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Universal response curve for nanowire superconducting single-photon detectors

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Using detector tomography, we investigate the detection mechanism in NbN-based superconducting single photon detectors (SSPDs). We demonstrate that the detection probability uniquely depends on a particular linear combination of bias current and energy, for a large variation of bias currents, input energies and detection probabilities, producing a universal detection curve. We obtain this result by studying multiphoton excitations in a nanodetector with a sparsity-based tomographic method that allows factoring out of the optical absorption. We discuss the implication of our model system for the understanding of meander-type SSPDs.

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I. INTRODUCTION

Nanowire superconducting single photon detectors (SSPDs)¹ have high detection efficiency,² low dark counts, low jitter, and a broadband absorption spectrum.³ This makes them suitable for many applications including quantum optics,^{4–7} quantum key distribution,^{8,9} optical coherence domain reflectometry,¹⁰ and interplanetary communication.¹¹ These detectors typically consist of a thin nanowire (~ 4 nm \times 100 nm) of superconducting material, such as NbN,¹ TaN,¹² NbTiN,¹³ Nb,¹⁴ or WSi,² which is typically fabricated in a meander shape to cover an active area of 25–1600 μm^2 .¹⁵ The absorption of a single photon in the nanowire results in the creation of a region with a nonequilibrium concentration of quasiparticles. When the nanowire is biased close to the critical current, this perturbation causes a transition from the superconducting to the resistive state, producing a voltage pulse in the external circuit.

While progress has been made in understanding the detection process, many crucial features of the process are still unknown. In this publication, we investigate the detection process by means of a model system: an NbN nanodetector¹⁶ (see Fig. 1). This detector has a single cross section of wire as its active element, defined by a bowtie-shaped constriction. We investigate this system with sparsity-based detector tomography. The tomographic method does not require a model of the device, which makes it ideally suited as a tool for investigating the working principle of a detector of which the working mechanism is not fully understood yet.

It has long been known that at lower bias current, the detector operates in a regime where multiple photons are necessary to break the superconductivity.^{1,17} In a nanodetector, the geometry is such that many multiphoton processes play a strong role.^{16,18} This enables us to probe the response of the device to excitations at different energies simultaneously. The role of detector tomography is to extract the effects of the various multiphoton excitations.

In this paper, we investigate the detection process by combining tomography and a nanodetector. With this combination, we can probe the system in a way that is independent of the incoupling efficiency of light into the detector. Moreover, because we tune the energy of the excitation via the number of photons at the same wavelength, we are insensitive to

wavelength-dependent effects in the setup. This combination allows us to focus on the fundamentals of the detection process. We demonstrate that for intrinsic detection probabilities ranging from 0.3 to 10^{-4} , the detection probability depends only on a specific linear combination of bias current and excitation energy. Thus we obtain a universal detection curve for our model system of an SSPD: for each bias current and excitation (photon) energy, the detection probability is given by a point on this single curve. This universal curve stretches from the regime where photodetection is almost deterministic (given that the photon is absorbed into the active area) to the regime where fluctuations in the wire are thought to play a role in assisting the detection process.

II. EXPERIMENT

All experiments in this work were performed on a nanodetector (see Fig. 1). The nanodetector consists of 4-nm-thin NbN wire on a GaAs substrate, shaped into a 150-nm-wide bowtie geometry. The device was fabricated via a combination of dc magnetron sputtering,¹⁹ electron-beam lithography, reactive ion etching, and evaporation of the metal contacts.¹⁶ In previous work,^{16,18} it was shown that such a detector has multiphoton regimes based on the bias current. The physical mechanism behind these multiphoton regimes is that at relatively low-bias currents, multiple photons are required to supply a sufficient perturbation for the superconductivity to be broken.

The device was cooled in a two-stage pulse-tube/Joule-Thompson cryocooler to a temperature of approximately 1.2 K. The nanodetector was illuminated using a lensed fiber mounted on cryogenic nanomanipulators. At this temperature, the overall system detection efficiency for single photons was 1.5×10^{-4} around our working point at $I_b = 20 \mu\text{A}$ ($I_c = 29 \mu\text{A}$). This low efficiency is attributable to the mismatch between the device active area and the size of the illumination spot. The device was operated in a voltage bias regime, using a low-noise voltage source (Yokogawa GS200) in series with a 10 Ω resistor. The detector was biased through the dc port of a bias tee, and the RF signal was amplified in a 45 dB amplifier chain.

