Hierarchical systems

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1 Introduction

1.1 Gravity

Gravity is one of the four fundamental forces of nature. It plays a crucial role in the Universe at all scales, from the onset of the Big Bang, at the present day, and in the future. On Earth, gravity ensures that objects are not flung into space. In the Solar system, it binds the planets to the Sun, which itself was formed by the gravitational collapse of a giant gas cloud. Most stars are bound by gravity in galaxies like the Milky Way and smaller clusters like globular clusters. The largest known gravitationally bound and orbiting systems are clusters of galaxies, with scales on the order of Mpc, and crossing times as long as Gyr. On the other hand, at the time of writing, ESO's Rosetta spacecraft is bound to and orbiting comet 67P at a mere distance of 10 km.

Despite the crucial role of gravity on these disparate scales, its fundamental nature is not yet completely understood. On the scale of galaxies (kpcs), measurements of the velocities of galaxies in galaxy clusters, and the rotation rate of stars suggest the presence of additional unseen, i.e. dark matter (Zwicky 1933; Rubin et al. 1980), or a modification of the laws of gravity on these scales, giving rise to Modified Newtonian Dynamics, or MOND (Milgrom 1983). On the other extreme end of the length scale spectrum of the Planck length (1.6×10^{-35} m), classical general relativity (GR) currently cannot be reconciled theoretically with quantum physics.

Between these extremes of length scales, gravity is generally well described in terms of the theory of GR. To extraordinary accuracy, pulse timing measurements of pulsars in binary systems have confirmed predictions by GR of the apsidal precession and the shrinkage and circularisation of the binary orbit due to the emission of gravitational waves (GWs) (Hulse & Taylor 1975; Burgay et al. 2003; Antoniadis et al. 2013). Until very recently, the existence of GWs was only proven indirectly through the orbital decay of these pulsar binaries. In a spectacular discovery, the first direct detection of GWs by the ground-based LIGO detector was announced on February 11 2016 (Abbott et al. 2016). The source, GW150914, was a stellar black hole binary (with masses of ≈ 36 and $\approx 29\,\mathrm{M}_\odot$) that merged to a single black hole of $\approx 62\,\mathrm{M}_\odot$, releasing $\approx 3\,\mathrm{M}_\odot$ of mass-energy in the form of GWs. The observed waveform was in extremely good agreement with the predictions of GR.

The GW150914 system clearly resides in the regime of *strong* gravitational fields. In the limit of *weak* gravitational fields, GR reduces to the Newtonian laws of motion and gravity, which describe well the dynamics of most planetary and stellar systems. In some cases, it is necessary to include in the Newtonian description corrections from GR, which are usually well described in terms of the post-Newtonian (PN) framework (Einstein et al. 1938). For example, in the Solar system, GR corrections contribute to the apsidal precession of Mercury, and together with Newtonian gravitational perturbations from other planets, amount precisely to the measured apsidal precession rate.

1.2 History of the N-body problem

The Newtonian laws of motion and gravitation have been the subject of research by mathematicians and (astro)physicists for centuries. In the 17th century, Isaac Newton, in his *Principia* (Newton 1687), famously solved his laws of motion and gravity for two-body systems, providing a theoretical understanding of Kepler's laws of planetary motion (Kepler 1609). In modern notation, these laws are formulated mathematically as

$$m_i \ddot{\boldsymbol{R}}_i = \sum_{j \neq i}^{N} \frac{G m_i m_j (\boldsymbol{R}_j - \boldsymbol{R}_i)}{||\boldsymbol{R}_j - \boldsymbol{R}_i||^3}, \tag{1.1}$$

where m_i and $\boldsymbol{R}_i$ are the mass and the position vector of body i, respectively. Although seemingly simple, Newton's law of gravity turn out to be notoriously hard to solve for systems with more than two bodies. In the 18th century, progress was made by Euler and Lagrange for specific three-body configurations (notably, the circular restricted three-body problem). In the context of the Solar system, Laplace and Lagrange developed approximate solutions for the long-term evolution of the planetary orbits. Although very useful, these solutions were approximate and apply only to planetary systems with a central massive body with nearly circular and coplanar orbits. Also, strong interactions between planets, which could lead to collisions or ejections, are not taken into account, as well as orbital resonances.

