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Topics in the arithmetic of del Pezzo and K3 surfaces

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geboren te Salerno, Italië,
in 1988

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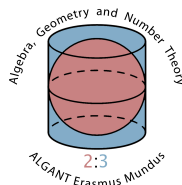
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This work was part of the ALGANT program and was carried out at Universiteit Leiden and Università degli studi di Milano.



A Rino, Paola, e Gaetano.

The mathematician's patterns, like
the painter's or the poet's, must be
beautiful; the ideas, like the colours or
the words, must fit together in a
harmonious way.
Beauty is the first test:
there is no permanent place in the
world for ugly mathematics.

A mathematician's apology,
G. H. HARDY,
1940.

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