On multifield inflation, adiabaticity, and the speed of sound of the curvature perturbations

Atal, V.

Author: Atal, Vicente
Title: On multifield inflation, adiabaticity and the speed of sound of the curvature perturbations
Issue Date: 2016-03-08

1

Introduction

Our knowledge of the content and history of the physical cosmos has seen an unprecedent revolution in the last century. In these years we have seen how the scientific method was finally capable of providing insights to many questions that were previously only of the realm of the metaphysical experience and speculation. As a result a widely accepted cosmological model has been established, which describes a vast and evolving Universe since fractions of a second after the so-called Big-Bang until today. The history of this revolution is a good lesson on how physics is nothing more than the description of the observed natural world and that without an *observed* world, it is very difficult -if not impossible- to make definitive scientific claims. The onset of this revolution was, first, the discovery of the recession of distant galaxies in the 1920's, and later on the measurement in the 60's of a uniform cosmic microwave background radiation. The realization that the Universe is dynamical, and that the microwave radiation is a relic from a primordial era of its evolution, were the clues that finally converted cosmology into a scientific discipline.

These transformations were possible not only by the advent of new technologies which made observations possible, but also because the necessary mathematical tools to describe spacetime and its internal constituents were at hand. The advent of General Relativity in the beginning of the 20th century (exactly 100 years ago when writing this introduction) made it possible to make definite predictions which could finally be under the scrutiny of data. This fortunate conjunction between the presence of data and a variety of well motivated models for the Universe eventually made it possible to arrive at what we call today the Standard Cosmological Model.

While these developments are undoubtedly tremendous steps towards an understanding of the Universe, the Standard Cosmological Model may well be considered just a parametric model of our ignorance. Indeed, we have little idea of what the physical origin of many of its free parameters is. As it is, the main contributions to the energy density of our present Universe come from two unknowns forms of energy, dark energy and dark matter, that together sum up to 96% of the total present energy density. While dark matter may be embedded in rather minimal extensions of the Standard Model of particles, understanding the physical origin of dark energy (in particular its incredibly small value compared to the Planck scale) may well need yet another paradigmatical revolution. Furthermore, if the Universe is an evolving system we need to understand its initial conditions. Contrary to other areas of physics, we do not have the experimental privilege to *create* Universes, and study their subsequent evolution as a function of fixed initial conditions. We can however infer them from its present stage of evolution, and the likeliness or unlikeness of the initial conditions is also a measure on how satisfactory our cosmological model is. As we will see later, we will have to deal with the fact that the initial conditions that result in our present observed Universe may not seem natural.

While we cannot replicate the evolution of the Universe in the laboratory, the Universe can in itself be understood as a major experiment in which extremely high energy processes have occurred. This is the "poor man's laboratory" through which we can observe the spectrum of particles present at energies which are beyond any direct experimentation. We expect these observations to be essential for resolving the problems mentioned above, knowing that any resolution will be an important step forward in the construction of a theory able to reconcile all of the fundamental forces and particles into one single consistent picture.

The question of initial conditions, and the possibilities of accessing high energy states is going to be, in a very broad sense, the framework for this thesis.

1.1 The homogeneous and isotropic model of the Universe

The first step leading to our current cosmological model is the observation that at sufficiently large scales the Universe looks homogeneous and isotropic. This can be implemented in General Relativity while still allowing for a dynamical Universe, by describing spacetime with the so-called Friedmann-Lemaître-Roberston-Walker (FLRW) metric, given by[1]:

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1-\kappa r^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right) \ . \tag{1.1}$$

In this model, κ is the curvature of the three-dimensional space, and $a(t)$ -called the scale factor- parametrizes the dynamics of the metric. On large scales, where local

[1]As usual done in cosmology, we work in units in which the speed of light $c = 1$.

inhomogeneities induce small displacements and velocities compared to the background cosmic flow, the physical distance R between two observers is $R(t) = a(t)\chi$, where χ is defined as the *comoving* (or coordinate) distance. The comoving distance between these observers is constant[2], and then all the time dependence of their physical distance is set by the scale factor $a(t)$. In a flat metric ($\kappa = 0$) and for simplicity only considering radial separations, their comoving distance is simply $\chi = \Delta r$. As only $a(t)$ determines the evolution of large scale physical distances, a history of the Universe might be understood as a history of the scale factor $a(t)$.

The time dependence of the scale factor is intimately related to the constituents of the Universe. In particular, Einstein's equations couple the scale factor and its time derivative to the density and pressure of the different 'matter' constituents. For a single fluid with pressure p and density ρ, the relevant equations are:[3]

$$H^2 \equiv \frac{\dot{a}(t)^2}{a^2} = \frac{\rho}{3} - \frac{\kappa}{a^2} \ , \tag{1.2}$$

$$\dot{\rho} + 3\frac{\dot{a}(t)}{a}(\rho + p) = 0 \ , \tag{1.3}$$

where a dot represents a derivative with respect to cosmic time. The first equation is known as the Friedmann equation, while the latter is the continuity equation, ensuring energy conservation. We have further defined the Hubble parameter H, whose magnitude and evolution will set an important cosmological scale. The Friedmann equation (1.2) can be written in terms of the density parameter $\Omega \equiv \rho/\rho_c$ (or equivalently the curvature density parameter $\Omega_\kappa \equiv 1 - \Omega$), where $\rho_c \equiv 3H^2$ is known as the critical density, as

$$1 - \Omega = \Omega_\kappa \tag{1.4}$$

$$= -\frac{\kappa}{(aH)^2} \ . \tag{1.5}$$

The density parameter Ω determines whether the Universe is closed ($\Omega > 1$), flat ($\Omega = 1$) or open ($\Omega < 1$).

Additionally, from equations (1.2) and (1.3) we can derive an equation for the acceleration of the scale factor, which will later prove important:

$$\frac{\ddot{a}}{a} = -\frac{\rho}{6}(1 + 3w) \ . \tag{1.6}$$

Here we have further introduced the equation of state parameter w which links the pressure to the density of a perfect fluid, as $p = w\rho$. Each different constituent of the

[2]Observers moving with the background cosmic flow are also called comoving observers

[3]Throughout all this thesis we work in units in which the reduced Planck mass $m_{\rm pl} = M_{\rm pl}/\sqrt{8\pi} \equiv 1$. In units in which $c = \hbar = 1$, the Planck mass is given by $M_{\rm pl}^2 = G^{-1}$, where G is the gravitational constant.

Universe has a different equation of state. For example, normal matter is effectively presureless (because of its small velocity compared to speed of light), so it is described by an equation of state parameter $w = 0$. Radiation instead is described by $w = 1/3$. An element that does not dilute as the Universe expand, i.e. that has a constant energy density, satisfies $w = -1$. This is the cosmological constant, and its value for the energy density is usually called Λ, i.e. $\rho_{\text{c.c.}} = \Lambda$.

The first observation that pointed towards a dynamical history of the scale factor $a(t)$ was the observation of the recession of distant galaxies[4]. Distant galaxies recede from each other at a velocity v that is proportional to their distance d. This is the famous Hubble law:

$$v = H_0 d \ . \tag{1.7}$$

The factor of proportionality H_0 is the present expansion rate, and its value is constrained with percent level precision by present cosmological data. The Hubble law has striking consequences. If galaxies are moving away from each other, it immediately follows that galaxies were closer as we go back in time. As can be deduced from eq. (1.6), if the Universe is filled with normal matter during all its evolution ($w > 0$) this process is never reversed. Using the known abundances of the different elements that contribute to the total energy density of the Universe, we can arrive at the conclusion that our observable Universe was contracted to a spacetime singularity 13.8 Gyr ago. In reality, however, we have no clue as to whether the Universe is infinitely old or not: neither the equations of General Relativity nor the matter content of the Standard Model of particles should be extrapolated back to the singularity. In particular, unknowns forms of energy may emerge, and a quantum theory of gravity would be needed. This will not be an impediment for loosely setting the starting time of the cosmic clock at the *would be* singularity, having in mind that we should not necessarily attribute to it any deep physical meaning.

