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We can be heroes.

– David Bowie

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Preface

We can change the world. Although the processes that determine life on earth are not controllable, the impact of human kind is major. We have evolved to an intelligent species that knows exactly how to employ nature as a provider for our survival. Our natural thirst for knowledge has brought us an understanding of many of nature's processes and by virtue of that, prosperity. The population thrives on our innovations and scientific development. In particular since the 18^{th} century, science has flourished and has established to be a valued instrument in the never-ending urge to improve our well-being. And it has been successful; the average life expectancy of humans rises steadily. Scientific discoveries have enabled us to combat diseases and to conquer natural disasters. Intelligent constructing prevents buildings from collapsing during earthquakes, our ships can overcome heavy storms, and, relying heavily on the quality of dykes and levees, we can even live on land that is below sea level. But even on a smaller scale, science delivers; through artificial fertilizer that enhances the growth of our crops, and sunscreen that prevents our skins from the malignant effects of the sun.

However, as beneficial as scientific innovations have been to the growth and health of our population, the human impact on nature is also palpable from much more alarming viewpoints. We arrogantly consider ourselves the most important residents of our planet. To facilitate mass consumption in the world, territories of other species are seized, driving them to extinction; the use of heavy chemicals has contaminated soil, making it unfruitful and the fossil fuel industry that still provides for most of our technology contributes heavily to global warming and all its inherent issues. We seem to have forgotten that our relation to the planet is in fact a symbiosis; the earth is essential to our existence.

The need for a transition of focus of the sciences is hence more urgent than ever. In order to provide for our ever growing community, we need to aim for sustainable development and choose to acknowledge our 'Gaia' as a whole instead of focusing mostly on the people that occupy this planet. In this task there lies an important role for the mathematician, being perhaps the most fundamental of all scientists. Driven by questions that are posed by experts from ecology, biology or other applied sciences, applied mathematicians identify in complex systems the important mechanisms that are responsible for the process in question. These types of processes frequently refer to, for example, growth or spread with a certain regularity. While other applied scientists often contribute by observing a phenomenon and describing it meticulously, applied mathematicians take another approach. They are not bothered by the exact particulars that define the system, but pay attention to the important driving forces. In many cases, it turns out that the answer a mathematician finds to a specific question, is generally applicable to a much wider range of problems. As an example, think of a meadow with rabbits and foxes. Foxes eat rabbits, but rabbits reproduce faster, so an evident question to ask is what will happen to both populations as time proceeds. To study this, we design a phenomenological model that describes the change of the populations per time unit using two equations; one for foxes and one for rabbits. For the foxes, it incorporates how many foxes are born in that time unit due to reproduction and a diet of fluffy rodents, and a negative term that describes the natural death per time unit. For the rabbits, the equation includes a growth term from reproduction and a negative term that describes not only the natural death, but also death by fox per time unit[1]. One could wonder whether the quality of the grass or the size of the meadow should not be taken into the equation[2], which exemplifies the nontriviality of modeling in the first place. But even when we suppose that only reproduction and the foxes' appetite are of importance, several approaches to this problem are possible. One way is to experimentally measure how many rabbits a fox eats, and what their average life span is, in order to estimate all the numbers that make up the equations. Next, a computer can use these equations to tell you exactly how many foxes and rabbits will survive, given the exact number of both populations at a certain starting point. A mathematician, however, will not be bothered by exactly how many rabbits are born per time unit or how many rabbits are on a fox's menu. Instead, they label all those estimated numbers with a letter so that it becomes a parameter; something that is still variable. Subsequently, we manipulate the abstract equations and draw conclusions that depend on those parameters. Such a conclusion might be that rabbits survive provided that their reproduction rate is at least twice that of a fox. The beauty of this approach is that to a mathematician the actual numbers do not matter. Moreover, a mathematician is less susceptible to drawing biased conclusions because he/she is not troubled by the exact interpretation of the parameters. Experimental data is prone to small

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[1] This particular model from population dynamics is referred to as the Lotka-Volterra system, proposed separately by Lotka and Volterra in the beginning of the 20$^{\text{th}}$ century, see [105, 162]

[2] pun intended

errors, and a mathematician's results are not affected by that. In fact, the results are so general that they can be applied to any type of predator-prey interaction, other than foxes and rabbits. Furthermore, a mathematician provides results with a certain mathematical rigor. That means that within the laws of the equations, the conclusions come with proofs and are not open to interpretation.

Of course there are disadvantages to this level of abstraction, too. In very complex systems, it is sometimes hard to translate abstract results back to their original application. Also, the rigorous analysis of a mathematician is often nearly impossible in large scale systems with many interaction processes. An essential step in the mathematical modeling of a natural phenomenon is therefore to construct a model that is stripped down from all unnecessary intricacies so that it is manageable, but remains to capture the determining processes. This in itself is a nontrivial task, but there are some ideas to follow. First, we could inspect for any large discrepancies between quantities in the system. Returning to the fox-rabbit model, instead of using general parameters for the reproduction rate of both species, we could include the fact that one is much faster than the other. Such extra information can simplify the model significantly and makes it more controllable. A second approach is to see whether there are some conserved quantities in the system. This could be physical laws like conservation of mass or energy, but it could also be less obvious. In general, any type of extra structure that we lay upon or identify in the system, has the ability to simplify it.

The imperative problems that the expansion of our population brings are obviously hard. The mere issue of prioritizing this large class of questions makes starting to work on them nearly impossible; the diversity in scale and impact is enormous. Not to mention the obstacles that one needs to overcome to model these types of large scale systems. So even though a long history of innovation provides enough confidence that we can indeed change the world and find a way to sustain our planet, it is not going to be easy. The need for interdisciplinary collaborations is evident and the role of an applied mathematician should not be underestimated. Because even when at first sight a model may only bring utter confusion and total chaos, the correct framework and a trained mathematician's eye can provide insight. After all, mathematicians even have a strict definition for chaos.