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High sensitivity SQUID-detection and feedback-cooling of an ultrasoft microcantilever

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We measure the motion of an ultrasoft cantilever, carrying a ferromagnetic particle, by means of a superconducting quantum interference device (SQUID). In our scheme, the cantilever motion modulates the magnetic flux in the SQUID due to the coupling with the magnetic particle. For the cantilever fundamental mode, cooled to temperatures below 100 mK, we achieve a dimensionless coupling factor as large as 0.07, displacement sensitivity of $200\,\mathrm{fm}/\sqrt{\mathrm{Hz}}$, and subattonewton force sensitivity. We demonstrate the outstanding combination of very low displacement and force noise by feedback-cooling the cantilever mode to an effective mode temperature of 160 μK.
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In recent years, mechanical resonators, in particular micro and nanomechanical resonators, have been coupled to a variety of quantum devices and ultrasensitive displacement sensors, based for instance on optomechanical, microwave, electromechanical, magnetomechanical, and quantum point contact detection techniques.[1] Applications of ultrasensitive mechanical resonators range from the detection of weak forces, for instance in magnetic resonance force microscopy (MRFM)[2] or gravitational wave detection,[3] to the test of quantum mechanics in macroscopic objects.[4]

A topic that has become increasingly popular is the quest of cooling mechanical resonators to the ground state, which is considered an enabling step in order to prepare a mechanical resonator in nonclassical states. The most remarkable achievement in this sense has been the cryogenic cooling of a 6 GHz resonator and its strong coupling to a superconducting qubit, which has enabled the first demonstration of nonclassical mechanical states.[5] On the other hand, other techniques have been proposed to cool resonators with lower frequency. Sideband cooling to the ground state has been recently demonstrated using microwave[6] and optomechanical[7] cavities. A related technique is active feedback-cooling, based on high precision measurement and control of the mechanical resonator. Feedback-cooling can be applied to a wider range of detectors and in particular it is more suitable for low frequency resonators. Indeed, very large cooling factors and extremely low temperatures have already been achieved through feedback,[8–13] for resonator frequencies in the range 100 Hz-2 MHz. Cooling ultrasoft low-frequency resonators close to the ground state might in principle allow the preparation of well-separated macroscopic quantum superpositions and therefore enable tests of quantum mechanics at macroscopic level, including alternative wavefunction collapse models.[4]

The efficiency of feedback-cooling can be expressed in the following way,[9] in terms of a minimum achievable temperature $T_{\min}$ or a minimum number of phonons $N_{\min}$

$$N_{\min} = \frac{k_B T_{\min}}{\hbar\omega} = \frac{1}{2\hbar}\sqrt{S_f S_x}. \tag{1}$$

Here, S_f and S_x are one-sided power spectral densities, respectively, of the force noise driving the resonator and the detector displacement noise. In particular, approaching the ground state requires $\sqrt{S_f S_x} \approx \hbar$, which is achieved only when the force noise S_f is dominated by the detector backaction, and the detector itself is quantum limited.[14,15]

We have recently demonstrated a scheme to measure the motion of a mechanical resonator by using a superconducting pick-up coil connected to a superconducting quantum interference device (SQUID) to detect a ferromagnetic particle attached to the resonator.[16] Here, we demonstrate an improved version of this technique, in which the ferromagnetic particle is directly approaching the SQUID loop without an intermediate pick-up coil. This configuration allows to reach a much stronger magnetomechanical coupling, which translates into much better displacement sensitivity and feedback-cooling efficiency.

A scheme of the setup is shown in Fig. 1(a). A cantilever mechanical resonator with a ferromagnetic particle with magnetic moment $\vec{\mu}$ attached to its end (from now on, the "magnet") is brought near the superconducting loop of a SQUID. The magnet couples a magnetic flux $\Phi(x)$ in the SQUID, so that a displacement of the cantilever end x will cause a flux change $\Phi_x x$. Here, $\Phi_x = \partial\Phi/\partial x$ is the magnetomechanical coupling. It can be calculated as $\Phi_x = \vec{\mu}\cdot\partial\vec{b}/\partial x$, where $\vec{b} = \vec{B}/I$ is the magnetic field generated in the dipole location by a probe current I flowing in the SQUID loop. The latter formula can be rigorously derived by means of reciprocity arguments.[17] The displacement detection noise spectral density is given by $S_x = S_\Phi/\Phi_x^2$, where S_Φ is the SQUID flux noise spectral density and scales inversely with the square of the magnetomechanical coupling.