The device was illuminated with a Fianium supercontinuum laser, whose pulse duration was specified to be 7 ps. It is crucial for this experiment that the pulse duration is shorter than the

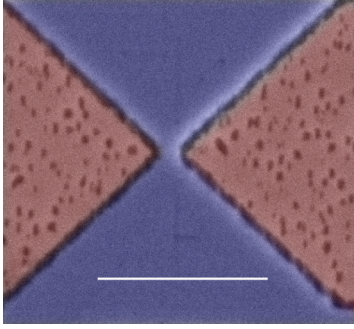


FIG. 1. (Color online) False-color SEM image of the detector. The active part of the detector is the narrow bridge in the center of the image. The blue parts represent the thin layer of NbN, the red parts are the GaAs substrate. The scale bar has a length of $1 \mu\text{m}$.

lifetime of an excitation, which was measured to be several tens of picoseconds.^{20–22} If the pulse duration is longer than that, it is possible to have a pulse that produces two excitations which are far enough apart in time that one has died out before the second is created; this will therefore not result in a multiphoton excitation.

We confirmed that our laser produces coherent states, measuring $g^{(2)}(\tau = 0) = 0.98 \pm 0.01$ in a separate experiment. Furthermore, we measured that the intensity fluctuations in the laser are below 2%. Hence the laser is suitable for tomography.²³ The detector was illuminated with narrow-band light at wavelengths of 1000, 1300, and 1500 nm ($\Delta\lambda = 10$ nm). In our experiment, we vary the intensity and wavelength of the input light, at various bias currents. At each of these settings, we record the count rates in a 0.1 s time window and repeat the experiment 10 times per measurement setting. In the current regime investigated in the present experiment, the detector has negligible dark counts (<1 per second).

III. TOMOGRAPHY OF MULTIPHOTON EXCITATIONS

In order to distinguish the effects of the various photon numbers in the laser pulses, we make use of a sparsity-based tomographic protocol. We give here a brief summary of this protocol (for a full description, see Ref. 18, where we introduced this technique). We illuminate the detector with a range of coherent states, and record the detection probability R_{click} . We make use of two properties of coherent states: first, that a coherent state under attenuation remains coherent, second, that the decomposition of the coherent state in the Fock basis is completely determined by the mean photon number, which can be determined by measuring the intensity.²⁴

Each illumination intensity probes the detector with a different linear combination of photon number states, introducing different combinations of multiphoton excitations. In particular, we model the detection probability R_{click} by

$$R_{\text{click}} = 1 - e^{-\eta N} \sum_{n=0}^{\infty} (1 - p_n) \frac{(\eta N)^n}{n!}, \quad (1)$$

where η is the incoupling efficiency and N is the mean photon number of the incident coherent state. The linear efficiency appears separately, since our protocol enables us to distinguish linear processes—such as incoupling to the NbN film—from

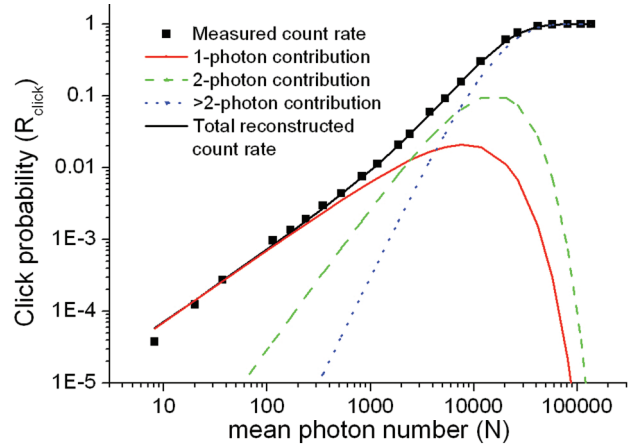


FIG. 2. (Color online) Illustration of the tomographic protocol. The black squares indicate the measured count rate as a function of input power, at $\lambda = 1500$ nm and $I_b = 17 \mu\text{A}$. The red (solid) and green (dashed) lines show the contribution to the count rate of single photons and photon pairs, respectively. The blue (dotted) line shows the contribution of higher numbers of photons. The black line shows the sum of all the photon contributions, indicating that our tomographic reconstruction successfully reproduces the observed count rates. From this fit, we reconstruct the set of detection probabilities p_n and the linear efficiency η , which together fully describe the behavior of the detector.

