**Electronic shot noise in fractal conductors**
Groth, C.W.; Tworzydlo, J.; Beenakker, C.W.J.

**Citation**

**Version:** Not Applicable (or Unknown)
**License:** Leiden University Non-exclusive license
**Downloaded from:** https://hdl.handle.net/1887/64314

**Note:** To cite this publication please use the final published version (if applicable).
Electronic Shot Noise in Fractal Conductors

C. W. Groth, J. Tworzydlo, and C. W. J. Beenakker

1Instituut-Lorentz, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands
2Institute of Theoretical Physics, University of Warsaw, Hoża 69, 00-681 Warsaw, Poland

(Received 13 February 2008; published 30 April 2008)

By solving a master equation in the Sierpiński lattice and in a planar random-resistor network, we determine the scaling with size $L$ of the shot noise power $P$ due to elastic scattering in a fractal conductor. We find a power-law scaling $P \propto L^{d_f-2-\alpha}$, with an exponent depending on the fractal dimension $d_f$ and the anomalous diffusion exponent $\alpha$. This is the same scaling as the time-averaged current $I$, which implies that the Fano factor $F = P/(2eI)$ is scale-independent. We obtain a value of $F=1/3$ for anomalous diffusion that is the same as for normal diffusion, even if there is no smallest length scale below which the normal diffusion equation holds. The fact that $F$ remains fixed at 1/3 as one crosses the percolation threshold in a random-resistor network may explain recent measurements of a doping-independent Fano factor in a graphene flake.

Diffusion in a medium with a fractal dimension is characterized by an anomalous scaling with time $t$ of the root-mean-squared displacement $\Delta$.

By solving a master equation in the Sierpiński lattice and in a planar random-resistor network, we determine the scaling with size $L$ of the shot noise power $P$ due to elastic scattering in a fractal conductor. We find a power-law scaling $P \propto L^{d_f-2-\alpha}$, with an exponent depending on the fractal dimension $d_f$ and the anomalous diffusion exponent $\alpha$. This is the same scaling as the time-averaged current $I$, which implies that the Fano factor $F = P/(2eI)$ is scale-independent. We obtain a value of $F=1/3$ for anomalous diffusion that is the same as for normal diffusion, even if there is no smallest length scale below which the normal diffusion equation holds. The fact that $F$ remains fixed at 1/3 as one crosses the percolation threshold in a random-resistor network may explain recent measurements of a doping-independent Fano factor in a graphene flake.

\begin{equation}
P = 2 \int_{-\infty}^{\infty} dt \langle \delta I(t)^2 \rangle
\end{equation}

and by the Fano factor $F = P/(2eI)$. The Pauli principle enforces $F < 1$, meaning that the noise power is smaller than the Poisson value $2eI$—which is the expected value for independent particles (Poisson statistics).

The investigation of shot noise in a fractal conductor is particularly timely in view of two different experimental results [20,21] that have been reported recently. Both experiments measure the shot noise power in a graphene flake and find $F < 1$. A calculation [22] of the effect of the Pauli principle on the shot noise of undoped graphene predicted $F = 1/3$ in the absence of disorder, with a rapid suppression upon either $p$-type or $n$-type doping. This prediction is consistent with the experiment of Danneau et al. [21], but the experiment of DiCarlo et al. [20] gives instead an approximately "doping-independent" $F$ near 1/3. Computer simulations [23,24] suggest that disorder in the samples of DiCarlo et al. might cause the difference.
Motivated by this specific example, we study here the fundamental problem of shot noise due to anomalous diffusion in a fractal conductor. While equilibrium thermal noise in a fractal has been studied previously [25–27], it remains unknown how anomalous diffusion might affect the nonequilibrium shot noise. Existing studies [28–30] of shot noise in a percolating network were in the high-voltage regime where inelastic scattering dominates, leading to hopping conduction [31], while in the low-voltage regime of diffusive conduction we have predominantly elastic scattering.

We demonstrate that anomalous diffusion affects $P$ and $\bar{I}$ in such a way that the Fano factor (their ratio) becomes scale-independent as well as independent of $d_f$ and $\alpha$. Anomalous diffusion, therefore, produces the same Fano factor $F = 1/3$ as is known [32,33] for normal diffusion. This is a remarkable property of diffusive conduction, given that hopping conduction does not produce a scale-independent Fano factor [28–30]. Our general findings are consistent with the doping independence of the Fano factor in disordered graphene observed by DiCarlo et al. [20].

To arrive at these conclusions, we work in the experimentally relevant regime where the temperature $T$ is sufficiently high that the phase coherence length is $\ll L$ and sufficiently low that the inelastic length is $\gg L$. Quantum interference effects can then be neglected, as well as inelastic scattering events. The Pauli principle remains operative if the thermal energy $kT$ remains well below the Fermi energy, so that the electron gas remains degenerate. We neglect Coulomb interactions between the electrons. In the application to graphene, this requires that the electron and hole puddles have a conductance large compared to $e^2/h$ (no Coulomb blockade).

