Hierarchical systems

Hamers, A.S.

Author: Hamers, Adrian Sven
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Summary

We have studied the gravitational dynamical evolution of hierarchical systems with various hierarchies, in various astrophysical contexts. We began by considering systems of the 'multiplex' type, dominated by an SBH (Chapters 2 and 3). Subsequently, we focussed on simplex-type systems, beginning with the general dynamics of quadruple systems (Chapter 4), and continuing with an application to circumbinary planets (Chapter 5). Finally, we generalised our methods to hierarchical simplex-type systems with an arbitrary hierarchical structure and an arbitrary number of bodies (Chapter 6), and applied these methods to the production of HJs through high-eccentricity migration in multiplanet systems (Chapter 7).

Chapter 2 – Relativistic dynamics around an SBH

In this chapter, we studied the long-term gravitational dynamics of orbits around a supermassive black hole (SBH). We developed a special-purpose N-body code, TPI, that allowed us to efficiently integrate these orbits including general relativistic (GR) corrections, and taking into account interactions with a large ($\sim 10^3$) number of 'field stars', which are assumed to move in (nearly) fixed Keplerian orbits. We have shown that TPI agrees with other, more accurate but slower codes.

Using TPI, we performed simulations of the Galactic Centre (GC) to test models for the origin of the S-stars with 4800 field stars close to the SBH. Such simulations were previously not feasible with other direct N-body codes. We assumed that the S-stars were deposited initially on to orbits of very high eccentricity, which is expected for the tidal disruption of a stellar binary. We have found that the cumulative eccentricity distribution of the S-stars evolves to $N(e) \sim e^{2.6}$ on a time-scale of $7 \pm 0.1\,\mathrm{Myr}$, which is consistent with observations. Our results suggest a lower limit on the typical age of the S-stars of $\sim 7\,\mathrm{Myr}$ and $\sim 25\,\mathrm{Myr}$ assuming burst and continuous formation of the S-stars, respectively.

In addition, from our simulations we extracted first- and second-order diffusion coefficients in the normalized angular-momentum variable $\ell \equiv \sqrt{1-e^2}$. These coefficients describe the long-term angular momentum evolution. We identified three angular-momentum regimes, in which the diffusion coefficients depend in functionally different ways on ℓ. In the regime of low ℓ, the diffusion coefficients are well described in terms of non-resonant relaxation, i.e. relaxation due to two-body encounters. In the regime of high ℓ, the diffusion coefficients are well described in terms of resonant relaxation (RR), i.e. torques arising from correlated encounters. In the intermediate regime, near the 'Schwarzschild barrier' (SB) where relativistic precession is important, we identified a new regime of 'anomalous relaxation', in which diffusion is driven by a combination of RR and rapid relativistic precession. In particular, we showed that the SB can be associated with a rapid drop of the diffusion coefficients with decreasing ℓ. We presented analytic expressions, in terms of physical parameters, that describe the diffusion coefficients in all three angular-momentum regimes. Also, we applied our results to obtain the steady-state distribution (i.e. extrapolated to infinity) of the orbital angular momentum.

Chapter 3 – Planetesimals in the GC

Continuing with the previous topic of orbits around an SBH, we studied the dynamics of planetesimals in the GC, and focussed on the tidal disruption of planetesimals by the SBH, possibly producing near-infrared and X-ray flares, which are observed from the SBH in the GC, Sgr A*, on an approximately daily basis. We assumed that the planetesimals were either formed in a large-scale cloud bound to the SBH (scenario 1), or in debris discs around stars (scenario 2). In the latter case, we found that the tidal force of the SBH is effective at stripping the planetesimals from their parent stars at distances $r \lesssim 0.5\,\mathrm{pc}$ from the SBH. Stripping by gravitational encounters with other stars is effective at stripping nearly all planetesimals within the radius of influence, $\approx 4\,\mathrm{pc}$, after $\sim 100\,\mathrm{Myr}$.