nonlinear processes.¹⁸ The p_n are the quantities of interest in further analysis: they describe the probability of a detection event, given the absorption of n photons in the active area of the detector.

Figure 2 illustrates this protocol as applied to a single experimental run for a given bias current. We vary the incident power, observe the detection probability, and apply the tomographic protocol to find the contributions from the various multiphoton excitations. The black squares indicate the measured count probability, approaching 1 as the detector saturates. The red, green, and blue lines indicate the contribution from one photon, two photons and higher photon numbers, respectively. Only a limited number of multiphoton excitations is resolvable, and this number depends on bias current. The rest is lumped into a remainder term containing the limit of high photon numbers and is not used in further analysis. The fact that at various powers different multiphoton processes are dominant enables us to recover them all from a single experiment. Furthermore, since the linear efficiency η only rescales the effective incident photon number, but does not alter its shape (corresponding to a simple shift in Fig. 2), we are also able to distinguish finite incoupling effects from effects due to multiphoton excitations.

IV. RESULTS

Figure 3 shows the reconstructed detection probabilities p_n , as a function of bias current and three different wavelengths. For each wavelength and current, we independently perform the tomographic procedure outlined in Sec. III and obtain a full set of parameters p_n . We observe that as the current is lowered, the detector makes a transition from being a one-photon threshold detector to a two-photon threshold detector, and

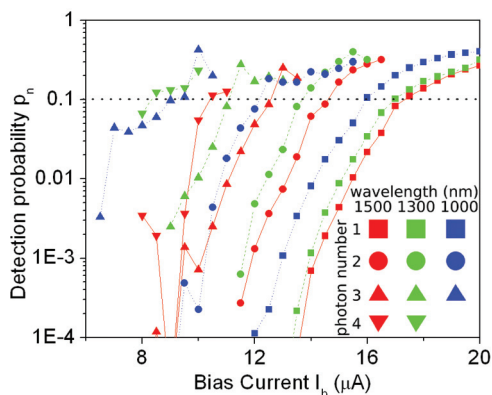


FIG. 3. (Color online) Current dependence of the nonlinear parameters p_n , as a function of wavelength and photon number. The probability p_n of a detection event at a given wavelength and photon number is plotted as a function of the current. The plots are color coded by wavelength. The shape of the symbols indicates the photon number (see legend). The connecting lines are a guide to the eye. The dotted line indicates the threshold level ($p = 0.1$) used to obtain Fig. 4.

so on. Furthermore, we observe that the response curves at different photon numbers and wavelengths have the same shape. We note that as the excitation energy becomes higher and the photon number larger, the points on our curves become more scattered, indicating that the tomography procedure becomes less accurate.

Figure 4 shows the bias current required to reach a detection probability of 10%, as a function of total excitation energy. In order to obtain this figure, we took a surface of constant $p_n(E, I_b) = 0.1$ in Fig. 3 (indicated by a dotted line), and

