Spin dynamics in general relativity

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Chapter 6 concludes my theoretical research in connection with the possible experiments. It provides the list of possible problems and directions, as an extension for the future exploration.

6

Conclusion

With our new formalism much physics remains to be explored. The following applications and generalizations are to be made:

- First of all it would be interesting to consider the effect of spin on the emission of gravitational waves from the above established orbits like circular and bound planar-orbits. We have also found these orbits in the Reissner-Nordstrom geometery [48]. Computing gravitational waves for particle in these orbits should be straightforward.

- There exist other types of orbits in spherically symmetric backgrounds. When the spinning particle comes from infinity - orbits the centre and moves to infinity it performs *scattering orbits*. When it comes from infinity and plunges directly into the centre this is known as *plunging orbits*. These orbits and the gravitational waves from the particle obeying these orbits are yet to be computed.

- As we have established the circular orbits for a non-minimal hamiltonian including gravitational Stern-Gerlach force, one can obtain the ISCO for a specific gravimagnetic ratio κ (depends on the object) and other possible orbits like bound non-circular, scattering and plunging orbits and their gravitational waves.

- The supermassive black holes in the galactic centres are of Kerr nature and there are many stars/compact objects orbiting them. Therefore extending our formalism to the Kerr metric is very essential (though it is very difficult analytically) to understand the practical dynamics of such systems. The first step would be calculating the circular orbits in the equatorial plane.

- The gravitational self-force can be incorporated within our formalism. The approach goes beyond the test particle approximation by including the self-field effects which modify the leading-order geodetic motion of a small mass moving in the vicinity of the background geometry [109].

- Finally our approach is based on the point particle approximation. Refining our methods to finite size bodies by including higher order mass multipoles would be helpful in understanding the comparable mass systems studied with ground based detectors.

All the above items are important and left for future investigation. It is desirable to develop a complete theoretical framework along these lines for Extreme Mass Ratio Binaries to be compared with the observations, such as those planned by the eLISA mission scheduled for 2034. Its advantage is that in contrast to the PN methods our scheme is fully relativistic; in fact it can work even in the plunge regime beyond the ISCO, as shown for example by the work in ref. [110].

A

Deviation Equations in Schwarzschild space-time

The deviation equations for the most general case i.e., the non-planar motion, of spinning particles in Schwarzschild space-time is given here.

A.1 The Orbital deviations:

$$\begin{aligned}
\frac{d\delta u^t}{d\tau} =& -\frac{2M}{r(r-2M)}\,\delta\left(u^t u^r\right) - \frac{2M}{mr^2(r-2M)}\,\delta\left(u^r\Sigma^{tr}\right) \\
&+ \frac{M}{mr}\,\delta\left(u^\theta\Sigma^{t\theta} + u^\varphi\Sigma^{t\varphi}\right) \\
&+ \left[\frac{4M(r-M)}{r^2(r-2M)^2}\,u^t u^r + \frac{2M(3r-4M)}{mr^3(r-2M)^2}\,u^r\Sigma^{tr}\right. \\
&\quad \left. - \frac{M}{mr^2}\left(u^\theta\Sigma^{t\theta} + u^\varphi\Sigma^{t\varphi}\right)\right]\delta r,
\end{aligned}$$

$$\begin{aligned}
\frac{d\delta u^r}{d\tau} =& -\frac{M(r-2M)}{r^3}\,\delta\left(u^{t\,2}\right) - \frac{2M(r-2M)}{mr^4}\,\delta\left(u^t\Sigma^{tr}\right) + \frac{2Mu^r}{r(r-2M)}\,\delta u^r \\
&+ (r-2M)\,\delta(u^{\theta\,2}) + \frac{M}{mr}\left[\delta\left(u^\theta\Sigma^{r\theta}\right) + \delta\left(u^\varphi\Sigma^{r\varphi}\right)\right] \\
&+ (r-2M)\,\sin^2\theta\,\delta(u^{\varphi\,2}) + \left((r-2M)u^{\varphi\,2}\,\sin 2\theta\right)\,\delta\theta \\
&+ \left[\frac{2M}{r^4}(r-3M)\,u^{t\,2} - \frac{2M(r-M)u^{r\,2}}{r^2(r-2M)^2} + u^{\theta\,2} + \sin^2\theta\,u^{\varphi\,2}\right. \\
&\quad \left. + \frac{2M}{mr^5}(3r-8M)u^t\Sigma^{tr} - \frac{M}{mr^2}\left(u^\theta\Sigma^{r\theta} + u^\varphi\Sigma^{r\varphi}\right)\right]\delta r,
\end{aligned}$$