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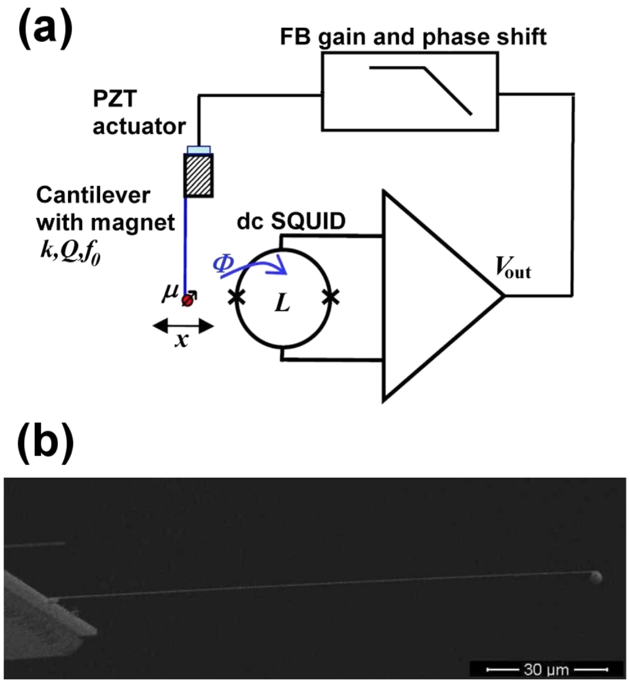

FIG. 1. (a) Schematic diagram of the experiment. (b) Electron microscope micrograph of the cantilever with the magnet attached to its free end.

We can define a dimensionless coupling factor β by the expression $\beta^2 = \Phi_x^2/kL$, where k is the cantilever spring constant and L is the SQUID inductance. β^2 can be thought as the ratio between the magnetic energy $\Phi_x^2 x^2/2L$ coupled into the SQUID loop inductance and the total mechanical resonator energy $kx^2/2$. In a quantum mechanical picture, if L were part of a quantum LC resonator coupled to the mechanical resonator, then $\lambda = \beta\hbar\sqrt{\omega_1\omega_2}$ would be the energy coupling in the interaction hamiltonian. Here, ω_1 and ω_2 are the frequencies of mechanical and electrical systems.

In general, large coupling β can be obtained by making the SQUID loop as small as possible, with thin linewidth, and placing the magnet close to the SQUID at a distance lower or comparable to the loop size. The functional dependence of the coupling on the magnet position and orientation with respect to the SQUID loop can be calculated by means of a magnetostatic model.

In our experiment the resonator is an ultrasoft micromachined silicon cantilever, of the type developed for MRFM,[18] shown in Fig. 1(b). It has a very low spring constant, $k = 90$ μN/m. A NdFeB alloy magnetic particle with diameter 3.0 μm is attached to the cantilever and magnetized as described in Ref. 16. The magnetic dipole $\vec{\mu}$ is oriented parallel to the motion of the cantilever in its fundamental mode. The SQUID is a gradiometric microsusceptometer based on Nb/AlOx/Nb technology. The diameter of each loop is 30 μm, the linewidth is 4 μm, and the total SQUID inductance has been experimentally estimated as $L = 250$ pH. A feedback coil and a field coil are integrated in the circuit. The Josephson junctions are located quite far from the SQUID loop and the magnet, roughly 150 μm, so that the junction critical current is not affected by the magnet static field. The SQUID is operated with a commercial SQUID electronics in two-stage mode with a SQUID array as second stage. The measured SQUID flux noise during the experiment was $\sqrt{S_\Phi} = 1.0$ $\mu\Phi_0/\sqrt{\mathrm{Hz}}$. This noise level is about a factor of 2 higher than the intrinsic SQUID susceptometer noise. We attribute the excess noise to the non-optimal working point of the second stage SQUID array, which was caused by a failure in the wiring of the array flux bias line.

The SQUID chip is mounted on a custom made three-dimensional piezo fine-stage with a range of 2 μm at cryogenic temperature. The cantilever is oriented perpendicular to the SQUID chip surface, in order to avoid snap-to-contact, and is mounted on a custom made three-dimensional coarse approach based on piezo rotators,[19] with a range up to 1 mm. The combined use of both stages allows for an easy alignment of the magnet above the SQUID loop and for the optimization of the magneto-mechanical coupling. The alignment can be performed at low temperature, starting with an initial misalignment as large as 300 μm, using the magnetic flux coupled into the SQUID by the magnet as a guide. A small piezoelectric actuator placed underneath the cantilever chip allows both to drive the cantilever for mechanical characterization and to apply a feedback force. The assembly is mounted on mechanical suspensions cooled in a commercial cryo-free pulse-tube dilution refrigerator.[20] During the experiment reported here, the base temperature of the suspended mass was about 28 mK.

