# Spin dynamics in general relativity

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**Citation**

Saravanan, S. (2016, July 7). *Spin dynamics in general relativity*. *Casimir PhD Series*. Retrieved from https://hdl.handle.net/1887/42443

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**Title**: Spin dynamics in general relativity
**Issue Date**: 2016-07-07

Chapter 1 introduces gravity in the Newtonian framework, stating its merits and short comings. Then by describing Einstein's general relativistic idea of gravitation, I define gravitational waves and its properties. The indirect evidence of gravitational waves has been explained with binary pulsar PSR B1913+16. The first direct detection of gravitational waves: GW150914, conforms the existence of black holes and binary black holes. With these motivations, I describe my system of research – Extreme Mass Ratio Systems and its scope of experimental detection with the evolved Laser Interferometer Space Antenna.

# 1

# Introduction

## 1.1 Gravitation

Of the four fundamental forces of nature, gravity is the weakest. For instance, the gravitational force between the proton and electron is $10^{40}$ times smaller than the electric force that binds these particles together in atoms. However gravity is a universal force. Newton's law of gravitation was the first major physical theory which attempts to describe gravity. According to Newton's theory, two bodies, irrespective of whether they are on the Earth or in the heavens, whether they are in the state of motion or rest, always mutually attract each other with a force directly proportional to the product of their mass and inversely proportional to the square of their mutual distance

$$F = G\frac{mM}{d^2}, \tag{1.1.1}$$

where $F$ is the force between the masses; $G$ is the universal gravitational constant, whose value is $6.674 \times 10^{-11} Nm^2/kg^2$; $m$ and $M$ are two masses, and $d$ is the distance between the centers of the masses. This implies, the gravitational force propagates in space at an infinitely great speed. This is the weak point of Newton's theory, because it means that something is simultaneously having an effect somewhere, where it is not present, and this is a physical impossibility. Despite this weakness, it still provides an excellent basis for explaining and calculating the planetary movements.

This absurd idea of "action at a distance" emerging in Newton's theory was not resolved, until Einstein in 1915. Einstein described space and time as different aspects of reality in which matter and energy are ultimately the same. With this he describes gravitation very accurately in this 4-dimensional universe (3 spatial dimension + 1 time dimension) in which we are living in. The presence of large amounts of mass or energy distorts space-time – in essence causing the fabric to "warp" and we observe this as gravity.

Freely falling objects – whether a soccer ball, a satellite, or a beam of starlight – simply follow the shortest space-time path (geodesic) in this curved space-time.

Therefore, the planets are moving in "straight lines" in the curvature produced by the sun and it appears as if they are in circular or elliptical motion around the sun. This is the central idea of general theory of relativity [3].

Thus the Newtonian idea of a gravitational force acting at a distance between bodies was replaced by the idea of a body moving in response to the curvature of space-time. Indeed Newton's theory of gravity is not completely wrong. It is a correct approximation to Einstein's theory when space-time curvature is negligible and the velocities of masses are much smaller than the velocity of light.

Newton's theory forms an excellent basis for describing weak gravitational regimes like in earth or solar system. In this regimes the general relativistic corrections to the Newton's theory are very small. But general relativity also predicts new strong gravitational phenomena like bending of light, black holes, gravitational waves and the big bang.

## 1.2 Gravitational Waves

Accelerated mass varies space-time and the change propagates as ripples in space-time curvature with the speed of light known as gravitational waves. Gravitational waves are analogous to the electromagnetic waves, the oscillations in the electric and magnetic fields produced by the accelerated charges.

Mass in motion is the source of gravitational waves. In turn, gravitational waves can be detected through the motion of masses produced as the ripple in space-time curvature passes by. When a gravitational wave passes through a ring of particles it changes their relative positions, depending on the wave's polarisation [4]. Here we have shown the particle's motion produced by a wave with "+" polarisation (top line) and " × " polarisation (bottom line).

