Solitary Waves and Fluctuations in Fragile Matter

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Title: Solitary waves and fluctuations in fragile matter
Issue Date: 2013-11-05

SUMMARY

In this thesis, we study energy transport and fluctuations in simple models of *fragile* matter : a unique state of matter that has a vanishingly small window of linear response since one or both of its elastic moduli (shear and bulk) are nearly zero. As a consequence, even the tiniest perturbations travel as non-linear waves. In addition, most models of fragile matter have an amorphous structure. It is the interaction of the non-linear waves with the underlying disorder and the resulting fluctuations, that constitutes the unifying theme explored in this thesis.

There are at least two seemingly distinct sources of fragility: a local source stemming from the strongly non-linear interaction potential between particles so that one can not expand around a potential minimum to define a spring constant, and a second, global source, whereby the collective response of the sample can be considered weakly linear.

As a model of the first kind, we consider a two dimensional packing of soft frictionless elastic disks that are just touching their nearest neighbours. The interaction potential between elastic disks is given by the nonlinear Hertz law that has no harmonic part. Consequently, for a packing in this state, the bulk modulus is vanishingly small and the smallest compressions imparted at the edges leads to nonlinear solitary like waves.

As a model of the second kind, we consider a two dimensional random network of harmonic springs where each node has on average around four nearest neighbours. Here, despite the contact interaction being harmonic, the network has a vanishingly small shear modulus. Consequently, even the tiniest shear strains elicit non-linear waves. There are many important similarities and differences between the nature of non-linear waves and the role played by disorder in the two models described above, which we are gradually beginning to understand.

In Chapters 2-4, we focus on models of the first kind, where we consider two dimensional packings of soft elastic frictionless disks such that, upon overlap, two disks interact with a non-linear potential given by the Hertz law. We begin our research work by first considering an ordered hexagonal packing of disks that are just in contact with their nearest neighbours, so

that the packing has a vanishing bulk modulus. Consequently, an impulse imparted at one of the edges leads to non-linear solitary waves that can be described by the one dimensional Nesterenko equation of motion whose solitary wave solutions are by now well established. We then introduce a simple model of disorder by creating an interface between two hexagonal packings with differing particle masses, and study the interaction of the solitary wave with the interface using a combination of simulations and simple analytical models, in particular, by invoking a quasi-particle approximation to the solitary wave.

We then study the propagation of the solitary like wave in a hexagonal packing with a random distribution of masses. We find that disorder effectively acts as a source of viscosity that causes the solitary-wave amplitude to decay as it propagates through the medium. For small variation in the mass distribution, we find two distinct regimes of attenuation: an initial exponential decay of the solitary-wave amplitude followed by a longer time, power law decay. We understand the exponential decay by invoking the quasi-particle approximation to the solitary wave. However, the decay in the power-law regime signals a new regime, where the solitary-wave ceases to exist. By contrast, we find the emergence of a new type of weakly non-linear shock-like front whose amplitude decays as a power law.

This observation provides our first link to the physics of energy attenuation in a strongly disordered medium such as in a hexagonal packing with a large variance in the distribution of masses and jammed amorphous packings of soft frictionless disks. In both cases, an initial impulse soon transitions into a triangular shock front whose amplitude decays as a power law with a rate that is independent of the amount of disorder.

The attenuation of the impulse has some interesting consequences. We find that disorder causes the energy, initially localized in an impulse, to be distributed throughout the amorphous packing (of finite size). In a system with no intrinsic mechanism to dissipate energy, the particles therefore continue to fluctuate forever. This we imagine as a granular analogue of temperature where the passage of the impulse effectively fluidizes the amorphous packing and where the energy of the initial impulse plays the role of temperature. As a consequence, the emergent mechanical state now has a finite bulk modulus (we started with an amorphous packing with a vanishingly small bulk modulus).

In Chapter 5, we study a simple one-dimensional model of fragile matter that is explicitly coupled to a source of thermal fluctuations. As the simplest toy model, we adopt a one-dimensional chain of strongly non-linear Hertzian springs coupled to a heat bath. We then study the propagation of an impulse along this chain and find that the dynamics of the solitary wave quasi-particle mimics that of a Brownian particle.

In Chapter 6-7, we shift our focus to models of fragile matter of the second kind, i.e, a two dimensional disordered network of harmonic springs. Here the loss of rigidity (vanishing shear modulus) is a collective phenomenon. By shearing one edge of the sample at a uniform rate we study the dynamics of the diffusing and propagating shear fronts. We find that the initial response of the network causes a super-diffusive spreading of the energy away from the boundary. However, after a critical time, the super-diffusive spreading transitions into a propagating shear front whose width continues to grow diffusively. Disorder in these networks thus directly manifests in the very large widths of the propagating shear fronts.

For large shearing rates, we find the onset of non-linearly propagating fronts. However, even though we maintain a constant influx of energy at the boundaries, we do not see steadily propagating nonlinear fronts. Instead, the front widths in the nonlinear regime continue to grow super-diffusively while the shearing rate dictates the speed of the propagating fronts. The super-diffusive spreading in both the linear and nonlinear regimes has very interesting connections with the nature of the fluidized states we found for the amorphous packing of disks interacting with the Hertz law potential, and is consistent with more generic transport properties of lower dimensional fluids.

Finally, we conclude with some initial observations on the emergence of rigidity in random spring networks as an interplay between the network geometry and the coupling to a source of quantum and thermal fluctuations.