In an expanding Universe, as we go back in time the same amount of matter was circumscribed to a smaller volume. This means that the energy density, and therefore the overall *temperature* of the Universe were higher. For example, far enough in the past, the energy density would have been so high that the common building blocks of our present Universe, like stars and galaxies, could not exist. As temperature is proportional to some (negative) power of the scale factor, a *thermal* history of the Universe can be derived. This is very useful since many important physical processes depend on temperature (for a detailed account, see [4]). In particular, below certain critical temperatures gauge symmetries can be spontaneously broken and phase transitions occur,

[4]While usually attributed to Hubble in 1929 [1], already in 1927 Lemaître was able to *derive* the linear relation between velocity and distance from General Relativity, and test it against data [2]. For an historical account on this discovery, see e.g. [3].

as for the electroweak and quantum chromodynamics sectors of the Standard Model of particles. Other processes like particle and antiparticle annihilation also depend on temperature (in this case, whether it is below or above the particles' rest mass), as well as the interaction rates between different particles. All of these elements make the thermal history of the Universe a very peculiar process. Considering the fundamental particles and their interactions, it was shown very early [5] that a Hot Big Bang scenario predicts the creation of light atomic elements. This is the subject of Big Bang nucleosynthesis (or BBN), and its predictions for the relative abundances of the primordial elements (from H to Li^7) is one of the most famous and -relatively- well tested predictions of the Hot Big Bang model.

We can begin a (very) brief thermal history of the Universe 1 sec after the "singularity", when the temperature was about 10^{10} K: back then the energy density of the Universe was dominated by a plasma consisting of electrons, protons, neutrons and photons, all of them in thermal equilibrium. As Thomson scattering between photons and electrons was very efficient, this plasma was homogeneous and opaque (in the sense that photons could not free stream for long distances). On the other hand species like neutrinos and dark matter, having smaller cross sections, were decoupled from the plasma. At a temperature of 10^9 K, or 100 s after the singularity, some of the protons and neutrons bound in the form of atomic nuclei, in particular He^4. Eventually, when the temperature dropped to 10^3 K, 380.000 years after the singularity, atomic nuclei could bind with electrons, forming the first atoms. This process is called recombination. Soon after, with a reduced density of free electrons, Thomson scattering became inefficient: photons could travel freely, and the Universe became *transparent*. This is known as the time of decoupling, and it is the furthest event we can directly observe with photons. The photons that we observe today that scattered for the last time at recombination emerged from a thin shell that we call the surface of last scattering. After decoupling, gravity became the major driving force, making atomic clouds -mostly composed by hydrogen- collapse to form the first stars and galaxies.

At the time of decoupling the plasma was at a finite and homogeneous temperature, and then the photons that emerged from this plasma, having travelled without further interactions, should be at a finite and homogeneous temperature today. This temperature (T_0) is directly related to the temperature at decoupling ($T_{\rm dec}$) by noticing that temperature drops as a^{-1}, such that $T_0 = T_{\rm dec} a_{\rm dec}/a_0$, where $a_{\rm dec}$ and a_0 are the scale factor today and at decoupling respectively. These arguments led Alpher and Herman [6] to predict that if the Universe has been expanding there should be, today, a uniform bath of radiation at a temperature of ~ 5 K. This is the famous Cosmic Microwave Background (CMB).

The observational milestone that strongly supported this Big-Bang picture (as was pejoratively named by one of its firmer detractors), was the discovery of the CMB. In 1965, Penzias and Wilson [7, 8] measured a constant and isotropic radiation, consistent with a CMB blackbody spectrum at a temperature $T_0 = 3.5 \pm 1$ K. In 1993, the COBE satellite [9] determined that the CMB indeed follows a blackbody spectrum by measuring the intensity of the signal at different wavelengths. Its temperature was measured to be $T_0 = 2.7$ K, which is consistent with the first measurement of Penzias and Wilson. Crucially, the same satellite also measured the presence of small inhomogeneities (of the order of 10^{-5} with respect to the homogeneous 2.7 K component), when comparing the temperature at different directions in the sky [10].

The discovery by COBE of these small inhomogeneities is, with the discovery of the late time accelerated expansion of the Universe, among the major recent breakthroughs in observational Cosmology. The importance of the primordial inhomogeneities is two-fold. First, they are nothing less than the initial density perturbations from which galaxies and clusters of galaxies formed. To understand the process of structure formation necessarily implies that we need to know how to characterize its initial conditions. Secondly, the initial conditions for large scale structure formation are also the final state of an earlier stage of evolution. In this sense, they are a portal to access the Universe at times way before the generation of the CMB map. These are the reasons why understanding the initial inhomogeneities is one of the major goals of modern Cosmology, and why so much effort has been made in order to measure these inhomogeneities with better precision and at a wider range of scales. The latest space mission (able to cover the full sky) designed for this purpose was the Planck satellite [11]. In figure (1.1) we show the CMB map of inhomogeneities, as measured by this experiment. By computing n-point correlation functions from this map it is possible to extract quantitative statistical information. The power spectrum, which measures the correlation in temperature between two directions separated by an angle θ in the sky, is shown in figure (1.2)

1.1.1 The contents of the Universe

Observations of the CMB have made it possible to construct a minimal model with 6 free parameters, able to consistently describe the main features of our observed Universe. In this model the Universe has been expanding for 13.8 Gyr until today. It is composed by matter, radiation, neutrinos and a cosmological constant (denoted by Λ, which as we explained above, is a form of energy that does not dilute as the Universe expands). Matter itself is composed by atoms and their internal constituents, and an elusive form of matter called dark matter. The presence of dark matter has only been inferred by its gravitational effects, so little is known about its internal constitution (for example, on

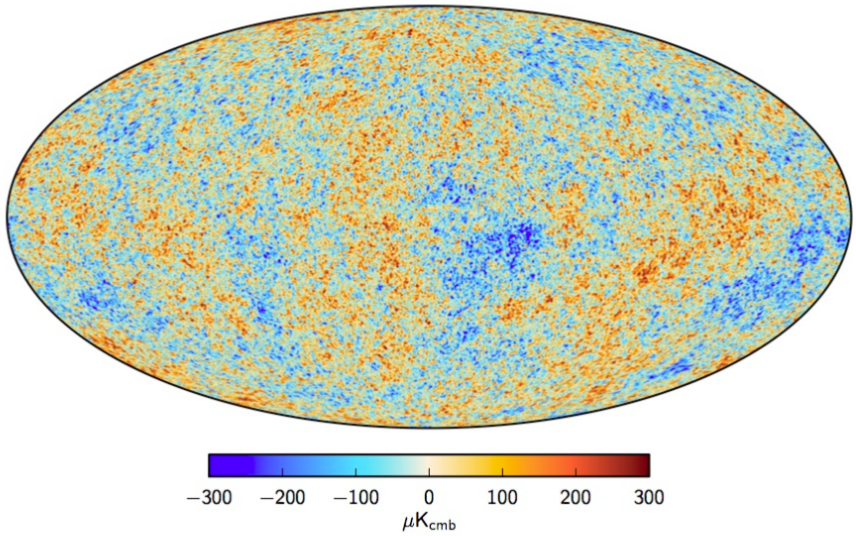

FIGURE 1.1: CMB intensity map. Colours represent inhomogeneities of the order of 10^{-5} with respect to the homogeneous 2.7 K component.