Fig. 1.1 implies that a single wave cycle of a gravitational wave changes the ring ($R$ being the radius) into an ellipse with semi-major axis $R + dR$ and semi-minor axis $R - dR$, back through a ring into the same ellipse rotated by $90°$ and finally back to a ring. The strength of a gravitational wave is determined by how rapidly the quadrupole moment of its source is changing:

$$h \simeq \frac{G}{c^4} \frac{d^2Q/dt^2}{D} \tag{1.2.1}$$

where $h$ is the strain, the strength of a gravitational wave, $Q$ is the quadrupole moment of the source and $D$ is the distance from source to observer and $c$ is the speed of light.

In principle any accelerated mass produces gravitational waves, for example a falling apple. But the quantity $\frac{G}{c^4} = 8.26 \times 10^{-45} kg^{-1}(m/s^2)^{-1}$ is very tiny, therefore we need very large masses undergoing extreme accelerations to produce detectable gravitational waves. Thus we look for most energetic phenomena in the universe like big bang, supernovae explosion or compact binary coalescence [5].

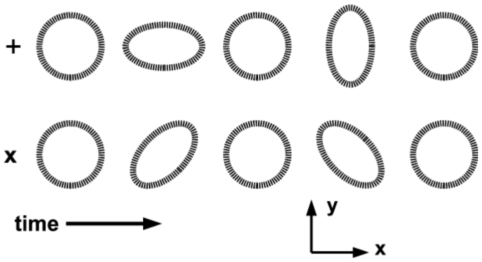


**Figure 1.1**. The effect of a gravitational wave on a ring of particles. The wave is traveling in the z-direction (perpendicular to the page). The upper and lower parts are the effects of a "+" and " × " polarised wave, respectively.

## 1.3 PSR B1913+16: Indirect Evidence of Gravitational Waves

Even after several years of general relativistic predictions of gravitational waves, their existence was not universally believed. The very first convincing experimental evidence was given by Russell Hulse and Joseph Taylor in 1974 [6] in connection with the discovery of binary pulsar PSR B1913+16. The observed system must be composed of neutron stars, at least one of which is a pulsar. We observe a pulse of radio waves every time the bright spot sweeps around to face Earth [7]. The pulsar has a rotational period of 59 ms and its frequency varied with a period of 7.75 hours; apparently it is a member of a binary system with high eccentricity [8, 9].

After several years of observation [10, 11], a variety of relativistic effects has been recognized: orbital precession, advance of periastron, gravitational redshift, and the time-dilation and so on. It is found that both the objects in the system were neutron stars (incredibly dense objects the burned-out core often left behind after a supernovae) with masses around 1.4 $M_\odot$ (solar mass). But the most exciting prediction was that they found the orbital period was decreasing by about 75 millionths of a second per year. This could not be understood unless the dissipative reaction force associated with gravitational waves produced is included. Thus the two neutron stars gradually fall closer to each other and their orbital speed increases steadily because it emits energy as gravitational waves and this is in excellent agreement with the rate predicted by the general relativity as shown in the Fig. 1.2.

The frequency of the gravitational waves from the Hulse-Taylor binary system are too low for the existing ground based detectors to detect the signal. But the rate of orbital decay as predicted by the general relativity is in perfect agreement with the experimental observation is the very first strong evidence for the existence of gravitational waves [12, 13]. This discovery of Hulse and Taylor has opened a new window to study gravitation and they were awarded Nobel prize in 1993.