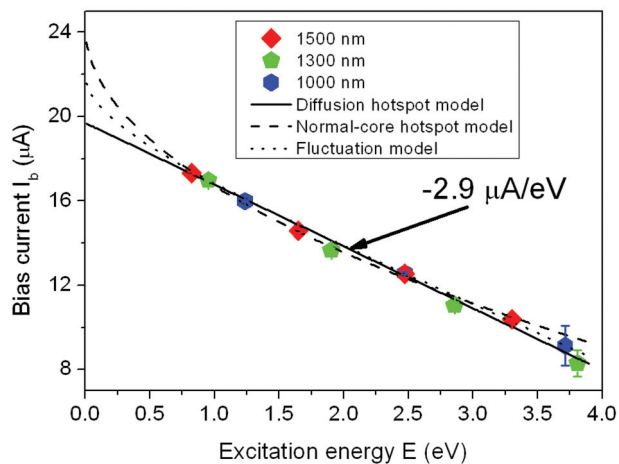


FIG. 4. (Color online) Scaling law for the nanodetector. From the data in Fig. 3, we find all points that have $p_n(E, I_b) = 0.1$ (indicated by the dotted line in that figure), where E is the overall excitation energy. In this graph, we plot the values of I_b and E that satisfy this condition. This graph shows that bias current and overall excitation energy have an approximately linear dependence. The fact that points at various photon numbers all fall on the same line demonstrates that the nanodetector is only sensitive to the overall energy of the excitation. The three lines show the fits of the three microscopic models to the data. Apart from the two rightmost points, the errors on these data points are ~ 100 nA.

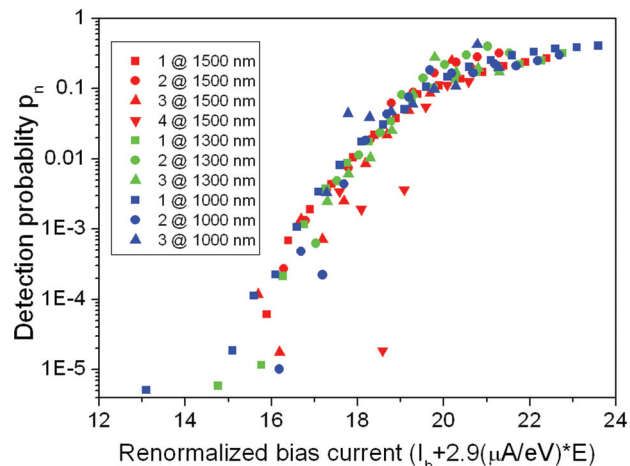


FIG. 5. (Color online) Universal response curve for the nanodetector. To obtain these curves, we rescale the curves reported in Fig. 3 by the scaling law demonstrated in Fig. 4.

plotted the bias current at which the detector has 10% probability of responding to an energy E , where $E = nh\nu$ is the total energy of the n photons absorbed by the detector. This figure demonstrates that there is a scaling law between bias current and overall excitation energy. We determine the scaling constant γ to be $\gamma = -2.9 \pm 0.1 \mu\text{A}/\text{eV}$ ($= -1.8 \times 10^{13} \text{ Wb}^{-1}$ in SI units) for our detector. Furthermore, this figure shows that the detection probability is independent of the way in which the excitation is composed of different photons: only the overall energy determines the detection probability. We note that we have used only a small fraction of the data present in Fig. 3 to obtain the data presented in Fig. 4.

We compare three models from literature to our data. We find that over the range of the experiment, all three models are consistent with our data. The three models are a hotspot-based model, a hotspot-based model in which diffusion plays a large role, and a fluctuation-assisted model. For a full description of the three models, see Appendix. These models distinguish themselves not only by different detection mechanisms, but also by different scaling between bias current and energy at constant detection efficiency.

Figure 5 presents the main result of our paper: a universal detection curve for a single line-segment of an NbN SSPD. In Fig. 5, we apply the scaling law, which was derived from the points around $p = 0.1$ to the entire data set. We find that all the curves of detection probability as a function of rescaled bias current superimpose over more than three orders of magnitude in the detection probability. This shows that the photoresponse of our detector depends only on this specific combination $I_b + \gamma E$ of bias current and excitation energy. We stress that this universal curve can only be obtained through detector tomography, which allows separation of the effects of multiphoton excitation and finite linear efficiency.

The data presented in Fig. 5 show that the scaling behavior, which we obtained at $p_i = 0.1$ in Fig. 4 is universal for all values of p . Since we have used only the points in Fig. 3, which lie around to obtain the result in Fig. 4, we do not *a priori* expect the curves to superimpose when we apply the scaling law to the entire data set. In such a procedure, only the points that are used to obtain the scaling factor are guaranteed to superimpose.