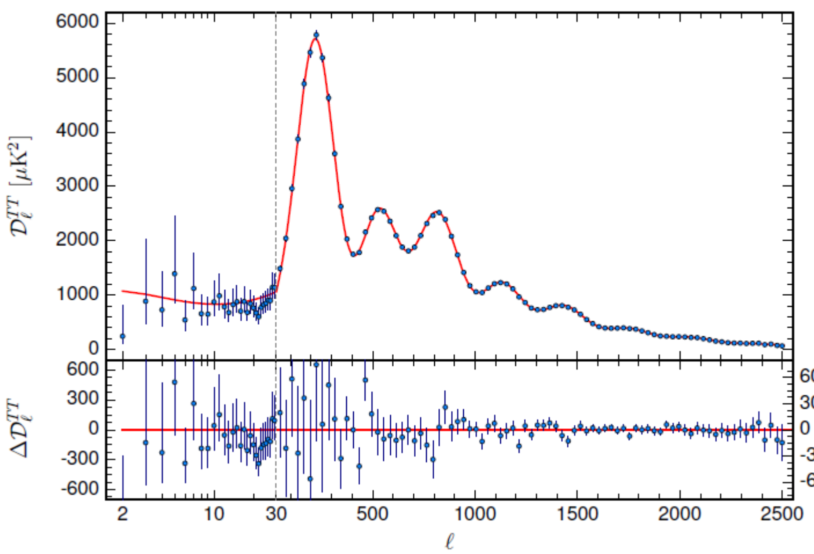

FIGURE 1.2: Power spectrum of the CMB temperature map. The horizontal axis represents the angular scale $l \sim 180/\theta$ (a logarithmic scale is used in the interval $l = [2, 49]$), while the vertical axis is the intensity of the power spectrum. The first peak is located at $l \sim 220$. The red curve is the theoretical prediction of the best fit ΛCDM model (to be explained in the following section), while blue dots are the data points taken from the map in figure 1.1. Figure taken from [12].

whether is charged under a certain gauge group). The present conclusion is that most of the matter should be composed of cold -slowly moving- dark matter (CDM). The so-called normal hierarchy is assumed for neutrinos (with an effective number $N_{\text{eff}} = 3.046$, and well approximated by a single massive neutrino), and the radiation density can be derived from the homogeneous component of the CMB temperature. As the energy density of the present Universe is dominated both by a cosmological constant and cold dark matter, this model is called ΛCDM[5]. The Universe is just a box in which these elements (and the size of the box) evolve according to their relative abundance and initial conditions. The minimal cosmological model assumes the Universe is flat ($\kappa = 0$), and its 6 parameters are:

- Ω_b: Fraction of so-called baryonic matter. It represents atoms and their internal constituents (so, in this definition, leptons also make part of baryonic matter).
- Ω_c: Fraction of cold dark matter. This is a type of matter that has, if any at all, very suppressed interactions with Standard Model particles. We can infer its presence only by its gravitational effect.
- τ: Optical depth to the epoch of reionization. When the first stars formed, new ionizing photons where produced, so that free electrons were again present to eventually interact with CMB photons. Roughly speaking τ measures the strength of that secondary interaction (or, equivalently, the number of additional scattering events).
- θ_{MC}: Approximation to the angular scale of the sound horizon, defined as the sound horizon at the surface of last scattering divided by the distance to the surface of last scattering. The sound horizon is the distance sound waves of the photon-baryon fluid could have travel in the time up to recombination. θ_{MC} roughly corresponds to the angular size of the first peak in the two-point correlation function that can be seen in fig. 1.2.
- A_{S}: In order for inhomogeneities to be seen in the CMB, an initial amplitude for these inhomogeneities should have been present. A_{S} is the amplitude of the initial two-point function (to be defined more precisely later) at a given scale.
- n_s: The amplitude of the initial fluctuations may depend on the scale. The quantity n_s is the spectral dependence of the two-point function, when parametrized as a power law ($n_s = 1$ being scale invariance).

Given these parameters, it is possible to reproduce to a great accuracy the CMB data as well as the abundances of the primordial elements. We show in table 1.1 their best fit values as deduced from the Planck 2015 analysis [12]. Apart from the first six parameters, we have added the values of three additional parameters that, while they are not part of the baseline cosmological model, are very important for the subject of this thesis.

[5] It is very important to stress that, *within this model*, the universe is dominated by these constituents. Different cosmologies may lead to very different values for different parameters. The ΛCDM model continues to be preferred since it has the best compromise between number of parameters and goodness of fit to the data.

Parameter	Best fit value at 68% limits
$\Omega_b h^2$	0.02230 ± 0.00014
$\Omega_c h^2$	0.1188 ± 0.0010
$100\theta_{MC}$	1.04093 ± 0.00030
τ	0.066 ± 0.012
$\ln\,(10^{10} A_{\rm S})$	3.064 ± 0.023
n_s	0.9667 ± 0.0040
Ω_k	0.000 ± 0.005
r	< 0.09
β_{iso}	< 0.03

TABLE 1.1: Best fit parameters of the 6 parameter ΛCDM model. The factor h appearing in front of $\Omega_{c,b}$ denotes the current expansion rate, as a fraction of 100 km s^{-1} Mpc^{-1}. The last three parameters are important extensions to the 6 parameter ΛCDM model.

These are the tensor-to-scalar ratio, r, which is the ratio of the initial tensor to scalar perturbations, the curvature density of the universe Ω_k (defined in 1.4) and β_{iso}, the initial isocurvature fraction. We will later define them more precisely (in particular r and β_{iso}), and their importance will become more transparent as we go further on.
In order to solve any dynamical system it is important to set the initial conditions, so we should not be surprised if we need some parameters to describe them. We should be surprised, however, if the initial conditions consistent with the observations are a very special subset of all the possible initial conditions. This is exactly what happens in the base ΛCDM cosmology. This issue is present at different levels:

- The first observation that should call our attention is the fact that the CMB can be described by a single temperature, i.e., that the CMB is a bath of radiation, of the size of the observable Universe, in thermal equilibrium. This would not be an issue if there were the means of reaching thermal equilibrium in the very early Universe, which would demand the very different patches of the Universe to be in causal contact and with large interactions among its constituents. The size of a causal patch is known as the particle horizon, and it is given by the distance light could travel from an initial time t_0 to any given time $t_\star$. The comoving particle horizon at $t = t_\star$ is given by

$$h = \int_{t_0}^{t_\star} \frac{dt}{a(t)} = \int_{a_{ini}}^{a_\star} \frac{d\ln a}{aH} \; . \tag{1.8}$$

Taking $t_0 = 0$, $t_\star$ to be the time of decoupling and assuming the scale factor is determined by the ΛCDM model, one finds that only patches separated by less than 2° in the sky could have been in causal contact. This mean that patches of the CMB separated by more than 2° were never in their history in causal contact, and could not by any thermodynamical mechanism reach the very same temperature. This apparent acausal correlation is also seen at the level of the perturbations, since a non-zero correlation for the two-point function is also measured on such large scales (see fig 1.2). This is known as the horizon problem.

- Another issue concerns the observation that the Universe is flat to an incredible accuracy. If, today, the energy density associated with the curvature is measured to be negligible, it means that the curvature energy density was exponentially smaller in the past. This can be seen by considering how the curvature parameter, Ω_κ, evolves with time. From equation (1.5) this is given by

$$\frac{d\Omega_\kappa}{d\log a} = \Omega_\kappa(1-\Omega_\kappa)(1+3w) \ . \tag{1.9}$$

Unless the Universe is exactly flat ($\Omega_\kappa = 0$), the curvature density parameter depends on time. Moreover, for $(1+3w) > 0$, which is the case in a Universe dominated by radiation and/or matter, small departures from flatness are said to be *unstable* since any small initial curvature density Ω_κ (positive or negative) will grow as the size of the Universe grows. Today we have an upper bound $|\Omega_k| < 10^{-2}$ which means that $|\Omega_k| < 10^{-18}$ at BBN era, and exponentially smaller as we go back in time. As there is no *a priori* reason in General Relativity to choose $\kappa = 0$ among all the possibilities, we may be puzzled by the fact that such value was initially tuned to zero to such great accuracy.