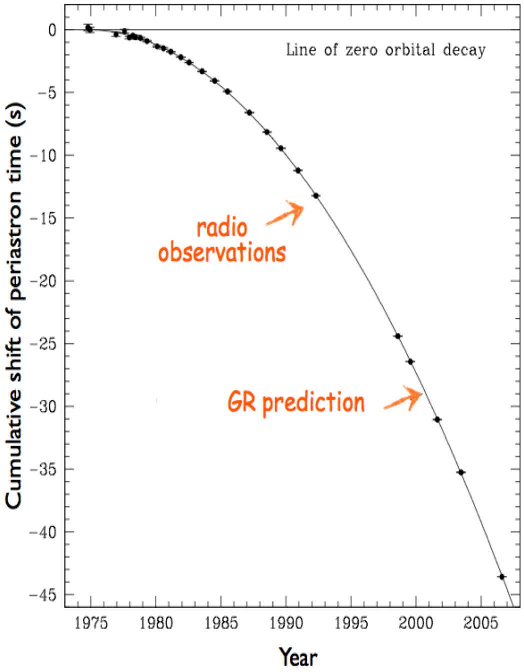


**Figure 1.2**. The orbital period of binaries PSR B1913+16 decreases because the system loses energy as gravitational waves. Since the system is relativistic, the effect is very strong here. The measure of this decrease in orbital period is due to the steady shift over time of the time of the pulsar's periastron (closest approach to its companion). The points are the observed data points over several decades and the solid line is the general relativity prediction.

## 1.4 GW150914: The Direct Detection of Gravitational Waves

According to general relativity, whenever a sufficient mass is compressed into a very small volume such that the gravitational pull at the surface is too large, even light cannot escape once it enters into the surface. Such objects are called black holes. Black holes can be identified with minimum number of properties like mass, spin and charge.

Coalescence of black hole binaries are the most promising sources of gravitational radiation [14, 15]. According to general relativity the coalescence happens in three phases: in-spiral, merger and ringdown (Fig. 1.3). During the evolution there is loss in the energy and angular momentum of the system; as they are carried away by the gravitational waves. Therefore the orbit shrinks at the rate predicted by general relativity, and is already confirmed by the observation in the Hulse-Taylor binary system.

> *"The black holes of nature are the most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time. And since the general theory of relativity provides only a single unique family of solutions for their descriptions, they are the simplest objects as well."*
>
> *– S. Chandrasekhar, The Mathematical Theory of Black Holes [16]*