We used the Fokker-Planck equation in energy space taking into account these stripping effects and other possibly important effects such as relaxation due to massive perturbers, and determined the disruption rate of planetesimals by the SBH. We found that the disruption rate at $t = 10\,\mathrm{Gyr}$ is $\approx 0.6\,\mathrm{d}^{-1}$, which is roughly consistent with the observed rate of the flares of once per day. Our rates are insensitive to the model assumptions, in particular the initial distribution of planetesimals, i.e. scenarios (1) or (2). In the case of scenario (2), the assumed number of planetesimals per star $N_{\mathrm{a}/\star} = 2 \times 10^7$, is inferred from observations of debris discs around stars in the solar neighbourhood. In contrast, in the case of scenario (1), i.e. formation in a large-scale cloud, this implies that the number of planetesimals formed is strongly correlated with the number of stars, which requires finetuning of $N_{\mathrm{a}/\star}$. We favoured the more natural explanation that planetesimals in galactic nuclei similar to the GC are formed no differently than planetesimals around stars in the solar neighbourhood.

Chapter 4 – Secular evolution of hierarchical quadruple systems

In the remaining chapters, we changed tack to hierarchical systems of the simplex type, i.e. systems consisting of nested binary orbits. We first considered hierarchical quadruple systems in the '3+1' configuration, i.e. a hierarchical triple orbited by a fourth body. We assumed that the system is sufficiently hierarchical, i.e. that the ratios x_i of the binary separations are small, and we expanded the Hamiltonian in terms of the x_i. Subsequently, we orbit-averaged the expanded Hamiltonian, and implemented the equations of motion into a computer code, SECULARQUADRUPLE.

Subsequently, we studied the secular evolution of highly hierarchical systems that are well described by the lowest order terms in the Hamiltonian, and we characterised the evolution in terms of ratios of Lidov-Kozai (LK) time-scales applied to different binary pairs. Let the innermost, intermediate and outermost binaries by denoted with A, B and C, respectively. For these systems, we found that the global dynamics can be qualitatively described in terms of the (initial) ratio of the LK time-scales applied to the AB binary and BC binary pairs, $\mathcal{R}_0 \equiv P_{\mathrm{LK,AB,0}}/P_{\mathrm{LK,BC,0}}$.

If $\mathcal{R}_0 \ll 1$, then the torque of binary B on A dominates compared to the torque of binary C on binary B, and therefore binaries A and B remain coplanar if this was initially the case. If binaries A and B are initially inclined, LK eccentricity oscillations in binary A are not much affected by the presence of the fourth body. Eccentricity oscillations in binary B due to the secular torque of binary C are efficiently quenched due to short time- scale precession induced on binary B by binary A.

If $\mathcal{R}_0 \gg 1$, then the torque of binary C on binary B dominates compared to the torque of binary B on binary A. Initially, the inclination of binary B changes, whereas this is not the case for binary A. This induces a mutual inclination between binaries A and B, even if they are initially not inclined. However, rapid precession of binary B compared to the LK time-scale for the AB pair prevents any significant eccentricity oscillations in binary A, and even quenches LK oscillations if binaries A and B are initially inclined.

Lastly, if $\mathcal{R}_0 \sim 1$, complex LK eccentricity oscillations can occur in binary A that are strongly coupled with the LK eccentricity oscillations in binary B. The latter are also affected compared to the situation in which binary A were replaced by a point mass, although this is typically a much smaller effect. Even if binaries A and B are initially coplanar, the induced inclination can result in high-amplitude eccentricity oscillations in binary A. These extreme eccentricities could have significant implications for strong interactions such as tidal interactions, gravitational wave dissipation, and collisions and mergers of stars and compact objects.

Chapter 5 – Explaining the lack of circumbinary planets around short-period binaries

In this chapter, we applied SECULARQUADRUPLE to explain a recent curious result in observations of transiting circumbinary planets by the *Kepler* spacecraft. Assuming that the occurrence rate of circumbinary planets is not related to the binary period, then about twice more planets are expected to have been observed around short-period binaries ($\sim 3-6\,\mathrm{d}$) compared to their longer period counterparts. However, so far, no planets have been observed around these short-period binaries, whereas 10 have been found around longer-period binaries. Therefore, it there is likely an intrinsic lack of (observable) circumbinary planets around short-period binaries.