1.2 Inflation

The theory of inflation was proposed as a solution to these problems, and it can be understood as a theory for the initial conditions of the ΛCDM cosmology [13–17]. Its real achievement is that it pushes back the problem of initial conditions to $\sim 10^{-30}$ s after the singularity[6]. It is not difficult to see that both the horizon and curvature problem are linked to the fact that the comoving Hubble radius, defined as $r_H = (1/aH)$, increases as the scale factor increases. Then, one solution to the problem of the initial conditions is to impose that at some earlier period the comoving Hubble radius decreased

[6]Inflation also needs initial conditions which may or not be considered natural. Whether they are or not or whether they should be at all is of course a very important question.

with time. This would in turn mean the Universe was accelerating, as

$$\frac{d}{dt}\frac{1}{aH} < 0 \quad \rightarrow \quad \ddot{a} > 0 \ . \tag{1.10}$$

Inflation is just a period in which these conditions are fulfilled. This simple behaviour naturally generates the initial conditions for the primordial plasma: On the one hand, in an inflationary period the integrand in (1.8) rapidly diverges in the past, making the particle horizon substantially bigger as we go back in time. This can make all the observable Universe to have been in causal contact. Additionally, the curvature density converges at late times to zero as can directly be seen from its definition. A successful solution of these problems needs at least 60 e-folds of inflation, by which we mean that the ratio of the scale factor at the beginning and at the end of inflation is $\sim e^{60}$. As we will see shortly, this very singular causal structure, i.e. a shrinking Hubble radius, will also provide an elegant mechanism to generate the initial perturbations for the matter distribution in the Universe.

From equation (1.6) it is clear that an exotic type of matter should be dominating the energy density of the Universe such that an accelerated expansion can take place. Neither radiation nor baryons nor CDM can achieve such an expansion, since $w_{\gamma,b} > -1/3$. One possibility, though, is that the energy density is in the form of a cosmological constant, since $w_\Lambda = -1$. Ending inflation would demand tunneling to a different vacuum with a smaller cosmological constant, as for example, the one we measure today. While apparently satisfactory, this first order phase transition would cause bubble nucleation, spoiling the isotropy and/or the spectrum of the primordial perturbations [18]. These problems can be avoided by considering instead a scalar field classically rolling down in a potential. If the potential is flat enough, the scalar field would *resemble* a cosmological constant and produce an accelerated expansion. However, and contrary to the first scenario, in this case inflation can ends smoothly if, after the required e-folds of inflation, the potential becomes steep. To see how this could happen, consider a scalar field $\phi(\mathbf{x}, t)$, canonically coupled to gravity. It is then described by an action of the form

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \ . \tag{1.11}$$

Here $g_{\mu\nu}$ is the spacetime metric and R and g are respectively the Ricci scalar and the determinant of this metric. We have further introduced $\partial_\mu\phi \equiv \partial\phi/\partial x^\mu$, where μ ranges from zero (time) to three (space). In a FLRW metric the equation of motion for the homogeneous component of the field, $\phi = \phi(t)$, is given by

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0 \ , \tag{1.12}$$

where $V_{,\phi} = dV/d\phi$. The Hubble parameter H is determined by the Friedmann equation (1.2), which in this case reads

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \ . \tag{1.13}$$

Equations (1.12) and (1.13) determine the joint evolution of the background component of the scalar field $\phi(t)$ and the scale factor $a(t)$. A shrinking Hubble radius is equivalent to demanding that the Hubble parameter varies slowly. Indeed, it is easy to show that

$$\frac{d}{dt}\frac{1}{aH} < 0 \quad \rightarrow \quad \epsilon \equiv \frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{H^2} < 1 \ . \tag{1.14}$$

A slow variation of H ($\epsilon \ll 1$) implies then that the kinetic energy is a small fraction of the total energy, or equivalently, that the dynamics are dominated by the potential energy (which is changing slowly). In this case the scale factor grows almost exponentially

$$3H^2 \sim V(\phi) \qquad \rightarrow \qquad a(t) = e^{H(t_{ini}-t)} \ . \tag{1.15}$$

The requirement of having at least 60 e-folds of inflation demands that the condition $\epsilon \ll 1$ is maintained for a sufficient amount of time. The proper dimensionless variable quantifying the rate of change of ϵ is known as η, as is defined as

$$\eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \ . \tag{1.16}$$

Then, both ϵ and η, the so-called slow-roll parameters, must satisfy the following conditions:

$$\epsilon \ll 1 \qquad \text{and} \qquad |\eta| \ll 1 \ . \tag{1.17}$$

Indeed, we can define an entire hierarchy of slow-roll parameters $\epsilon_{n+1} \equiv \dot{\epsilon_n}/(H\epsilon_n)$, where $\epsilon_1 = \epsilon$ and $\epsilon_2 = \eta$. For smooth potentials it is in general a good approximation to treat ϵ and η as constants, which is equivalent to neglecting all the tower of slow-roll parameters with $n > 2$. This is not however a necessary condition for having an overall exponential expansion since higher order slow-roll parameters may become large for a very small amount of time (i.e. $\epsilon < 1$ may still be satisfied while, locally in time, $|\eta| > 1$). This regime will have important observable consequences that we will address in Chapter 3. For the moment, we stick to the simple case where the slow-roll conditions (1.17) are satisfied all along the inflaton trajectory and the higher order slow-roll parameters are negligible.

In order to know whether a given potential $V(\phi)$ can sustain inflation or not, we may write the slow-roll parameters as a function of V. At leading order in slow-roll (i.e. for

ϵ,$|\eta| \ll 1$), ϵ and η are given by

$$\epsilon = \frac{1}{2}\left(\frac{V_\phi}{V}\right)^2 \qquad \text{and} \qquad \eta = -2\frac{V_{\phi\phi}}{V} + 2\left(\frac{V_\phi}{V}\right)^2 \ . \tag{1.18}$$

We finish this section by saying that we may have considered different definitions for η. For example, at leading order in slow-roll it is also true that $|V_{\phi\phi}/V| \ll 1$. Furthermore, in this regime $\epsilon - \ddot{\phi}/H\dot{\phi} - V_{\phi\phi}/V = 0$, such that it is also true that $|\ddot{\phi}/H\dot{\phi}| \ll 1$. Then, as is usually found in the literature, one may rather impose that

$$\eta_V \equiv \frac{V_{\phi\phi}}{V} \ll 1 \qquad \text{or} \qquad \eta_\phi \equiv \frac{\ddot{\phi}}{H\dot{\phi}} \ll 1 \ . \tag{1.19}$$

The different definitions of η are related through the following equations:

$$\eta = -2\eta_V + 4\epsilon \qquad \text{and} \qquad \eta_V = \epsilon - 2\eta_\phi \ . \tag{1.20}$$

In the next section, we study how do perturbations evolve on top of the inflationary background.

1.2.1 Primordial perturbations

Undoubtedly, the big success of inflation is that it provides a mechanism for generating the primordial density fluctuations from which the structure of the Universe was created [19] (for a pedagogical review, see [20]). In this model, the primordial perturbations are nothing more than a combination of the quantum fluctuations of the scalar field and spacetime metric[7]. The peculiar causal structure of an accelerating Universe is responsible for converting these microscopic quantum fluctuations into classical and macroscopic density perturbations.

We begin by writing the most general perturbation of the inflaton field and metric. One possible parametrization is the following:

$$\phi(t) \to \phi(t) + \delta\phi(x,t) \ , \tag{1.21}$$

$$g_{\mu\nu} \to (-1+2\Phi)dt^2 + 2a(t)B_i dx^i + a(t)^2\left[(1-2\Psi)\delta_{ij} + E_{ij}\right]dx^i dx^j \ . \tag{1.22}$$

where Φ, B_i, Ψ and E_{ij} are functions of both space and time. Perturbations of the metric can be classified according to their transformation properties under spatial rotations. In particular, they can be decomposed into scalar, vector and tensor modes, and it

[7]As we will argue later, it is not even important that inflation is driven by a scalar field. The only important information is that the Universe undergoes a quasi de Sitter expansion.

can be shown that these different components are decoupled at the linear level. In principle, we can directly expand the action (1.11) to second order using the expansions (1.21) and (1.22) and treat every mode independently and up to second order. We will however find that the equations of motion are overconstrained. To see this, let us note that the perturbations defined in (1.21) and (1.22) are not invariant under a generic coordinate transformation, which means that, taken individually, they are not physical. For example, by redefining time as $t \to \tilde{t} = t + \xi^0$, the perturbation to the scalar field becomes

$$\delta\phi(t,x) \to \delta\phi(t,x) + \xi^0 \dot{\delta\phi}(t,x) \ . \tag{1.23}$$