The difficulty of finding a general solution to the 3- or N-body problem led to the establishment in 1885 of a prize by the journal *Acta Mathematica* in honour of the 60th birthday of King Oscar II of Sweden and Norway. The problem was posed as follows.

Given a system of arbitrarily many mass points that attract each according to Newton's law, under the assumption that no two points ever collide, try to find a representation of the coordinates of each point as a series in a variable that is some known function of time and for all of whose values the series converges uniformly.

Although he did not solve this specific problem, Poincaré received the prize for his work on the three-body problem. In particular, he showed that no solutions exist in terms of known 'simple' analytic functions, and in the process, he laid the foundation for chaos theory.

The actual solution to King Oscar's prize as it was originally formulated, although restricted to $N = 3$, was found by Sundman (Sundman 1912). For the case of three bodies, he derived a solution in terms of a convergent power series of $t^{1/3}$. Although formally a solution, it did not give physical insight into the three-body problem. In addition, the series converges extremely slowly, making it not useful for practical applications — in order to be useful for astronomical observations, one would need at least $10^{8\times10^6}$ terms (Beloriszky 1930).

Nearly 80 years later, Sundman's solution for $N = 3$ was generalised to arbitrary N by Wang (1991). As before, the solution does not give much physical insight, and converges extremely slowly.

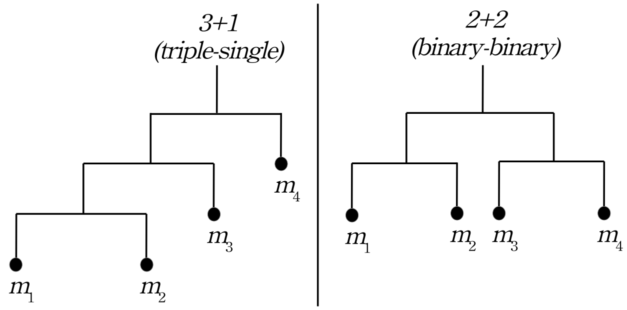


Figure 1.1: A schematic representation of two hierarchical quadruple systems in a mobile diagram. Left: a '3+1' or 'triple-single' configuration. Right: a '2+2' or 'binary-binary' system.

Although no physically meaningful solutions exist to the general N-body problem, analytic or semi-analytic results can be obtained for certain restricted systems. In particular, in this thesis, we will consider systems which are ordered in a hierarchy, i.e. hierarchical systems.

1.3 Hierarchies in nature

Planetary and stellar systems are often ordered in a hierarchical fashion. This can be explained intuitively by noting that on short time-scales, i.e. time-scales comparable to the orbital periods or crossing times, non-hierarchical systems are usually unstable, or stable but unstable to perturbations[1]. For example, in the Solar system, all planets orbit a common centre, the Sun (more correctly, the centre of mass of the solar system). Their orbits are sufficiently separated to ensure stability, at least on time-scales that are short compared to the lifetime of the Sun. The Solar system also contains 'deeper' hierarchical layers: most planets are orbited by moons, which in turn can be orbited by lighter objects (e.g. the Apollo spacecraft around the Moon).