$$\frac{d\delta u^\theta}{d\tau} = \frac{M(r-2M)}{mr^4}\,\delta(u^t\Sigma^{t\theta}) + \frac{M}{mr^2(r-2M)}\delta(u^r\Sigma^{r\theta}) - \frac{2M}{mr}\,\delta(u^\varphi\Sigma^{\theta\varphi})$$

$$- \frac{2}{r}\,\delta(u^r u^\theta) + \sin\theta\,\cos\theta\,\delta(u^{\varphi\,2}) + \left(\cos 2\theta\, u^{\varphi\,2}\right)\,\delta\theta$$

$$+ \left[\frac{2}{r^2}u^r u^\theta - \frac{M}{mr^5}(3r-8M)u^t\Sigma^{t\theta} + \frac{M(3r-4M)}{mr^3(r-2M)^2}u^r\Sigma^{r\theta}\right.$$

$$\left. + \quad \frac{2M}{mr^2}u^\varphi\Sigma^{\theta\varphi}\right]\delta r.$$

$$\frac{d\delta u^\varphi}{d\tau} = \frac{M(r-2M)}{mr^4}\,\delta(u^t\Sigma^{t\varphi}) - \frac{M}{mr^2(r-2M)}\,\delta(u^r\Sigma^{r\varphi}) + \frac{2M}{mr}\,\delta(u^\theta\Sigma^{\theta\varphi})$$

$$- \frac{2}{r}\,\delta(u^r u^\varphi) - (2\cot\theta\, u^\varphi)\,\delta u^\theta - \left(2\cot\theta\, u^\theta\right)\delta u^\varphi + \left(2\csc^2\theta\, u^\theta u^\varphi\right)\,\delta\theta$$

$$+ \left[\frac{2}{r^2}u^r u^\varphi - \frac{M}{mr^5}(3r-8M)u^t\Sigma^{t\varphi} + \frac{M(3r-4M)}{mr^3(r-2M)^2}u^r\Sigma^{r\varphi}\right.$$

$$\left. - \quad \frac{2M}{mr^2}u^\theta\Sigma^{\theta\varphi}\right]\delta r,$$

A.2 The spin-dipole deviations:

$$\frac{d\delta\Sigma^{t\varphi}}{d\tau} = -\frac{M}{r(r-2M)}\delta(u^t\Sigma^{r\varphi}) - \frac{(r-M)}{r(r-2M)}\delta(u^r\Sigma^{t\varphi})$$

$$- \ \mathrm{ctg}\,\varphi\,\delta(u^\theta\Sigma^{t\varphi}) - \frac{1}{r}\delta(u^\varphi\Sigma^{tr}) + \ \mathrm{ctg}\,\theta\,\delta(u^\varphi\Sigma^{t\theta})$$

$$- \left[\frac{1}{r^2}u^\varphi\,\Sigma^{tr} + \frac{2M(r-M)}{r^2(r-2M)^2}u^t\Sigma^{r\varphi} + \frac{1}{r^2}\left(1 + \frac{2M(r-M)}{(r-2M)^2}\right)u^r\Sigma^{t\varphi}\right]\delta r,$$

$$\frac{d\delta\Sigma^{tr}}{d\tau} = (r-2M)\,\delta(u^\theta\Sigma^{t\theta}) + (r-2M)\,\sin^2\theta\,\delta(u^\varphi\Sigma^{t\varphi})$$

$$+ \ \left(u^\theta\Sigma^{t\theta} + \sin^2\theta\, u^\varphi\Sigma^{t\varphi}\right)\delta r + \left((r-2M)\,\sin 2\theta\, u^\varphi\Sigma^{t\varphi}\right)\delta\theta,$$