If we choose ξ^0 such that $\xi^0 = -\delta\phi(t,x)/\dot{\delta\phi}(t,x)$, then in the new coordinate system $\delta\phi(t,x) = 0$. This means that the perturbation to the scalar field is not a physical quantity in itself, since we can always choose a coordinate system in which it vanishes. However, we do not have the freedom to choose one single coordinate transformation in which all the perturbations vanish simultaneously. This is because the most general coordinate transformation is parametrized by only three degrees of freedom, two scalars and one vector, and that the perturbations in (1.21) and (1.22) are parametrized by 5 scalar, 2 vector and 1 tensor degrees of freedom. Thus, we do not have sufficient functions in our coordinate transformation to make all the perturbations in the metric and scalar field to disappear, and we are left with only 3 scalar, 1 vector and 1 tensor degrees of freedom (which may be later reduced by the equations of motion). There are two ways of dealing with the ambiguity of choosing the coordinate system. The first is to fix it from the beginning, which is known as *fixing a gauge*. For example, the coordinate system defined previously, in which the inflaton's perturbation vanishes, is called the *comoving gauge*[8]. There are indeed infinite ways of fixing a gauge, but in general there will be more appropriate ones depending on which calculation one wants to perform. The second possibility is to work with gauge invariant variables, which are linear combinations of the perturbation in (1.21) and (1.22) that do not transform under coordinate transformations. There is also an infinity of gauge invariant variables but some of them are more useful than others. In particular we will work with the so-called comoving curvature perturbation $\mathcal{R}$, defined as

$$\mathcal{R} \equiv \Psi + H\frac{\delta\phi}{\dot{\phi}} \ . \tag{1.24}$$

This variable is called the comoving curvature perturbation because in the comoving gauge (where $\delta\phi = 0$), it reduces to the spatial curvature of the metric. As we will see later in this section, this variable becomes constant in time for modes whose wavelength

[8]Formally, the comoving gauge is defined as the gauge in which the perturbation to the stress-energy tensor δT_{0i} vanishes. In slow-roll inflation, this is equivalent to having $\delta\phi = 0$.

exceed the Hubble radius, a crucial behavior that allows us to connect the quantum perturbations during inflation with the perturbations of the CMB.

In general, any field theory may be classified by its predictions for the n-point correlation functions of whatever observables the theory possesses. One of the predictions of the canonical single field models of inflation (as the one we have considered) is that the statistics of the curvature perturbations are nearly gaussian. Indeed, for single-field and canonical models of inflation, the smallness of the curvature perturbations and the flatness of the potential ensure that interactions other than quadratic in the field variables are highly suppressed. This means that the only relevant n-point correlation function is the two-point function (in an exactly gaussian theory all the n-point correlation functions are either zero -for n odd-, or a function of the two-point function -for n even-). We start then with the action for the comoving curvature perturbation at second order, which is given by

$$S_2 = \int d^4x\, a^3\epsilon \left\{ \dot{\mathcal{R}}^2 - \frac{1}{a^2}\left(\nabla\mathcal{R}\right)^2 \right\} \ , \tag{1.25}$$

At this point we may introduce an important time re-parametrization, known as conformal time and defined as $d\tau = adt$. In this new coordinate system, the canonical perturbation v is given by

$$v \equiv z\mathcal{R} \qquad \text{with} \qquad z^2 = 2a^2\epsilon, \tag{1.26}$$

where the scale factor, at first order in slow-roll, takes the following form:

$$a(\tau) = -\frac{1}{H\tau}(1+\epsilon) \ . \tag{1.27}$$

Let us note that τ takes negative values, from $-\infty$ in the far past to 0 at the end of inflation. In order to find a solution to the equation of motion we expand the field in Fourier modes $v(\mathbf{x}, \tau)$ as

$$v(\mathbf{x},\tau) = \int d^3\mathbf{k}\ v_{\mathbf{k}}\,(\tau)\, e^{i\mathbf{k}.\mathbf{x}} \ . \tag{1.28}$$

The mode function $v_{\mathbf{k}}(\tau)$ satisfies the Mukhanov-Sasaki equation [21, 22]:

$$v''_{\mathbf{k}} + \left(k^2 - \frac{z''}{z}\right) v_{\mathbf{k}} = 0 \ , \tag{1.29}$$

where $k^2 = \mathbf{k}.\mathbf{k}$, and prime denotes derivatives with respect to conformal time. There are two independent solutions of the previous equation, only dependent on k, so we might further expand $v_{\mathbf{k}} (= v_k)$ and write

$$v(\mathbf{x},\tau) = \int d^3\mathbf{k} \left[a^-_{\mathbf{k}} v_k(\tau) e^{i\mathbf{k}.\mathbf{x}} + a^\dagger_{\mathbf{k}} v^*_k(\tau) e^{-i\mathbf{k}.\mathbf{x}} \right] \ , \tag{1.30}$$

where $(a_{\mathbf{k}}^-)^* = a_{\mathbf{k}}^\dagger$ so that the solution $v(\mathbf{x},\tau)$ is real. The Mukhanov-Sasaki equation (1.29) has an exact solution for constant ϵ and η and the integration constants can be found by imposing boundary conditions and quantum commutation relations to the mode functions. First of all, we need to promote v_k to an operator and impose the quantum commutation relations to $\hat{v_k}$ and its conjugate momentum $\pi = v'$. This implies identifying $a_{\mathbf{k}}^-$ and $a_{\mathbf{k}}^\dagger$ as the usual creation and annihilation operators. Then, boundary conditions can be imposed in the far past ($k\tau \gg 1$), by assuming that every mode k begin its evolution in the vacuum state: high frequency modes do not feel the curvature of spacetime, and the expectation value of the Hamiltonian can be unambiguously minimized as in flat space. This is known as the Bunch-Davies vacuum [23]. Under these prescriptions, and in the most simple case of an exact de Sitter expansion, the full solution to equation (1.29) is given by:

$$v_k(\tau) = \frac{1}{\sqrt{2k}} e^{-ik\tau}\left(1 - \frac{i}{k\tau}\right) . \tag{1.31}$$

As we have shown, during inflation modes go from the ultraviolet (or subhorizon) to the infrared (or superhorizon) regime[9]. We might then trace the history of a given mode v_k by analysing the different asymptotic regimes of the solution (1.31). In the far past ($|k\tau| \gg 1$), the solution to the mode function is dominated by the oscillating part $\exp(-ik\tau)$. This is nothing more than the result of imposing the Bunch-Davies vacuum prescription. As inflation proceeds, a mode initially in the short wavelength regime eventually crosses the horizon and becomes superhorizon ($|k\tau| \ll 1$). Then, the dominant contribution to the solution (**??**) is the divergent factor $\propto 1/\tau$. This means that the curvature perturbation $\mathcal{R}_k$, related to v_k through eq. (1.26), becomes constant[10]. When inflation ends the Hubble radius starts to grow, and this superhorizon mode k eventually re-enters inside the horizon. When it does, it will start to oscillate with an initial amplitude that was fixed when that mode crossed the Hubble radius during inflation. This is precisely the way in which perturbations generated during inflation are connected with the initial conditions for the photon-baryon plasma.

Let us note that in this description we have implicitly assumed that a superhorizon mode k remains constant after inflation ends (which we have derived *only* for the case in which there is a de Sitter background expansion). This is an important assumption, since between inflation and the decoupling of the CMB photons there are many unknown processes which may in principle invalidate the simple predictions of our calculations. For

[9] As usually done in the literature, we will say that a mode with wavelength bigger than the Hubble radius is a superhorizon mode ($k > aH$), while a mode with wavelength smaller than the Hubble radius is a subhorizon mode ($k < aH$). This terminology is not accurate since the Hubble radius is neither a particle nor an event horizon.