We have characterized the cantilever fundamental mode by means of ringdown measurements. Far from the surface, the frequency is $f_0 = 4163$ Hz and the quality factor is $Q = 4 \times 10^4$. When the cantilever is close to the surface, we observe a position dependent frequency shift and additional damping, in part due to magnetic coupling to the insulator surface spins[21] and in part due to the diamagnetic shielding of the SQUID superconducting lines. We have experimentally determined a position with relatively large coupling at a distance of about 5 μm from the SQUID loop line, where surface-induced nonlinearities are not an issue. Here, the frequency was $f_0 = 4450$ Hz, while the Q factor was slightly temperature dependent, about $Q = 4 \times 10^4$ at 1 K and $Q = 2.8 \times 10^4$ at $T < 100$ mK.

Subsequently, we have characterized the cantilever brownian motion. Inset of Fig. 2 shows two spectra of the SQUID output signal acquired at two different bath temperatures, $T = 28$ mK and $T = 470$ mK. The noise spectrum is remarkably clean from spurious peaks, showing that vibrational and electromagnetic noise generated by the pulse-tube is efficiently attenuated by the mechanical suspensions. Measurements of the area under the Lorentzian noise peak at several bath temperatures show a linear behaviour for temperature higher than 200 mK, demonstrating that cantilever motion is thermal and allowing for an absolute calibration. For bath temperatures below 150 mK, the cantilever appears to decouple from the thermal bath, and its effective noise temperature saturates at approximately $T_0 \approx (90 \pm 10)$ mK. This saturation temperature is significantly higher than that ($T_0 = 25$ mK) observed in a previous experiment using a different setup, in which the cantilever motion was detected by a pick-up coil connected to a remote SQUID.[16] We have checked that the saturation temperature observed here does not depend significantly on the magnet-SQUID distance and coupling and on the SQUID working point. This suggests that the cantilever overheating is not dominated by SQUID Josephson radiation dissipated in the magnet. Instead, we

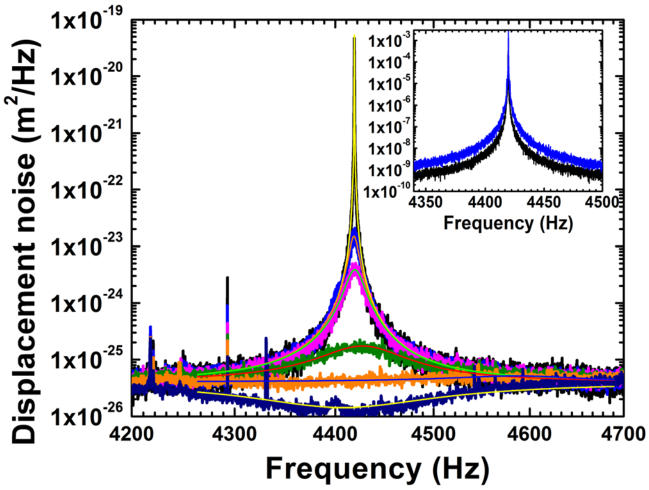

FIG. 2. Inset: SQUID output voltage noise, in $V/\sqrt{Hz}$, showing cantilever thermal noise, acquired at two different bath temperatures, 470 mK (top curve) and 28 mK (bottom curve). Main panel: feedback-cooling of the cantilever starting from an initial effective temperature $T_0 = 90$ mK, for different gain g. The noise spectra are calibrated in cantilever displacement and refer, from top to bottom, to $g = 0$, 60, 118, 560, 1032, 1960. The best fit of each spectrum with the model developed in Ref. 9 is also shown. The effective cantilever temperature extracted from the fitting model is, respectively, for the curves from top to bottom, $T = 90$, 8.2, 1.5, 0.76, 0.20, 0.16, 0.18 mK

observe a very slow trend to further cooling, with time constant of the order of several hours, suggesting that the saturation temperature is rather limited by a poor thermalization of the coarse approach stage which supports the cantilever chip. An optimized thermal design of the latter should then lead to a further reduction of T_0. We point out that this problem could not show up in the previous experiment of Ref. 16, because in that case the cantilever chip was rigidly mounted on top of the pick-up coil chip, rather than on a separate coarse approach stage.