Other, more exotic, hierarchies have been observed outside of the Solar system. For example, planets have been observed to orbit stellar binaries (circumbinary planets). A very useful tool in the study of hierarchical systems is the *mobile diagram*, a tree diagram in which the hierarchy of the system is depicted schematically[2]. An example is shown in the left part of Fig. 1.1, where a quadruple system of the 'triple-single' or '3+1' configuration is depicted. This could represent a circumbinary planet (m_3) orbiting a stellar binary ($m_1 + m_2$); the stellar binary itself is part of a stellar triple system (third star m_4). Another possible hierarchical configuration with four bodies is the 'binary-binary' or '2+2' configuration, depicted in the right part of Fig. 1.1. In the latter configuration, two binary systems are orbiting each other. One might expect that these configurations of four bodies are very rare and are 'freaks of nature', if existent at all. This is not the case, however; $\approx$ 1 per cent of all solar-type stellar systems within 67 pc of the Sun are hierarchical quadruple systems (Tokovinin 2014a,b). About 2/3 of those appear in the '2+2' configuration, and 1/3 in the '3+1' configuration.

[1] An exception in the case of a three-body system (i.e. star-planet-asteroid) is the occurrence of two stable points for asteroids orbiting the star in the same orbit as the planet, but advanced or delayed in phase (i.e. the 4th and 5th Lagrange points). Notable examples are the Trojan and Greek asteroids that share their orbit with Jupiter.

[2] This term was first introduced by Evans (1968).

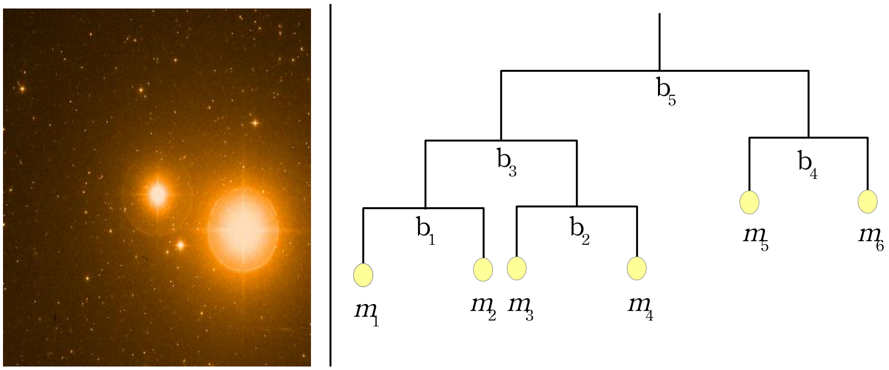


Figure 1.2: Left: image of Mizar and Alcor (image credit: ESO Online Digitized Sky Survey). Right: mobile diagram of the stellar sextuple system.

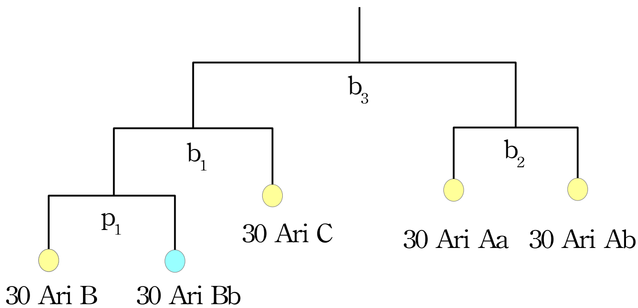


Figure 1.3: Mobile diagram of the stellar quadruple system 30 Ari with an additional planet, 30 Ari Bb.

Even more complicated stellar systems are known. For example, the famous visual binary Mizar and Alcor (Fig. 1.2, left) has been used as a vision test since antiquity (Allen 1899). Mizar itself is also a visual binary, known from as early as 1617, when Benedetto Castelli reported resolving it in a letter to Galileo Galilei (Ondra 2004; Siebert 2005). The components in Mizar, Mizar A and B, later turned out to be binaries themselves. In fact, Mizar A is the first spectroscopic binary known, found by Antonia Maury and reported by Pickering (1890). A few years later, the binary nature of Mizar B, also a spectroscopic binary, was discovered independently by Frost (1908) and Ludendorff (1908). Nearly a century later, Alcor was found to be a binary as well (Mamajek et al. 2010; Zimmerman et al. 2010). The mobile diagram of Mizar and Alcor, which is effectively a sextuple system, is shown in Fig. 1.2 (right). Another example is 30 Ari, which is a stellar quadruple system in the '2+2' configuration in which a planet has been discovered around one of the stars (Fig. 1.3).