[10] In the long wavelength regime the dominant contribution to v_k comes from its imaginary part. The real part is constant for v_k which means that it is a decaying perturbation mode for $\mathcal{R}_k$.

example, the energy density of the inflaton has to be transferred to matter and radiation, a process -called reheating- which we know very little about. A superhorizon evolution of the curvature perturbations would mean that we need to follow the evolution of those modes through these mysterious ages, which would in practice spoil the predictability of inflation. Importantly, it is possible to show that superhorizon perturbations remain constant independently of the background FLRW cosmology, provided the perturbations are adiabatic [24]. By adiabatic perturbations we mean that for a fluid composed of different elements (and in the case in which the total stress-energy tensor is the sum of the individual stress-energy tensors), all the individual components of the fluid (labelled i) are perturbed satisfying

$$\frac{\delta\rho_i}{\dot{\rho}_i} = \frac{\delta p_i}{\dot{p}_i} = \frac{\delta\rho_t}{\dot{\rho}_t} = \frac{\delta p_t}{\dot{p}_t}, \tag{1.32}$$

where t refers to the total energy density and pressure. For adiabatic perturbations, the local perturbation to the energy density (pressure) corresponds to a time shift of the background energy density (pressure). Single field models of inflation only produce this type of perturbations, and then the conservation of superhorizon modes is guaranteed.

The two-point quantum correlation function, evaluated in the vacuum, allows us to define the power spectrum $\mathcal{P}_\mathcal{R}$ as

$$\langle \mathcal{R}_\mathbf{k} \mathcal{R}_{\mathbf{k}'} \rangle \equiv (2\pi)^3 \delta^3(\mathbf{k} + \mathbf{k}') \mathcal{P}_\mathcal{R}(k) \ . \tag{1.33}$$

Using the solution (1.31) and (1.26) in the long wavelength limit, we find

$$\mathcal{P}_\mathcal{R}(k) = \frac{1}{8\pi^2} \frac{H^2}{\epsilon} \ . \tag{1.34}$$

The amplitude of the power spectrum at a given scale k depends on the value of H and ϵ at the moment in which $\mathcal{R}_k$ became frozen. As every mode k crosses the Hubble radius and becomes constant at a different moment (thus, for slightly different values of H and ϵ) the amplitude of the power spectrum has a mild scale-dependence. This can be parametrized by the spectral index n_s, giving

$$\begin{aligned} n_s &\equiv \frac{d \ln \mathcal{P}_\mathcal{R}}{d \ln k} \\ &= 1 - 2\epsilon - \eta \ . \end{aligned} \tag{1.35}$$

Higher order derivatives of the power spectrum will depend on higher order derivatives of the slow-roll parameters, which are suppressed in the simple case considered here. Considering only the first derivative of the power spectrum is equivalent to assuming that it is parametrized by a power law of the form

$$\mathcal{P}_{\mathcal{R}} = A_{\rm S} \left(\frac{k}{k_\star}\right)^{n_s - 1} \ , \tag{1.36}$$

in which $A_{\rm S} = H^2/(8\pi^2\epsilon)$, evaluated at an arbitrary scale $k = k_\star$. As we will see in the following chapters, if the slow-roll parameters are allowed to become large for some small amount of time during the inflationary trajectory (e.g. for the case of a very flat potential with a small and localized 'bump'), higher derivatives of the power spectrum may become important and a power law expansion of the form (1.36) may not be the most adequate parametrization for the power spectrum.

Apart from scalar perturbations, the spacetime metric also has vector and tensor degrees of freedom. While the vector modes decay rapidly as the scale factor increases, tensor modes are also conserved after crossing the Hubble radius. Calculating their power spectrum is a simpler task since only the metric contributes to the total tensor perturbation. Their power spectrum is given by

$$\mathcal{P}_h = \frac{2H^2}{\pi^2} \ . \tag{1.37}$$

Then, we can define the tensor-to-scalar ratio, r, as the ratio between the tensor and the scalar power spectrum

$$r \equiv \frac{\mathcal{P}_h}{\mathcal{P}_{\mathcal{R}}} = 16\epsilon \ . \tag{1.38}$$

Before linking these predictions to the observation of the CMB, we will briefly discuss one simple extension of the model presented here. In the previous example the speed of sound c_s of the scalar curvature fluctuations was, in units of the speed of light, equal to one. This can be immediately deduced by noticing that the dispersion relation in the mode function equation (1.29) is, in the short wavelength regime, $\omega = k$. We can easily generalize the predictions above to the case in which the speed of sound c_s is a free parameter [25]. At the level of the action, a speed of sound can be introduced by generalizing (1.25) to be of the form

$$S_2 = \int d^4x \, a^3 \epsilon \left\{ \frac{\dot{\mathcal{R}}^2}{c_s^2} - \frac{1}{a^2} \left(\nabla \mathcal{R}\right)^2 \right\} \ . \tag{1.39}$$

We can introduce a new variable s parametrizing the rate of change of c_s, given by

$$s \equiv \frac{\dot{c}_s}{c_s H} \ . \tag{1.40}$$

The Mukhanov-Sasaki equation now reads:

$$v''_{\mathbf{k}} + \left(c_s^2 k^2 - \frac{z''}{z} \right) v_{\mathbf{k}} = 0 \ , \tag{1.41}$$

where this time $z = 2a^2 \epsilon c_s^{-2}$. In the limit in which the speed of sound changes slowly, i.e, $s \ll 1$, we can follow the same steps as previously in order to calculate the power spectrum and spectral index of the curvature perturbations, and we get

$$\mathcal{P}_{\mathcal{R}} = \frac{1}{8\pi^2} \frac{H^2}{c_s \epsilon} \qquad \text{and} \qquad n_s = 1 - 2\epsilon - \eta - s \ . \tag{1.42}$$

When calculating the power spectrum we must have in mind that curvature perturbations will now become constant when a mode k exists the *sound* horizon, i.e., at times satisfying $c_s k \gg aH$. For a subluminal speed of sound, this happens before the freezing of modes with $c_s = 1$. The tensor-to-scalar ratio is now[11]

$$r = 16 \epsilon c_s \ . \tag{1.43}$$

At this point, the motivation for considering inflation with a reduced speed sound might only seem phenomenological but, as we will see in the following chapters, reduced speeds of sound may be a *portal* to access very high energy degrees of freedom.

In order to confront theory with observations we need to evolve the initial theoretical power spectrum to a power spectrum for the CMB photons at the time of decoupling. The power spectrum of the comoving curvature perturbations determine the initial conditions for the perturbations of the photon-baryon plasma. If the perturbations are adiabatic, the initial density contrast, $\delta_i = \delta\rho_i/\rho$ for photons (γ), baryons (b), CDM (c) and neutrinos (ν) are related to the initial curvature perturbation (in the long wavelength limit) as

$$\delta_b = \delta_c = \frac{3}{4}\delta_\gamma = \frac{3}{4}\delta_\nu = \zeta \ . \tag{1.44}$$

Once we impose the initial conditions, the evolution of the plasma is given by a set of Boltzmann equations. The perturbations must be further projected onto the celestial sphere, since we actually measure different temperatures at different angles in the sky. The physical evolution and the geometrical projection of the initial inhomogeneities will transform the initial flat quantum spectrum into a series of temperature peaks. This process can be modelled numerically with the help of publicly available codes as CAMB [28] or CLASS [29]. The structure of the peaks has been measured to a great accuracy by a number of experiments, the last one being the Planck satellite

[11]Contrary to curvature perturbations, the freeze-out time for tensor perturbations is not affected, as tensor modes (gravity waves) still travel at the speed of light. This difference in the time of freeze-out induces corrections to the tensor-to-scalar ratio which might be important if $c_s \ll 1$ [26, 27], but that we neglect in formula (1.43).

mission [11], which we already showed in figure 1.2. From these observations it is possible to determine the nature of the initial conditions, for example the amplitude A_S of the scalar fluctuations, the spectral index n_s and the tensor to scalar ratio r. The central values for A_S and n_s) and constraints (for r) can be read from table 1.1. Every single field potential has a very precise prediction for these values, as can be seen explicitly from eqs. (1.18). The contour plot of n_s and r, together with several theoretical predictions, are shown in figure 1.3. The constraints are specified for a specific scale in the CMB, $k_* = 0.002\ \text{Mpc}^{-1}$ which may have exited the Hubble radius 50 or 60 e-folds before the end of inflation. The predictions for the models are then specified for this range of e-folds.