We first briefly consider the case that the anomalous diffusion on long length scales is preceded by normal diffusion on short length scales. This would apply, for example, to a percolating cluster of electron and hole puddles with a mean free path $l$ which is short compared to the typical size $a$ of a puddle. We can then rely on the fact that $F = 1/3$ for a conductor of any shape, provided that the normal diffusion equation holds locally [34,35], to conclude that the transition to anomalous diffusion on long length scales must preserve the one-third Fano factor.

This simple argument cannot be applied to the more typical class of fractal conductors in which the normal diffusion equation does not hold on short length scales. As representative for this class, we consider fractal lattices of sites connected by tunnel barriers. The local tunneling dynamics then crosses over into global anomalous diffusion, without an intermediate regime of normal diffusion.

A classic example is the Sierpinski lattice [36] shown in Fig. 1 (inset). Each site is connected to four neighbors by bonds that represent the tunnel barriers, with equal tunnel rate $\Gamma$ through each barrier. (The tunnel rate is voltage-independent in the low-voltage limit considered here.) The fractal dimension is $d_f = \log_2 3$, and the anomalous diffusion exponent is $\alpha = \log_2 (5/4)$. The Pauli exclusion principle can be incorporated as in Ref. [37], by demanding that each site is either empty or occupied by a single electron. Tunneling is therefore allowed only between an occupied site and an adjacent empty site. A current is transported through the lattice by connecting the lower-left corner to a source (injecting electrons so that the site remains occupied) and the lower-right corner to a drain (extracting electrons so that the site remains empty). The resulting stochastic sequence of current pulses is the “tunnel exclusion process” of Ref. [38].

The statistics of the current pulses can be obtained exactly (albeit not in closed form) by solving a master equation [39]. We have calculated the first two cumulants by extending to a two-dimensional lattice the one-dimensional calculation of Ref. [38]. To manage the added complexity of an extra dimension, we found it convenient to use the Hamiltonian formulation of Ref. [40].
hierarchy of linear equations that we need to solve in order to obtain $I$ and $P$ is derived in the supplement [41].

The results in Fig. 1 demonstrate, first, that the shot noise power $P$ scales as a function of the size $L$ of the lattice with the same exponent $d_f - 2 - \alpha = \log_2(3/5)$ as the conductance and, second, that the Fano factor $F$ approaches $1/3$ for large $L$. More precisely (see Fig. 2), we find that $F - 1/3 \approx L^{-1.5}$ scales to zero as a power law, with $F - 1/3 < 10^{-4}$ for our largest $L$.

Turning now to the application to graphene mentioned in the introduction, we have repeated the calculation of shot noise and Fano factor for the random-resistor network of disordered graphene introduced by Cheianov et al. [13]. The construction of the network is explained in Fig. 3. While this model was inspired by a specific application [13], it is generic and representative for any model of two-dimensional percolation in the low-voltage limit. The results in Fig. 3 demonstrate, first, that the shot noise power $P$ scales with the same exponent $L^{-0.97}$ as the conductance $G$ (solid lines in the lower panel) and that the Fano factor $F$ approaches $1/3$ for large networks (upper panel). This is a random, rather than a deterministic, fractal, so there remains some statistical scatter in the data, but the deviation of $F$ from $1/3$ for the largest lattices is still $<10^{-3}$ (see the circular data points in Fig. 2).

In conclusion, we have found that the universality of the one-third Fano factor, previously established for normal diffusion [32–35], extends to anomalous diffusion as well. This universality might have been expected with respect to the fractal dimension $d_f$ (since the Fano factor is dimension-independent), but we had not expected universality with respect to the anomalous diffusion exponent $\alpha$. The experimental implication of the universality is that the Fano factor remains fixed at $1/3$ as one crosses the percolation threshold in a random-resistor network—thereby crossing over from anomalous diffusion to normal diffusion. This is consistent with the doping-independent Fano factor measured in a graphene flake by DiCarlo et al. [20].

A discussion with L. S. Levitov motivated this research, which was supported by the Netherlands Science Foundation NWO/FOM. We also acknowledge support by the European Community’s Marie Curie Research Training Network under Contract No. MRTN-CT-2003-504574, Fundamentals of Nanoelectronics.

Graphene is a single layer of carbon atoms, forming a two-dimensional honeycomb lattice. Electrical conduction is provided by overlapping π orbitals, with on average one electron per π orbital in undoped graphene. Electron puddles have a little more than one electron per π orbital (n-type doping), while hole puddles have a little less than one electron per π orbital (p-type doping).

An electron gas is called “degenerate” if the average occupation number of a quantum state is either close to unity or close to zero. It is called “nondegenerate” if the average occupation number is much smaller than unity for all states.

In hopping conduction, it is assumed that the voltage is sufficiently large to enable inelastic scattering (which leads to one-way hopping in the zero-temperature limit), while we take the low-voltage limit where only elastic scattering (two-way hopping) survives.