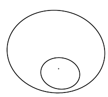
The hierarchies discussed so far are of the 'simplex' type composed of nested binaries, i.e. with each parent having only two children. Another type is the 'multiplex' type, in which each parent can have more than two children, and the children are not necessarily in hierarchical orbits with respect to each other. This latter means that crossing orbits and, therefore, close encounters, are possible.

Multiplex systems are observed in nature in addition to simplex systems. Most notably, in the Galactic Centre (GC), which is a distance of $\approx 8\,\mathrm{kpc}$ from the Sun, $\sim 10^6$ stars all orbit a supermassive black hole (SBH) of mass $\approx 4 \times 10^6\,\mathrm{M}_\odot$, at a distance ranging from only a few mpc ($\approx 206\,\mathrm{AU}$), to a few pc. These nuclear star clusters (NSCs) are known to exist in other galaxies as well; they may in fact be present in the centre of *every* galaxy.

Apart from dominating their motion on short time-scales, the SBH in an NSC can have dramatic effects on the lives of stars and other objects such as compact objects and planetesimals residing in such an environment. When passing within the tidal radius of the SBH, tidal forces can break up and disrupt a star, producing an energetic multiwavelength outburst known as a tidal disruption event, of which a handful have been observed so far.

Similarly, planetesimals with sufficiently large radius passing within the tidal radius of the SBH are separated into smaller fragments. These fragments, which are not bound by gravity but by molecular forces (and are therefore not broken up into even smaller pieces by the tidal force of the SBH), can subsequently be vaporised due to friction with the ambient gas around the SBH. This produces a potentially observable short-duration flare (Zubovas et al. 2012). The latter flares have been proposed to explain the near infrared and X-ray flares observed on a nearly daily basis from the SBH in the GC, Sgr A*.

More tightly bound compact objects orbiting the SBH such as white dwarfs, neutron stars and stellar-mass black holes are not tidally disrupted; however, the gradual emission of GWs can cause the compact object to spiral into the SBH. This process can take thousands of orbits to complete, producing a high signal-to-noise GW signal that is expected to be observable by future space-based GW detectors.



1.4 Secular evolution

1.4.1 Principles

As mentioned before, by necessity, hierarchical systems are stable on short times-scales, i.e. time-scales on the order of the orbital periods. On time-scales much longer than the orbital periods, the orbits can be interpreted as infinitesimally thin wires with a mass density that is proportional to the time spent at the corresponding point in the orbit. In the simplest case of a circular orbit (i.e. zero eccentricity), this implies a circle with uniform mass density. As the eccentricity of the orbit is increased, the time spent at apocentre increases relative to the time spent at pericentre, therefore the mass density is higher at apocentre.

These wires interact gravitationally and exert mutual torques on each other, thus transferring angular momentum between them. On the other hand, the orbital energies (i.e. the semimajor axes) remain constant. This implies that the eccentricities and the orientations of the orbits change. This type of long-term gravitational interaction is generally referred to as secular evolution. Depending on the type of system, secular evolution can have large implications. In the most extreme cases, eccentricities can be driven to nearly unity, e.g. leading to collisions or mergers of stars, planets or compact objects.