Since the curve is universal over more than four orders of magnitude in the detection probability, we have demonstrated that our results are independent of the arbitrary choice of the 10% criterion. The criterion only matters for the accuracy with which the curves can be superimposed: we find from theoretical simulations that the tomographic reconstruction is most accurate between $p_n = 0.1$ and 10^{-4} . This justifies the choice of our criterion.

V. DISCUSSION

In this section, we first compare our experimental method with that of previous studies on SSPDs. Then we discuss our experimental findings on the universal curve, quasiparticle conversion efficiency. Lastly, we discuss the phenomenon of dark counts in our detector.

A. Comparison with previous work

Previous investigations of the SSPDs detection mechanism were of a semiclassical nature, where only the efficiency and dark count rate were measured. By observing the exponent of the power-law dependence of count rate on input power, one can also infer the photon number detection regime semiclassically. However, such a characterization is limited to the observation that the detector is operating in a particular detection regime; a measurement of the p_i (i.e., how strongly the detector is in a particular regime) requires detector tomography. Since the width of each multiphoton regime is $\sim 2 \mu\text{A}$ for our experiment, the accuracy of the semiclassical method is rather limited. In order to characterize multiphoton processes beyond that resolution, detector tomography is an absolute requirement.

Most previous work focused on meander detectors, which is the geometry that is normally used in practical applications of SSPDs. In a meander, two photons that are absorbed in different places along the wire do not constitute a two-photon event, yet they may still produce one-photon events individually. By using a nanodetector, we sidestep any question of how the photons distribute themselves along the length of the wire, which was a major issue in measuring multiphoton effects in meander-type SSPDs.^{25,26}

Our present work probes the detection mechanism at various energies simultaneously. We are insensitive to incoupling losses, since they affect the various multiphoton processes equally. Furthermore, since we can perform excitations at different energies with the same wavelength, we are insensitive to any wavelength-dependent effects in the experiment, including wavelength-dependent absorption in the NbN layer.

B. Universal curve

The universal curve that we demonstrate in Fig. 5 is not predicted by any of current SSPD photodetection models. Typically, such models focus on calculating a single threshold bias current I_{th} , above which the energy of a photon is large enough to deterministically break the superconductivity. Above that current, the efficiency of the detector should be constant. We have shown in the present work that scaling behavior extends not just to a single threshold current, but to all combinations of currents and excitations in the present

experiment. Scaling behavior applies whether one is in the regime of high efficiency or not. This points to the fact that a single theory should describe detection events in SSPDs, both in the high- and low-efficiency regimes.

C. Quasiparticle conversion efficiency

The fact that only the overall energy of the excitation determines the response of the photodetector can be interpreted in terms of the cascade process that is generated by the initial excitation. This process, which is thought to involve both electrons and phonons in the film, and in which the mutual exchange of energy between the electron and phonon subsystem plays a key role, is still poorly understood. In the present work, we probe this cascade process with different initial excitations, and show that it is only the overall energy that determines the total number of quasiparticles, which are produced at the superconducting band edge. The fact that four excitations of a quarter of the energy produce the same number of QP as a single excitation with the full energy is evidence of the fact that the conversion efficiency by which the energy of the first QP is distributed over many others is independent of the initial energy.

D. Dark counts

We now turn to the phenomenon of dark counts. The most straightforward model is the following: one simply considers a dark count as an excitation with $E = 0$. Extrapolating the linear scaling law from Fig. 4 to $E = 0$ yields a current of $19 \mu\text{A}$. However, at this current, we do not observe a dark count probability of 10% as one would expect from the simple model; we only observe appreciable dark counts around the critical current of $29 \mu\text{A}$. The same discrepancy applies to the other two models. We can therefore say that the picture of a dark count as a zero-energy photodetection event is not supported by our data for any current detection model of SSPDs. The anomalous behavior of dark counts is a reminder of the danger of assuming a detection model, further demonstrating the relevance of our tomographic method. In particular, in this case, the tomographic method gives the first hints of substantial differences in detection mechanism between dark counts and light counts. We note that the nature of dark counts is still open to debate.^{27–29}

E. Outlook

The present work opens up the possibility of testing the various models of photodetection. This could be done by performing the present experiment in the mid-infrared. For this energy range, the predictions of the various models differ significantly (see Fig. 4). For example, at an excitation wavelength of $5 \mu\text{m}$, corresponding to 0.25 eV , the difference between the predictions of the various models is easily measurable; it is of the order of $1 \mu\text{A}$.

