Vortex Duality in Higher Dimensions

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Vortex duality in higher dimensions

PROEFSCHRIFT

TER VERKRIJGING VAN DE GRAAD
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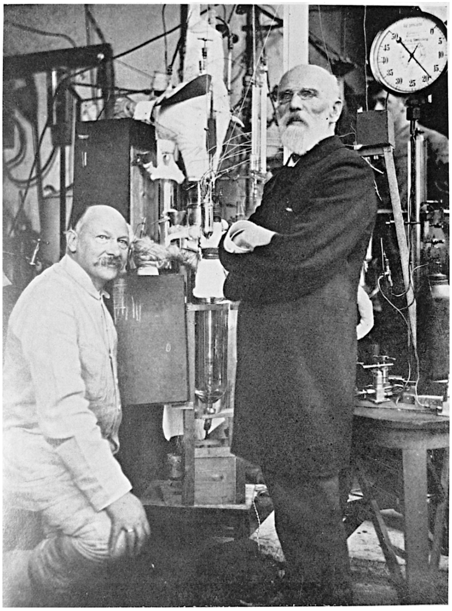
On April 8, 1911, in the physics laboratory located at 'het Steenschuur' in Leiden, Heike Kamerlingh Onnes and his coworkers Cornelis Dorsman, Gerrit Jan Flim and Gilles Holst, measured the instantaneous drop in resistivity of mercury when they cooled it below 4.2 degrees above absolute zero. They were the first people in the world to have beheld the phenomenon of superconductivity, and thereby the first macroscopic quantum fluid.

This year we have celebrated the centennial of this event. Superconductivity in its manifestation of the underlying quantum mechanical principles and its potential for world-changing applications does not cease to challenge the imagination. It is a privilege to be part of the continuing research on this fascinating topic in its place of birth.



H. Kamerlingh Onnes and J.D. van der Waals with the helium liquefactor (1911)

photo courtesy of the Leiden Institute of Physics

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