The simplest, and best-studied hierarchical system in which secular evolution can take place is the hierarchical three-body system (cf. the left part of Fig. 1.1, with m_4 removed). In this system, secular evolution is manifested in the form of Lidov-Kozai (LK) oscillations, named after its discoverers Lidov (Lidov 1962) and Kozai (Kozai 1962)[3]. If the system is highly hierarchical, i.e. if the orbital period of the 'outer' orbit, $P_{\mathrm{orb,out}}$, is much ($\gtrsim 10^2$ times) longer than the 'inner' orbital period, $P_{\mathrm{orb,in}}$, and if the outer orbit is not highly eccentric, then LK oscillations are highly regular. Furthermore, if the initial mutual inclination between the orbits, i_{tot}, lies within a certain range, $39.2^\circ \lesssim i_{\mathrm{tot}} \lesssim 140.8^\circ$, then the eccentricity of the inner orbit oscillates, reaching a maximum value that depends mainly on i_{tot}[4]. In this limit, the eccentricity of the outer orbit is unaffected. This can be understood intuitively by noting that the angular momentum of the outer orbit is much larger than that of the inner orbit; therefore, angular momentum exchanges have a much larger effect on the inner orbit. Assuming the inner orbit is initially circular, the maximum eccentricity reached in the inner orbit during the LK cycles is

$$e_{\mathrm{max,in}} = \sqrt{1 - \frac{5}{3}\cos^2(i_{\mathrm{tot}})}. \tag{1.2}$$

Equation (1.2) shows that $e_{\mathrm{max,in}}$ increases as i_{tot} approaches 90°. In other words, the larger the initial mutual inclination, the larger the maximum inner orbit eccentricity.

The eccentricity oscillations, as well as the oscillations of the orbital orientations, occur on a time-scale of the order of P_{LK}, where

$$P_{\mathrm{LK}} = \frac{P^2_{\mathrm{orb,out}}}{P_{\mathrm{orb,in}}} \frac{m_1 + m_2 + m_3}{m_3} \left(1 - e^2_{\mathrm{out}}\right)^{3/2}. \tag{1.3}$$

[3] Until a few years ago, LK oscillations were almost invariably referred to in the literature as 'Kozai' cycles, without reference to Lidov. Lidov was, in fact, the first to publish his results – in Russian – in 1961; the English version appeared in 1962. In 1962, Lidov presented his results at a conference that Kozai also attended. Kozai, likely guided by Lidov's work, published a paper on the secular evolution of asteroids around Jupiter later that year in 1962.

[4] Strictly, this is only for initially circulating orbits with zero initial eccentricity. In other cases, eccentricity oscillations are possible for lower inclinations.

Here, e_{out} is the eccentricity of the outer orbit, m_1 and m_2 are the masses of the bodies in the inner orbit, and m_3 is the mass of the tertiary body in the outer orbit. The precise period of the oscillations also depends on the initial orientation of the orbit; the latter dependence, which is much weaker compared to that of the other quantities, is not included in equation (1.3). Equation (1.3) shows that secular evolution takes place on a time-scale that is typically much longer than the outer orbital period. In addition to the orbital periods, the mass ratio and the outer orbit eccentricity are important factors in the period of the LK oscillations.

The above results apply only to highly hierarchical systems in which the tertiary body never closely approaches the inner binary (i.e. $P_{\text{orb,out}} \gg P_{\text{orb,in}}$ and $e_{\text{out}} \ll 1$). In this case, the system can be described by the lowest-order terms in the expansion of the Hamiltonian in terms of the small separation ratio $r_{\text{in}}/r_{\text{out}}$, known as the 'quadrupole' order[5]. At this order, and making another assumption relating to angular momentum conservation which is valid if $P_{\text{out}} \gg P_{\text{in}}$, the LK oscillations can be described analytically (Kinoshita & Nakai 1999, 2007). The properties equations (1.2) and (1.3) follow from these analytical solutions.

However, if the triple system is not extremely, but still moderately hierarchical ($10^1 \gtrsim P_{\text{orb,out}}/P_{\text{orb,in}} \gtrsim 10^2$), then LK oscillations are more complex, and they are typically no longer amenable to analytical solutions. In particular, the next-order terms in the expansion of the Hamiltonian, known as the 'octupole-order' terms, give rise to modulations of the quadrupole-order oscillations, on a time-scale that is given approximately by $P_{\text{LK}}/\epsilon_{\text{oct}}$, where the 'octupole parameter' $\epsilon_{\text{oct}} < 1$ is given by (Lithwick & Naoz 2011; Katz et al. 2011; Naoz et al. 2013a; Teyssandier et al. 2013; Li et al. 2014b)

$$\epsilon_{\text{oct}} = \frac{|m_1 - m_2|}{m_1 + m_2} \left[\frac{m_1 + m_2}{m_1 + m_2 + m_3} \left(\frac{P_{\text{in}}}{P_{\text{out}}} \right)^3 \right]^{1/2} \frac{e_{\text{out}}}{1 - e_{\text{out}}^2}. \tag{1.4}$$