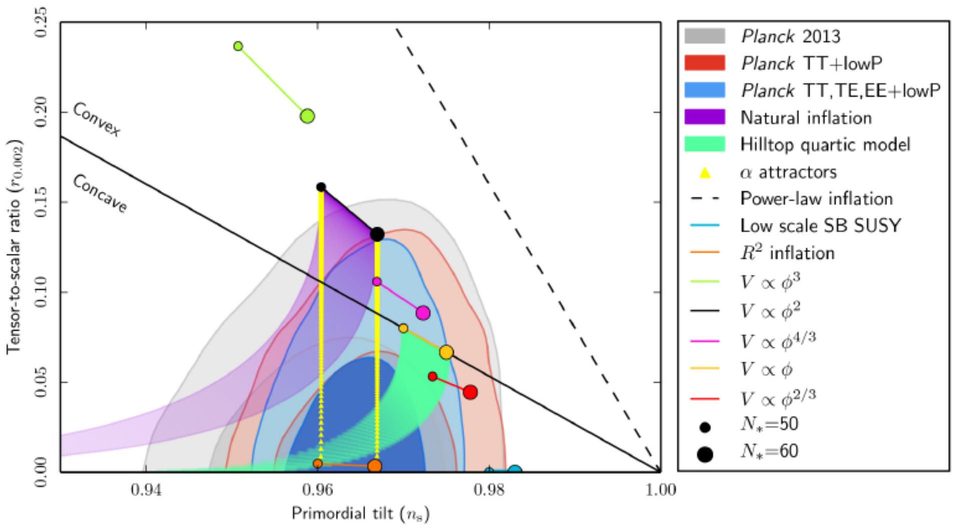

FIGURE 1.3: Marginalized joint 68% and 95% CL regions for n_s and r from Planck in combination with other data sets, compared to the theoretical predictions of selected inflationary models. From [30].

Among the theoretical predictions shown in figure 1.3, two of them are specially going to call our attention during this thesis. The first is the simplest inflationary model we can imagine, described by a quadratic potential (in black in fig. 1.3) [31]

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \ . \tag{1.45}$$

The second model is the so-called natural inflation potential (in purple in fig. 1.3) [32], described by the following potential:

$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{\phi}{f}\right) \right] \ , \tag{1.46}$$

where f is a constant parameter known as the axion decay constant. The fact that these two well motivated potentials are in tension with the data (a tension that is enhanced when considering the BICEP2/Keck Array experiment [33]) will be a motivation for testing the robustness of their predictions. We will carry out that work in Chapters 4 and 5.

1.2.2 Higher order correlation functions

As we shall also discuss in this thesis, modifications of the canonical single-field model of inflation can generate a large non-gaussian signal. First of all, from the point of effective field theory (EFT), higher order interactions are unavoidable. In the case of inflation [34] they can be calculated relying only on the background spacetime symmetries, so we expect them to be quite universal. In particular, the curvature perturbations of every single-clock inflationary model[12] are described by a unique Lagrangian, in which only the specific value for the coefficients of the different operators depends on the microscopic origin of the model.

A well studied non-gaussian signature is the three-point correlation function (for a review, see e.g. [35]). The simplest way of generating a three-point function is through an explicit third order interaction in the Lagrangian. The first thorough calculation of this signal was performed by Acquaviva et al. [36], and later Maldacena determined its explicit k-dependence [37]. From the EFT of inflation [34], the action of the curvature perturbation, up to third order, is given by:

$$S_2 = \int d^4x\, a^3 \dot{H} \left\{ - \left[\frac{\dot{\pi}^2}{c_s^2} - \frac{1}{a^2} \left(\nabla\pi\right)^2 \right] + 3\dot{H}\pi^2 \right\} \ , \tag{1.47}$$

$$\begin{aligned} S_3 = \int d^4x\, a^3 \Bigg\{ & \left[\dot{\pi}^2 - \frac{\left(\nabla\pi\right)^2}{a^2} \right] \left[\dot{H}\dot{\pi}\left(1 - c_s^{-2}\right) - \ddot{H}\pi \right] + 2\dot{H}\frac{\dot{c}_s}{c_s^3}\pi\dot{\pi}^2 \\ & - \frac{4}{3}M_3^4\dot{\pi}^3 - 3\ddot{H}\dot{H}\pi^3 \Bigg\} \ , \end{aligned} \tag{1.48}$$

where π is defined as the Goldstone boson of the broken time diffeomorphism which, at linear order, is related to the curvature perturbation as $\mathcal{R} = -H\pi$. First of all, let us note that the second order action derived from the EFT is the same as the one we wrote in section 1.2.1. At that point we derived the action for the specific case in which the curvature perturbations were identified with the scalar field and metric perturbations. The interesting point to notice is that in order to construct the EFT we do not really need to know the energy content of the Universe. More precisely, the action for the curvature perturbation is the same independently of whether the inflaton is a scalar

[12] As inflation describes a quasi de Sitter Universe, time translation is not a symmetry of the background evolution. By single-clock it is understood that time translation is broken by a single degree of freedom.

field, a composite field or any other exotic matter field (provided spatial diffeomorphism is preserved and there is only one degree of freedom dictating the background dynamics). The microscopic details of the theory are only going to be encoded in the values of c_s (which also determines s), M_3 (which is in general time dependent) and H (which has however more restrictions since it needs to support inflation).

While higher order correlation functions are unavoidable, in the simplest version of inflation they are highly suppressed. The first reason is because of the smallness of $\mathcal{R}$ (the amplitude of the two-point function translates into $\mathcal{R} \sim 10^{-5}$). Higher order operators are described by higher powers of $\mathcal{R}$, and so they are small with respect to the gaussian (quadratic) component. Moreover, also the coefficients of the higher order operators are suppressed since they are composed of higher order derivatives and higher order powers of the slow-roll parameters (e.g. through $\ddot{H}$ in (1.48)). These parameters are also small in canonical, and smooth, single-field models of inflation.

More precisely, the smallness of the three-point function is guaranteed provided some conditions are fulfilled. These condition are i) single-clock inflation: there is only one effective degree of freedom dictating the dynamics. ii) Bunch-Davies initial conditions: modes with wavelength much smaller than the Hubble radius effectively experience Minkowski spacetime. iii) Canonical kinetic terms: the inflaton has canonical kinetic terms with speed of sound $c_s = 1$. iv) The slow-roll parameters and their time variations are small during all the observable inflationary trajectory.

The interest in studying non-gaussianity comes from the fact that any detection of primordial non-gaussianity would very likely come from the violation of any of these conditions, which would be an incredibly important step towards understanding the nature of inflation.

As the three-point correlation function depends on three momenta (which are bounded to form a triangle in k-space because of translational invariance), its comparison with data is particularly difficult. It is then useful to concentrate on a few well motivated templates, which characterizes the shape of some ‘expected signals’. A template α can be generally defined as:

$$\langle \mathcal{R}(k_1)\mathcal{R}(k_2)\mathcal{R}(k_3)\rangle^{\alpha} = (2\pi)^3\delta(k_1 + k_2 + k_3) f^{\alpha}_{NL} \cdot F^{\alpha}(k_1, k_2, k_3) \; . \tag{1.49}$$

Every template α has a scale-dependent shape $F^{\alpha}(k_1, k_2, k_3)$ and a corresponding amplitude f^{α}_{NL}, which is a number that does not depend on k_i. The current data and techniques available for the analysis of the three-point function allow us to constrain the amplitude f^{α}_{NL} for a finite set of templates, that we have to choose beforehand. In this sense, the non-detection of a particular template α does not imply the absence of non-gaussianity, but rather constrains the presence of the non-gaussian signal described by that specific template. Furthermore, even if the three-point function is measured to be consistent with zero in every momentum configuration, non-gaussianities may appear in higher order correlation functions. This is the reason why complementary studies of non-gaussianities, e.g. based upon real-space statistics, are also very important.