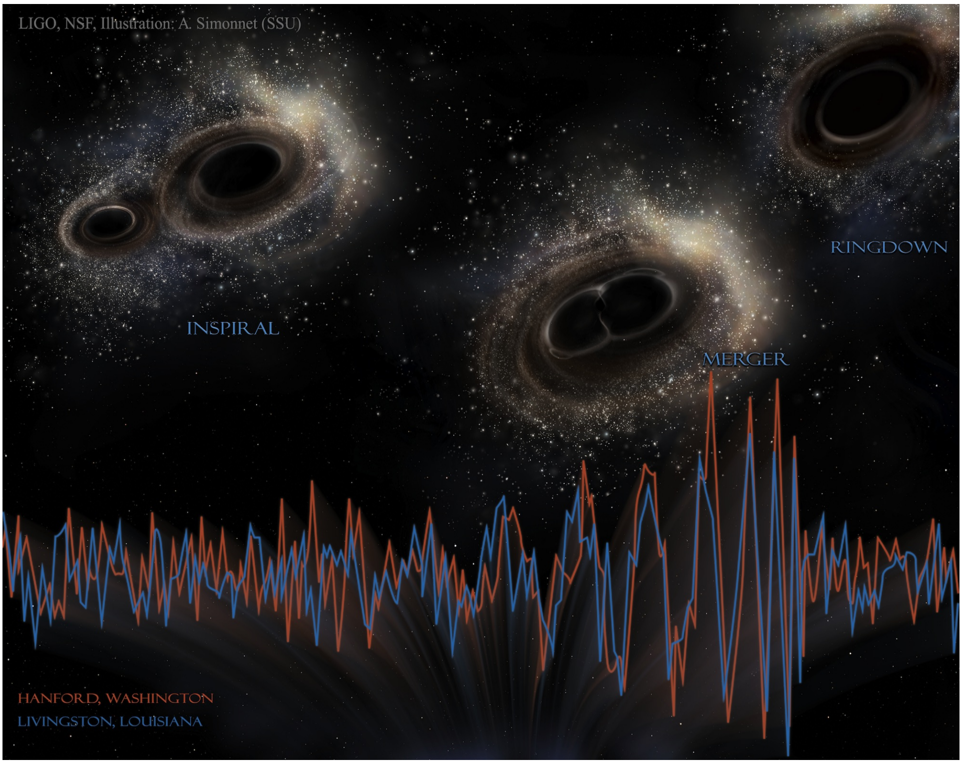


**Figure 1.3**. The evolution of binaries occur in three stages: in-spiral, merger and ring-down, as shown above. During the merger phase the system emits huge amount of gravitational waves. The orange coloured wave is the gravitational wave pattern observed in LIGO - Hanford and similarly the blue coloured wave is the pattern obeserved in LIGO - Livingston. These observations are named as GW150914 to indicate that the gravitational waves passed the detectors on 2015 September 14 (EST). Credit: LIGO / NSF / A. Simonnet (SSU)

After the completion of field equations in 1915, Einstein predicted the existence of gravitational waves in 1916. Historically searches for gravitational waves were started with the development of "Weber bar" detectors [17] and then Interferometric detectors since 1970 [18, 19].

After five decades of work, advanced Laser Interferometer Gravitational-Wave Observatory (advanced LIGO) is in operation now and made the *first* direct observation of gravitational waves [20, 21]. The event is named as GW150914 to indicate that the gravitational waves passed the detectors on 2015 September 14 (EST). The wave appeared first at Livingston, LIGO detector and then at Hanford, detector, a 7 ms later. This time difference is consistent with the fact that *gravitational wave travels at the speed of light*. The gravitational wave stretched and squeezed space-time with the frequency sweeping from 40 Hz to 260 Hz over 0.2s in the pattern of two black holes merging together (Fig. 1.4). The masses of two black holes are predicted to be 29 $M_{\odot}$ and 36 $M_{\odot}$. They merged to form a single black hole with a mass of 62 $M_{\odot}$. Thus the remaining 3 $M_{\odot}$ energy is released as gravitational waves during the inspiral and merger phases of these black holes. Then the remnant black hole has spin at a rate of 100 rotations per second. Thus the discovery implies the following conclusions:

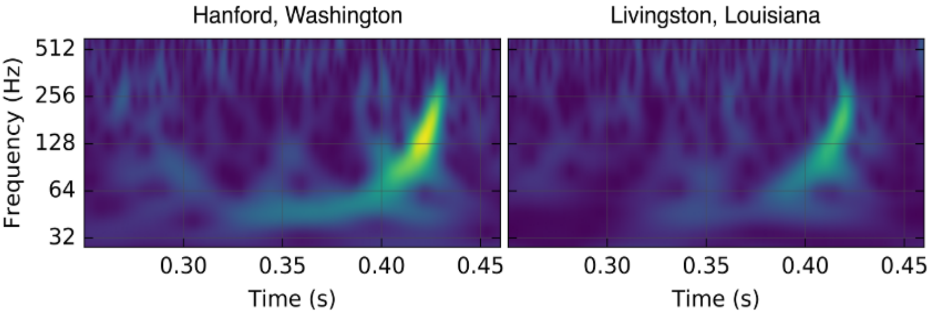


**Figure 1.4**. Pictures from the paper [20] reporting the GW150914 discovery. The frequency of gravitational wave oscillation plotted vertical, as a function of time plotted horizontally. The colors show the strength of the waves. Green and yellow represents the oscillations of gravitational wave; yellow represents the very strong oscillations during the merger phase. Blue colour due to noise in the detector. At both Hanford and Livingston, the green-yellow oscillations have precisely the form that we expect for gravitational waves produced by binary black holes in-spiralling and colliding.