From the calibrated cantilever temperature, and the estimated value of k and Q, we can infer, using the fluctuation-dissipation formula, the thermal force noise $\sqrt{S_f} = (0.8 \pm 0.1) \mathrm{aN}/\sqrt{\mathrm{Hz}}$. Furthermore, we can infer the absolute cantilever mean displacement fluctuation $\langle x^2 \rangle = k_B T/k$ and from this, the magnetomechanical coupling $\Phi_x = (5.3 \pm 0.5) \times 10^6 \quad \Phi_0/\mathrm{m}$, the dimensionless coupling $\beta = 0.07 \pm 0.01$, and the displacement noise floor $\sqrt{S_x} = (200 \pm 20)\ \mathrm{fm}/\sqrt{\mathrm{Hz}}$. This is about 4 orders of magnitude in energy better than our previous experiments with an intermediate pick-up coil[16] and about 2 orders of magnitude better than interferometric detection of ultrasoft cantilever at subkelvin temperature.[9] Despite the relatively large coupling factor, we estimate that the backaction force noise of the SQUID is still negligible, about 30 times lower than the thermal force noise. This is largely due to the relatively low quality factor of the cantilever.

Feedback-cooling is performed by using the SQUID signal to apply a feedback force to the cantilever through the piezo actuator. We apply a viscous feedback force by passing the feedback signal through a low-pass filter which allows for variable gain and nearly $-90°$ phase shift. Under purely viscous feedback, the quality factor is reduced from the intrinsic value Q_0 to an effective value $Q = Q_0/(1+g)$, where g is a normalized gain factor.[22] Fig. 2 shows the power spectral density of the cantilever thermal motion measured by the SQUID for different values of g. For low g, the measured cantilever noise is still Lorentzian and the mean energy is reduced to an effective value $k_B T \approx k_B T_0/(1+g)$. For high g, noise correlations introduced by the feedback modify the Lorentzian peak into a Lorentzian dip. For arbitrary gain g, we can use the model developed by Poggio *et al.*,[9] who have determined the analytic expression of the measured and the actual displacement spectral density in a similar situation. This model allows to determine the effective cantilever energy $k_B T$ even in the high g limit and predict the existence of a minimum in the effective energy as a function of g. In Fig. 2, the best fits of the experimental spectra at different gain g are shown. The gain g and the effective temperature T can be determined as fitting parameters. We find that the maximum cooling factor is achieved when the Lorentzian spectrum is completely whitened ($g = 1032$), and we determine the corresponding temperature as $T_{\min} = (160 \pm 10)\ \mu$K. This is equivalent to a mean number of phonons $N_{\min} \approx 760$. For even higher gain, the effective resonator temperature increases, due to the injection of displacement detection noise by the feedback, which generates an additional driving force.

Our result represents the lowest temperature achieved to date by feedback-cooling of a soft micromechanical resonator, improving by a factor of 20 over previous results.[9] This is a consequence of the simultaneous combination of ultralow force noise and displacement noise, the latter being a consequence of the high magnetomechanical coupling factor achieved in this experiment. Equation (1) states that a further progress will necessarily require a significant reduction both in S_f and S_x. In our scheme, S_x can be reduced in two ways. The first is to further increase the magnetomechanical coupling. This can be easily done in our scheme by optimizing the geometrical parameters. For instance, a factor of 8 can be gained by doubling the magnet diameter, as Φ_x scales with the magnetic moment μ and thus with the magnet volume. The second is to replace the SQUID with an even better magnetic flux sensor, like the recently demonstrated Josephson parametric amplifier (JPA), which is expected to be quantum limited.[23] On the other hand, S_f can be slightly improved by a better thermalization of the cantilever holder, but a more significant reduction will eventually require a radically different mechanical resonator, possibly with much higher Q. In this case, back-action from the detector can become dominant on the thermal noise, and the resonator noise will be completely determined by the detector. An interesting possibility, which can be naturally compatible with SQUID detection, is the recently proposed magnetomechanical resonator consisting of a μm size superconducting particle levitated in a trapping magnetic field.[24] The combination of an ultrahigh Q levitated resonator with a quantum limited amplifier may eventually allow ground state cooling of the center of mass of a micron size particle, enabling the creation of a quantum superposition of spatially separated states of a macroscopic object and test of wavefunction collapse models.[4,24]

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