50
3.B.1 Calculations for $G = SO(3)$ 51
3.C The I_h tensor order parameter 53

4 Classification of $3D$ point-group-symmetric order parameter tensors **61**
4.1 Construction of orientational ordering tensor 62
4.1.1 Warm up: Heisenberg ferromagnetic order and uniaxial nematic order 62
4.1.2 General $3D$ orientational order 63
4.1.3 Deriving the ordering tensors from the gauge theory 65
4.2 Minimal invariant tensors 66
4.2.1 Order parameter table 66
4.2.2 Structure of the ordering tensors 70
4.2.3 Point groups and ordering tensors 71
4.2.4 Determining the symmetry of a phase and phase transitions with ordering tensors 72
4.3 Examples and discussion 74
4.3.1 Continuous axial groups: unixial nematics 74
4.3.2 Biaxial nematics 77
4.3.3 Polyhedral nematics 80
4.4 Concluding remarks 84

5 Generalized biaxial phase transitions **85**
5.1 The structure of nematic order parameters and generalized biaxial transitions 86
5.1.1 Point groups and nematic order parameters 86
5.1.2 Generalized biaxial phases and transitions 88
5.2 Gauge theory realization of generalized biaxial transitions . 92
5.2.1 $O(3)/G$ lattice gauge theory for generalized nematics 93
5.2.2 Anisotropic couplings and generalized biaxial transitions 94
5.2.3 Temperature-anisotropy phase diagrams 98
5.3 Monte-Carlo simulations 100
5.3.1 Determination of the phases 102
5.3.2 Numerical phase diagrams 102
5.4 Concluding remarks 107
5.A Phase diagrams for axial nematics with high symmetries . . 109

6 Conclusion and outlook **113**

Bibliography **116**

Samenvatting **131**

Summary **135**

List of Publications **139**

Curriculum Vitæ **141**

Acknowledgments **143**