When ϵ_{oct} is large, typically $\epsilon_{\text{oct}} \gtrsim 10^{-3}$, the octupole-order terms are important, and this can give rise to 'flips' of the mutual orbital planes from prograde to retrograde, and vice versa. These orbital flips are associated with extremely high eccentricities in the inner orbit, in some cases as high as $1 - e_{\text{in}} \sim 1 - 10^{-7}$. In stellar triple systems with compact objects, these extreme eccentricities can lead to violent head-on collisions of carbon-oxygen (CO) white dwarfs (WDs) (Katz & Dong 2012). Such collisions are likely to result in a type Ia supernova (SNe Ia) explosion because of temperatures reaching $\sim 10^9$ K during the collision, high enough to trigger a thermonuclear runaway explosion. Although certainly spectacular, current estimates of the event rates are very low, i.e. $\sim 10^{-3}$ of the SNe Ia rate computed from binary population synthesis studies (assuming mergers of CO WDs in circular orbits after common-envelope evolution) (Hamers et al. 2013). The latter are about a factor 10 lower compared to observations.

Even when eccentricities are not high enough for direct collisions, in many cases they can be high enough to trigger strong orbital energy dissipation, notably due to tides raised on bodies, or due to the emission of GWs. Both processes are highly sensitive to the orbital eccentricity, and the dissipation rate peaks at the maximum eccentricities of the LK cycle. Either quickly or gradually, the orbit shrinks (i.e. $P_{\text{orb,in}}$ decreases) and is circularised, and the oscillations are quenched.

This quenching phenomenon can be understood by noting that as the inner orbital period decreases, the LK time-scale increases (cf. equation 1.3), whereas the precession rate of the

[5]Note: the term 'quadrupole' is easily confused with 'quadruple'.

inner orbit due to additional forces increases. Notable additional forces arise from GR and tidal bulges. Their associated precession rates increase with decreasing $P_{\mathrm{orb,in}}$. With an increasing LK time-scale and a decreasing precession time-scale due to additional forces, LK cycles are typically damped, or completely quenched.

1.4.2 Implications

This process of LK cycles combined with orbital energy dissipation has important astrophysical implications. In stellar triple systems, it is believed that LK cycles with tidal friction (LKCTF) drive the production of short-period binaries (orbital periods $\sim 3-6$ d) (Fabrycky & Tremaine 2007; Naoz & Fabrycky 2014). Such short-period binaries are believed not to have formed in isolation; in the case of LKCTF, the initial period can be as long as 10^4 d. This is consistent with observations of the period distribution of binaries, which shows a peak around $\sim 3-6$ d (Tokovinin et al. 2006; Tokovinin 2014a), and with the tertiary fraction of binaries, which is nearly unity for binaries with periods shorter than 3 d (Tokovinin et al. 2006).

Another important implication relates to the origin of hot Jupiter (HJ) planets, i.e. gas giant planets of order Jupiter mass that orbit their host star in extremely tight orbits, with periods between $\sim 3-10$ d. It is commonly thought that these planets cannot have formed in situ because of the low gas densities and high temperatures during the protoplanetary disk phase in this region so close to the star (however, e.g. recently Batygin et al. 2015 suggested that in situ formation due to core accretion is possible). Instead, they might have formed at more distant regions from the star, i.e. beyond the 'snow line' where volatiles condense into solid ice grains. The precise location of the snow line is uncertain and depends on various parameters (including the disk age), but it is typically a few AU from the host star (i.e. an orbital period of a few 100 days). This implies that the planet must have migrated by two orders of magnitude in orbital period.