$$
\begin{aligned}
\frac{d\delta\Sigma^{t\theta}}{d\tau} &= -\ \frac{M}{r(r-2M)}\,\delta(u^t\Sigma^{r\theta}) - \frac{(r-M)}{r(r-2M)}\,\delta(u^r\Sigma^{t\theta}) - \frac{1}{r}\,\delta(u^\theta\Sigma^{tr}) \\
&+\ \sin\theta\cos\theta\,\delta(u^\varphi\Sigma^{t\varphi}) + \left(\cos 2\theta\, u^\varphi\Sigma^{t\varphi}\right)\delta\theta \\
&+\ \left[\frac{2M(r-M)}{r^2(r-2M)^2}u^t\Sigma^{r\theta} + \frac{1}{r^2}\left(1+\frac{2M(r-M)}{(r-2M)^2}\right)u^r\Sigma^{t\theta} + \frac{1}{r^2}u^\theta\,\Sigma^{tr}\right]\delta r,
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\delta\Sigma^{r\varphi}}{d\tau} &= -\ \frac{M(r-2M)}{r^3}\,\delta(u^t\Sigma^{t\varphi}) - \frac{(r-3M)}{r(r-2M)}\,\delta(u^r\Sigma^{r\varphi}) -\ \mathrm{ctg}\,\theta\,\delta(u^\theta\Sigma^{r\varphi}) \\
&+\ (r-2M)\,\delta(u^\theta\Sigma^{\theta\varphi}) - \cot\theta\,\delta(u^\varphi\Sigma^{r\theta}) \\
&+\ \left[\frac{2M}{r^4}(r-3M)\,u^t\,\Sigma^{t\varphi} - \frac{1}{r^2}\left(1-\frac{2M(r-M)}{(r-2M)^2}\right)u^r\Sigma^{r\varphi} + u^\theta\,\Sigma^{\theta\varphi}\right]\delta r \\
&+\ \left(\csc^2\theta\, u^\theta\,\Sigma^{r\varphi} + \csc^2\theta\, u^\varphi\,\Sigma^{r\theta}\right)\delta\theta,
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\delta\Sigma^{r\theta}}{d\tau} &= -\ \frac{M(r-2M)}{r^3}\,\delta(u^t\Sigma^{t\theta}) - \frac{(r-3M)}{r(r-2M)}\,\delta(u^r\Sigma^{r\theta}) \\
&+\ \sin\theta\,\cos\theta\,\delta(u^\varphi\Sigma^{r\varphi}) - (r-2M)\,\sin^2\theta\,\delta(u^\varphi\Sigma^{\theta\varphi}) \\
&+\ \left(\cos 2\theta\, u^\varphi\,\Sigma^{r\varphi} - (r-2M)\,\sin 2\theta\, u^\varphi\,\Sigma^{\theta\varphi}\right)\delta\theta \\
&+\ \left[\frac{2M}{r^4}(r-3M)\,u^t\,\Sigma^{t\theta} + \frac{1}{r^2}\left(1-\frac{2M(r-M)}{(r-2M)^2}\right)u^r\Sigma^{r\theta}\right. \\
&\qquad\left. -\ \sin^2\theta\, u^\varphi\,\Sigma^{\theta\varphi}\right]\delta r,
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\delta\Sigma^{\theta\varphi}}{d\tau} &= -\ \frac{2}{r}\,\delta(u^r\Sigma^{\theta\varphi}) - \frac{1}{r}\,\delta(u^\theta\Sigma^{r\varphi}) + \frac{1}{r}\delta(u^\varphi\Sigma^{r\theta}) + \cot\theta\,\delta(u^\theta\Sigma^{\theta\varphi}) \\
&+\ \frac{1}{r^2}\left(-u^\varphi\,\Sigma^{r\theta} + u^\theta\,\Sigma^{r\varphi} + 2u^r\,\Sigma^{\theta\varphi}\right)\delta r + \left(\csc^2\theta\, u^\theta\,\Sigma^{\theta\varphi}\right)\delta\theta.
\end{aligned}
$$

Of course, any special case like planar orbits can be deduced from these equations, by using the respective conditions.