*(i). First direct detection of gravitational waves*

*(ii). First direct evidence of existence of black holes*

*(iii). First observation of binary black holes.*

This discovery give rise to a new branch of astrophysics: gravitational wave astronomy [22, 23]. Through which we can explore the dark side of the universe in the very broad spectrum which were inaccessible to us with electromagnetic astronomy.

Black holes and/or neutron stars, composed of stellar mass binaries are optimistic sources for ground based network of detectors [24–27] like advanced-LIGO, VIRGO, KAGRA, GEO 600 and LIGO-India. The observation of gravitational waves from such binaries will bring various information: event rate, binary parameters and even possible deviations from general relativity [28–30].

## 1.5 Laser Interferometer Space Antenna (eLISA) and sources

The existing ground based detectors are sensitive around 100 Hz. The universe is rich in strong sources of gravitational waves when we probe below these frequencies. But the seismic noise makes the ground based detectors insensitive for lower frequencies. Therefore we need observations from space. The space based detector eLISA is sensitive for frequencies from 0.1 mHz to 100 mHz [31,32], and it is planned to be launched by European Space Agency (ESA) in 2034. These frequencies corresponds to wide range of gravitational wave sources and its direct detection with eLISA will answer the very fundamental questions; mapping the present universe to all the way shortly after the Big bang. The mission has been named with the science theme *The Gravitational Universe* [33] by the ESA.

The electromagnetic observations clearly show that stars, black holes, and galaxies are ubiquitous components of the universe [34]. eLISA will study these objects in the gravitational wave spectrum. Thus it measures the amplitude of the strain in the space as a function of time. The following are the prospective sources (not limited to) of gravitational waves:

**Supermassive Binary Black Holes** Almost all bright galaxies (including our own Milky Way) host one or more massive central black holes. Their masses range from $10^4\ M_\odot - 10^7\ M_\odot$ and these are called as supermassive black holes. When galaxies coalesce (Fig. 1.5), these black holes will merge eventually [35], releasing huge amount of gravitational radiation during the process. Thus detecting these signals will not only test theories of gravity and black holes, but also reveal information about the evolution and merger history of galaxies [36].

**Ultra-Compact Binaries** The components of binaries could be compact objects like stellar mass black holes, neutron star or white dwarf. The Milky Way is full of these sources [37], but only a small fraction is observable in the radio and X-ray spectrum. Since the maximum loss of energy from these systems are always through gravitational waves, and it lies in the frequency range of eLISA [38], it should be possible to map all these objects soon.

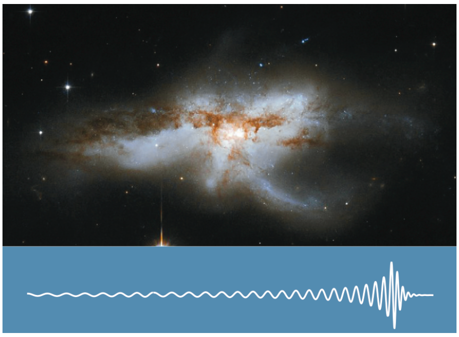
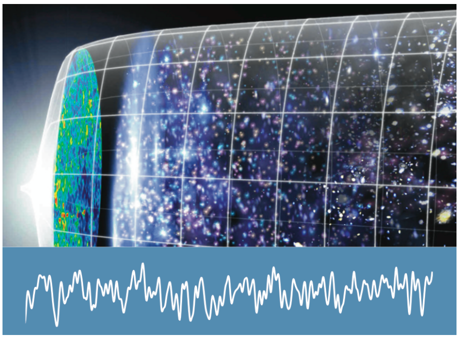


**Figure 1.5**. Coalescence of two supermassive black holes. The expected gravitational waveform has in-spiral, merger and ringdown phase. Merging galaxy NGC 6240 [35] has two giant black holes as reported by NASA's Chandra X-ray observatory. Credit: NASA / ESA / the Hubble heritage / A. Evans.