Multiphoton excitation has the practical advantage that the bandwidth of energy excitations that is offered can be extended by a factor equal to the number of photons in the highest excitation (in our case, four). This has applications in the situation where light of a particular wavelength is difficult to couple onto a cryogenic sample. In particular, the present work

opens up the possibility of studying NbN detector behavior in an energy range that corresponds to the near and medium UV range, using visible and NIR optics.

In a previous publication,²² we have introduced the notion of the nonlinear response function (NRF) $\eta(I_b, C)$, which measures the instantaneous detection probability, given that a bias current of I_b is present, and that there are C quasiparticles in the detector. The overall detection probability is then given by $R = \int_t \eta(t) I(t) dt$, where I is the instantaneous intensity. This function can be probed by various means such as a pump-probe experiment. The description in terms of an NRF is well matched to a tomographic experiment, as both are model-independent descriptions.

The holy grail of tomographic research on SSPDs would be to find the instantaneous detection probability as a function of the number of quasiparticles present at that instant. In the present experiment, we have achieved a step towards this goal: we have demonstrated the NRF to be of the form $\eta(I_b + \gamma E)$ over the energy range of the experiment, for short-pulse excitations.

VI. CONCLUSIONS

In conclusion, we have studied the physics of photodetection in a superconducting single photon detector. We have shown that the detection is based on the overall energy of the excitation. Furthermore, we have demonstrated a scaling law between overall excitation energy and bias current. From this, we find a universal response curve that depends only on a given combination of bias current and excitation energy. Thereby, we have shown that the known behavior of the detector extends into the multiphoton range. These results demonstrate that the tomographic method is a useful tool for investigating the fundamental physics of detection events in NbN SSPDs.

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APPENDIX: DETECTION MODELS IN SSPDS

1. Introduction

While big strides have been taken^{27–30} in understanding the fundamental physics of these detectors, many details of the detection mechanism in such detectors are still unknown. After the absorption of a photon, it is thought that the resulting high-energy electron destroys Cooper pairs that carry the bias current, producing a cloud of quasiparticles.^{31,32} This process results in a breakdown of the superconductivity, resulting in a

normal cross section. After such a resistive barrier has formed, the kinetic inductance of the device drives Joule heating in the normal state area,³³ causing it to grow. After that, the current drops to a negligible level³⁴ and the interplay between the cooling of the device and the restoration of the current determines whether the device returns to its previous state, ready to detect another photon.

Currently, there is no consensus on a microscopic model for the detection event in superconducting single-photon detectors. Below, we briefly introduce three microscopic models for detection events in the SSPD from literature. We refer to these three as the normal-core hotspot, the diffusion hotspot, and the fluctuation models. Each of these models predicts that the energy and current are exchangeable through a given scaling law, and their prediction will be compared with the experimental data below.

2. Three models

The *normal-core hotspot model* was introduced in the original papers reporting photodetection with SSPDs.^{1,35} In this model, it is assumed that the photon absorption creates a normal core inside the material. Current is then diverted around this core. If the current locally exceeds the critical current, superconductivity is destroyed and a normal-state slab is created, resulting in a detection event. In this model, the current required to pinch off the entire channel for a given input energy scales as the square root of the energy, since the hotspot is assumed to be a cylindrical object inside the wire. For a given energy, the bias current I_b needed to achieve maximal detection efficiency is then given by

$$E = \frac{w^2}{C^2} (1 - I_b/I_c)^2, \quad (\text{A1})$$

where E is the energy of the photon, I_b is the bias current, and I_c is the critical current. w is the width of the wire and C is a scaling constant, which is defined in this way for consistency with previous work.³⁶