We have shown that this lack can be explained by the secular influence of a circumbinary planet in hierarchical *triple star* systems. Short-period binaries are believed to have formed in stellar triple systems through LK eccentricity oscillations in the inner orbit induced by the tertiary, combined with tidal friction. In this case, the progenitor inner binary was wider, with an orbital period of up to $\sim 10^4$ d. We have shown that if there is a circumbinary planet around the progenitor inner binary, then the LK eccentricity oscillations in the inner orbit induced by the tertiary can be quenched by additional precession induced by the circumbinary planet. Thereby, the inner binary is 'shielded' from the torque of the tertiary star, and does not shrink to a tight orbit. However, this only occurs if the circumbinary planet is sufficiently massive, and if its orbit is sufficiently close to and coplanar with the inner binary. In other cases, the circumbinary orbit is stable but cannot efficiently shield the inner binary, or the circumbinary orbit is destabilized and is most likely ejected. In particular, if a low-mass circumbinary planet is initially inclined with respect to and far away from the inner binary, then planet shielding is typically ineffective, and the inner binary can shrink to a tight orbit. On the other hand, for a more massive planet in an initially approximately coplanar and tight circumbinary orbit, shielding typically prevents the inner orbit from shrinking.

Consequently, short-period binaries typically do not have massive ($m_3 \sim M_\mathrm{J}$) circumbinary planets on tight and coplanar orbits, which would have prevented orbital shrinkage. In contrast, if the circumbinary planet not massive ($m_3 \sim 10^{-2}\,M_\mathrm{J}$) and initially inclined with respect to and far away from the inner binary, then shrinkage is possible. This trend is consistent with the current *Kepler* observations, and suggests that the latter type of currently unobservable planets may be present around short-period binaries.

Chapter 6 – Secular evolution of hierarchical multiple systems

We generalised the method and the algorithm developed in Chapter 4 to systems consisting of nested binary (i.e. simplex) orbits, with an *arbitrary* number of bodies, and an *arbitrary* hierarchical configuration. We expanded the Hamiltonian in terms of ratios of binary separations, which are assumed to be small, and carried out orbital averaging. We implemented the formalism in an algorithm, SECULARMULTIPLE, making long-term integrations of hierarchical systems with complex hierarchies feasible, and presented first applications. In particular, we applied our method to the high-multiplicity systems Mizar and Alcor, and 30 Ari.

Chapter 7 – Hot Jupiters in multiplanet systems

We applied SECULARMULTIPLE to the formation of hot Jupiters (HJs) through high-eccentricity migration in multiplanet systems (systems with at least three planets). In this scenario, hot Jupiters are assumed to have been formed as Jupiter-like planets at $\sim 1\,\mathrm{AU}$. Subsequently, the orbital eccentricity was excited due to secular interactions with the other planets, strongly increasing the rate of tidal dissipation, and eventually producing a HJ in a circular orbit with a semimajor axis of $\sim 0.05\,\mathrm{AU}$.

First, we verified that our secular algorithm produces statistically correct results compared to direct N-body simulations. The latter are computationally much more expensive, however, therefore the secular method allows for an efficient exploration of the large parameter space of multiplanet systems. We investigated the conditions for exciting high eccentricities in multiplanet systems, and therefore potentially HJs. In addition to a sufficiently large angular momentum deficit (AMD), we found that a necessary condition for eccentricity excitation is that the ratio of LK time-scales $\mathcal{R}_0^{(ijk)}$ applied to adjacent orbit pairs (i,j) and (j,k) should be near unity. The strongest dependence applies to $\mathcal{R}_0^{(123)}$, i.e. the ratio applied to the innermost orbits.

Subsequently, we carried out a population synthesis study in which we combined grid and Monte Carlo sampling. We found that the number of HJs formed is typically small, and is strongly dependent on the assumed tidal dissipation strength, i.e. the viscous time-scale (or, equivalently, the time lag). Assuming the viscous time-scale derived previously by Socrates et al. (2012), we found no HJs at all, whereas at most ~ 0.01 of the systems lead to HJs if the assumed tidal dissipation strength is assumed to be 100 times more efficient. This can be explained by the ‘violent’ nature of secular eccentricity driving of the innermost planet, which tends to lead to the rapid tidal disruption of the planet, rather than the slower process of orbital shrinkage and circularisation due to tidal dissipation. Our results imply that high-e migration in multiplanet systems can explain at most 0.01 of the observed HJs, assuming 100 stronger tidal dissipation compared to Socrates et al. (2012).