The most studied templates for the three-point function are three (we refer the reader to [35] for a detailed account). Two of them come from noticing that, at first order in slow-roll, there are two free parameters in the cubic action (1.48), namely c_s and M_3^4. When c_s is a constant ($\dot{c_s} = 0$), c_s and $\tilde{c_3} \equiv 2M_3^4 c_s^2/\dot{H}$ control the amplitude of two different templates, named equilateral and orthogonal [38]. The third template, historically the first to be considered, is the so-called local template. It describes the non-gaussian signal for the case in which there is a non-linear relation between the inflaton and the observed curvature perturbations. It borrows its name from being defined in real space [39]. The main characteristics of the templates mentioned above are:

- Equilateral: This is the non-gaussian signal when c_s is a constant $c_s \neq 1$, and $\tilde{c_3} = \ddot{H} = 0$. The template peaks in the equilateral limit, which is the configuration in which the three momenta are equal, $k_1 = k_2 = k_3$. Models with constant reduced speed of sound will peak at this configuration.
- Orthogonal: This is a template designed to be *orthogonal* to the equilateral template, such that a basis is defined that covers all the relative contributions of c_s and $\tilde{c_3}$. This template also peaks in the equilateral but has an important contribution in the flattened limit (defined as $2k_1 = 2k_2 = k_3$).
- Local: The local configuration comes when the non-gaussian component of the curvature perturbation $\mathcal{R}$ is parametrized as a function of its gaussian component $\mathcal{R}^g$ as:
$$\mathcal{R}(x) = \mathcal{R}^g(x) + \frac{3}{5} f_{NL}^{\rm loc} \left(\mathcal{R}^g(x) - \langle \mathcal{R}^g(x)^2 \rangle\right) \ . \tag{1.50}$$
The parameter $f_{NL}^{\rm loc}$ controls the skewness of the probability density function. In momentum space this template peaks in the squeezed limit. This is an interesting configuration, in which one of the momenta is much smaller that the other two, $k_1 = k_L$ and $k_2 = k_3 = k_S$ with $k_S \gg k_L$ (S,L denoting short and long mode respectively). In [37], Maldacena showed that the amplitude of the bispectrum in the squeezed limit is proportional to the tilt of the power spectrum, as
$$\lim_{k_L \to 0} \langle \mathcal{R}(k_S)\mathcal{R}(k_S)\mathcal{R}(k_L)\rangle = -\mathcal{P}_\mathcal{R}(k_L)\mathcal{P}_\mathcal{R}(k_S) \frac{d}{d\ln k_S} \ln \left[k_S^3 \mathcal{P}_\mathcal{R}(k_S)\right] \ . \tag{1.51}$$
This is also know as the 'consistency condition', since any model violating this relation will have non trivial departures from the simplest single field inflationary

models. In particular, this configuration is bound to be undetectable in the simplest inflationary models since $d\ln\mathcal{P}_{\mathcal{R}}/d\ln k \sim (n_s - 1)$ is of order slow-roll in all the observable inflationary trajectory (see equation 1.35). Contracting cosmologies, multifield models of inflation, as well as single field models with features (e.g. oscillations) in the primordial power spectra can generate however a large local signal[13].

These configurations have been tested in the Planck 2105 data [40], and the following constraints has been derived (the amplitude of the three shapes being consistent with zero):

$$f_{NL}^{\rm local} = 0.8 \pm 5.0 \quad , \quad f_{NL}^{\rm equi} = -4 \pm 43 \quad , \quad f_{NL}^{\rm orth} = -26 \pm 21 \quad (68\%{\rm CL}) \tag{1.52}$$

These constraints can be related to constraints in c_s and $\tilde{c_3}$, from where we can deduce an upper bound on c_s, $c_s > 0.024$ at 95% CL (the two-dimensional constraint is shown in figure (1.4)).

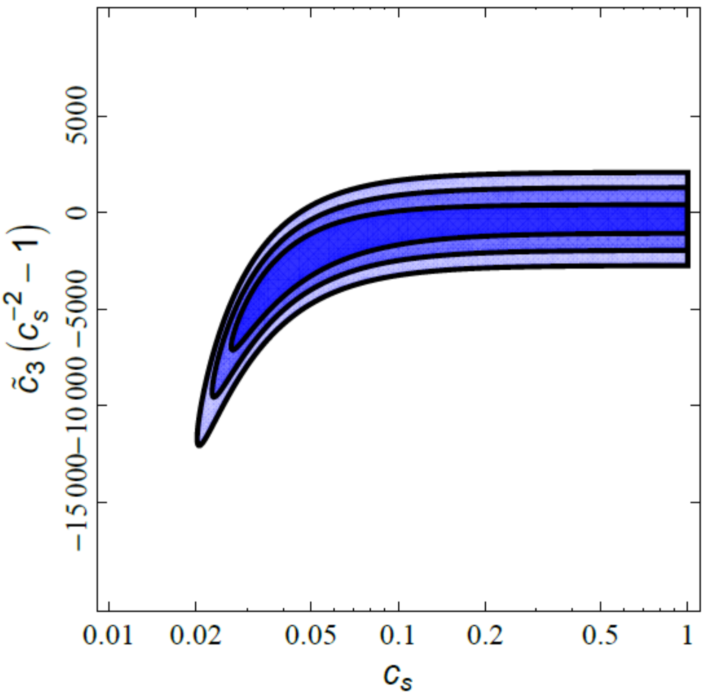

Figure 1.4: 68%, 95%, 99.7% confidence regions in the single field inflation parameter space $(c_s, \tilde{c_3})$, with $\tilde{c_3} \equiv 2M_3^4 c_s^2/\dot{H}$. Figure from [40]

[13] In some of these cases the consistency condition will be violated (by violating some of the assumptions necessary for its derivation) while in others not. In particular, let us note that single field models with oscillations in the primordial power spectrum still satisfy this relation. A primordial power spectrum which is overall flat but has small oscillations on top of it has big and oscillatory values for $d\ln\mathcal{P}/d\ln k$. This family of primordial power spectrum can also provide better fits to the data than the featureless power-law primordial power spectrum, and will be an important subject of this thesis.

In the general framework of EFT, the coefficients that appear in the low energy Lagrangian are functions of the 'UV' completion of the theory. For example, consider a 'UV' theory consisting of two fields: a massless field ϕ_l and a massive field ϕ_h (with mass M) with an interaction $g\phi_l^2\phi_h$. At energies well below M, this interaction is effectively described by a 4-point self interaction of the massless field of the form $(g^2/M^2)\phi_l^4$. The strength of this 4-point self interaction is then proportional to the UV parameters of the theory, in this case g and M. So, in the context of the EFT of inflation, a natural question arises: what are the possible UV completions of inflation that give rise to non-zero values for the parameters of the low energy EFT (in our case, c_s and M_3^4 when considering up to third-order interactions)? Are they bound to be very suppressed ($\propto 1/M^2$), as in the simple two-field model showed here? Or more generally, which values for these parameters can be interpreted in terms of the UV completion?

While we will not address these questions in full generality (see e.g. [41]), in this thesis we will consider a particular embedding of inflation in which this procedure is tractable. In particular, we will consider inflation happening in a two-field landscape, in which one of the fields is light and the other is heavy (with respect to the scale of inflation), and we will show explicitly how the coefficients of the low energy operators emerge when integrating out the heavy field. In particular, we will show that the speed of sound of the curvature fluctuations is linked to the angular velocity of the inflationary trajectory. The time dependence of the speed of sound will not only determine the regime of validity of the single-field low energy effective theory but it will also demand using new techniques for calculating the n-point correlation functions. Of course, this is not the only UV completion for which the EFT of the curvature fluctuations has a reduced speed of sound[14], but, as we will see explicitly, this particular embedding has enough richness to provide a better understanding of the subtleties of both decoupling in EFT and of multifield inflation. We will address these questions up to Chapter 4, while we will reserve the fifth and last chapter to study the case in which there are two light fields during some part of the inflaton trajectory.

[14] As in the Dirac-Born-Infeld (DBI) models of inflation [42, 43].