In the *diffusion-hotspot model*, which was introduced later as a refinement of the original normal-core hotspot model, the role of the diffusion of quasiparticles is taken into account, as well as the reduction of the critical current due to the quasiparticles.^{30,31} In this model, the relevant length scale is given by the diffusion length over a time characteristic for the cascade of quasiparticles. This expression, which was first derived in Ref. 31, predicts linear scaling between bias current and cutoff energy:

$$E = E_0 (1 - I_b/I_c), \quad (\text{A2})$$

where E_0 is an energy scale.³²

Fluctuation model. The previous two models predict a sharp cutoff of the detection efficiency as a function of photon energy, which is not observed in experiments.³² In order to explain the observed detection probability beyond the cutoff energy, fluctuation-assisted detection models have been proposed.^{27–29,31} All of these have in common that the role of the photon in the detection process is to depress the superconducting gap. Subsequently, a thermally activated fluctuation occurs, such as the depairing of a vortex-antivortex pair (VAP)³⁷ in the superconductor or the crossing of a single

vortex. This fluctuation must overcome an energy barrier $E(\Delta, I_b)$.²⁹ Expressions for such energy barriers typically contain a gap-dependent energy scale and a current-dependent geometric factor.³² The specifics of the geometric factor depend on the precise fluctuation process. For both the VAP and the single-vortex crossing models, we can linearize the current-dependence of the geometric factor over the range of currents that was used in the experiment, to obtain

$$A = (\Delta - \alpha\sqrt{E})(I_0 - \beta I_b), \quad (\text{A3})$$

where the constants I_0 and β are known from the linearization of the geometric factor, and A and α are the experimental fit parameters. Such a model predicts a hyperbolic interchange between the energy and current.

We note the following subtle point: to obtain Fig. 4, one must introduce an arbitrary cutoff criterion, which we have chosen to be $p_i = 0.1$ (see main text). This introduces an offset in the parameters of each model, in particular in the parameter that describes the behavior at $E = 0$. We have constructed the expressions for each model such that the choice of cutoff manifests itself as an offset in a single parameter, namely, the current scale appropriate for that model. The parameters reported below are independent of the choice of the cutoff criterion.

3. Comparison with experimental results

We apply the three detection models to the results in Fig. 4, and compare the results with the values from literature. For the normal-core hotspot model, we find $C = 47 \pm 1 \text{ eV}^{-1/2}/\text{nm}$, which should be compared to the values of $C = 11\text{--}20 \text{ eV}^{-1/2}/\text{nm}$ found in other experiments.³ For the diffusion

hotspot model, we apply the expression from Ref. 32, to find a theoretical value of $\gamma = -2.5 \mu\text{A}/\text{eV}$ for our sample and $\gamma = -3.5 \mu\text{A}/\text{eV}$ for the samples in that reference, which should be compared with the value of $\gamma = -2.9 \pm 0.1 \mu\text{A}/\text{eV}$ obtained experimentally. For the fluctuation model, we find $\alpha = 2.8 \times 10^{-4} \pm 0.05 \times 10^{-4} \sqrt{\text{eV}}$, which should be compared to a literature value of $\alpha = 6 \times 10^{-4} \sqrt{\text{eV}}$, for the experiment reported in Ref. 37. We note, however, that comparisons between different detectors are problematic. In particular, the conversion efficiency of the initial excitation to quasiparticles at the gap edge is a free parameter which varies from detector to detector.³²

The error analysis on the quantities given in the previous paragraph was based on the 50 nA accuracy of the current readout of our experiment, combined with error propagation on the interpolation formula used to obtain the intersection with the line $p_i = 0.1$. For low i , the former error dominates. At higher i , we are limited by the quality of our tomographic reconstruction. We calculate $\chi^{(2)}$ per degree of freedom to be 2.2, 2.9, and 2.1 for the normal-core hotspot, diffusion hotspot, and fluctuation models, respectively. These numbers do not enable us to conclusively rule out any of the models.

In the normal-core hotspot model, multiphoton events can only occur if the various normal cores are created at the cross section of the wire. Given the relative size of our detector and the size of the normal cores in this hotspot model, we would expect to see reduced count rates at high photon numbers due to the low probability of several photons being absorbed at exactly the same point in our detector. The fact that we do not observe such reduced count rates suggests that in our experiment, the size of the hotspot is comparable to the size of our detector.

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