**Figure 1.6**. The fossil gravitational waves are the only way to probe the early Universe all the way immediately after the Big Bang. The expected gravitational waveform is stochastic background (random noise). Credit: NASA / WMAP science team.



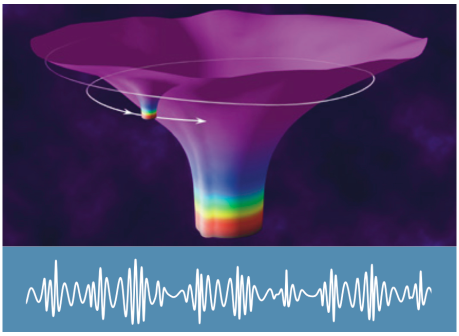


**Figure 1.7**. An artist impression of EMRS; a small black hole inspiralling around a supermassive black hole. The modelled waveform would have many harmonics as shown. Credit: NASA.

**The Big Bang and The Early Universe** Since the universe was transparent to gravity moments after the Big Bang and long before light (remember the first light still present, Cosmic Microwave Background was produced only about 300,000 years after the Big Bang), gravitational waves will allow us to observe further back into the history of the universe than ever before. And since gravitational waves are not absorbed or reflected by the matter in the rest of the universe, we will be able to see them in the form in which they were created. Immediately after the Big Bang, when the Universe was very young it underwent a period of very rapid expansion, as a result space-time got distorted which in turn produced gravitational waves between approximately $10^{-43}$ to $10^{-32}$ seconds after the Big Bang (Fig. 1.6). These relic gravitational waves from the early evolution of the universe may carry information about the origin and history of the universe.

**Extreme Mass Ratio Systems** The supermassive black holes in the galactic centres may be accompanied by one or more stellar mass compact objects like black hole, neutron star or white dwarf, of few solar masses are called Extreme Mass Ratio Systems (EMRS) (Fig. 1.7). eLISA will track the complex relativistic orbits of the stellar companion around a central black hole in the mass interval between $10^4\ M_\odot < M < 5 \times 10^6\ M_\odot$ for upto $10^4 - 10^5$ cycles [39, 40]. The waveforms emitted from these systems would inform us about the stellar mass compact object populations, mass spectrum and their spin. It will also describe the properties of space-time geometry around the central black hole [41, 42] and the formation of supermassive black holes at the galactic centers [43].

Infrared astronomy has given the *best empirical evidence* for the existence of a 4 million solar mass black hole in our Milky Way. Observation has not only tracked the 28 stars orbiting a supermassive black hole [44, 45]: Sagittarius A*, but also predicted its mass and distance (27,000 light years away from the solar system). The stellar orbits in the galactic centre show that the central mass concentration of four million solar masses must be a black hole, beyond any reasonable doubt [46].

Though most of the black holes in nature are spinning, we start modelling EMRS as a small spinning black hole or a neutron star orbiting around a static spherically symmetric - supermassive black hole. This is the subject of this thesis. The mass ratio between the smaller compact object and the central black hole is typically $\sim 10^{-5}$. Because of this extreme mass ratio, the curvature produced by the smaller object can be neglected. I introduce the necessary general relativistic tools for modelling EMRS in chapter 2 & 3. After this I discuss EMRS: a test mass orbiting a Schwarzschild black hole in chapter 3. Then I discuss the formalism for a spinning compact object in Schwarzschild space-time in chapter 4, which we have developed recently [47]. In chapter 5, I discuss the applications of our formalism and important aspects of the dynamics of EMRS [48]. The theoretical model I present in the rest of the thesis is esentially the preparation for eLISA observations. The first direct observation of EMRS and therefore supermassive black holes, through gravitational wave detection is expected immediately after the launch of eLISA mission. The estimated detection rates based on the best available models are 50 events for a 2 year mission [49].