Two main migration scenarios have been proposed: disk migration and high-eccentricity (or high-e) migration. In the case of disk migration, the planet migrates because of gas drag within the protoplanetary disk. There are a number of concerns with this type of migration. Firstly, theoretical models for disk migration do not predict a preferred orbital period at which migration stops. Observations, however, show a pile-up of HJs at periods near 5 d. Furthermore, the observed obliquities of HJs, i.e. the angles between the stellar spin and the orbit of the HJ, are found to be both consistent with zero, and ranging between 0 and 180 degrees, i.e. retrograde orbits are also possible. Such high-obliquity orbits are at odds with disk migration models, in which the obliquity remains close to, or exactly zero.

An alternative migration scenario for the formation of HJs is high-e migration. In this case, the eccentricity of the orbit of the planet is driven to high values by one or more external perturbers. The high eccentricities lead to strong tidal evolution and hence orbital energy dissipation and circularisation, eventually leading to a HJ. The eccentricity excitation could be due to planet-planet scattering interactions, or due to secular interactions induced by a stellar binary companion or other (inclined) planets. In the case of secular interactions, a pile-up is predicted, around 3 days, because of the stalling mechanism mentioned above. This is (approximately) consistent with observations.

However, there are problems with these secular high-e scenarios. Evidently, a third body (in addition to the star and the Jupiter-like planet) is required. If the third body is another Jupiter-like planet, then the former needs to be either in a very close, eccentric and/or inclined orbit with respect to the inner planet. This is inconsistent with observations of the majority of HJs, which show that HJs do not have any close-in planetary companions. Furthermore, high mutual planetary inclinations seem unlikely if the star is isolated (this could be different if there is a more distant binary companion which could have torqued and hence warped the protoplanetary disk). If the third body is a star (i.e. binary companion), then it can be further away and a high initial mutual inclination with respect to the (inner) planet seems more plausible. However, such binary companions are observed for only ~ 0.5 of the observed HJs (Ngo et al. 2015).

In an alternative scenario proposed by Wu & Lithwick (2011), the planetary system consists of three or more planets in orbits that are mildly eccentric and inclined (eccentricities on the order of $0.2-0.4$ and inclinations on the order of $5-20^\circ$). The planets need not be very closely separated; their semimajor axes could e.g. be 1, 6 and 16 AU. In this case, chaotic motion is likely to occur. Very high eccentricities can be excited, particularly in the innermost orbit, during highly irregular secular oscillations. The time-scale for reaching high eccentricities can be as long as a few Gyr. This process of 'secular chaos' is likely very important for determining the long-term evolution of planetary systems. For example, in the Solar system, the orbit of Mercury is unstable due to secular chaos on a time-scale of $\sim\ 5\,\mathrm{Gyr}$ relative to the current Solar system (Laskar 1994; Ito & Tanikawa 2002; Laskar 2008; Laskar & Gastineau 2009).

Secular chaos is an interesting scenario for producing HJs on time-scales of Gyr, without requiring close-in planets. This is currently consistent with observations, which cannot yet detect planets at these larger separations with good certainty. Although promising, this scenario of high-e migration merits further investigation, in particular with consideration of a larger range of parameter space compared to the work of Wu & Lithwick (2011), who considered a limited number of systems. In particular, it is unclear what range of HJs properties can be attained.

1.5 This thesis

This thesis deals with the dynamical evolution of hierarchical systems with various hierarchies, in various astrophysical contexts. We begin by considering systems of the 'multiplex' type, dominated by an SBH (Chapters 2 and 3). Subsequently, we focus on simplex-type systems, beginning with the general dynamics of quadruple systems (Chapter 4), and continuing with an application to circumbinary planets (Chapter 5). Finally, we generalise our methods to hierarchical simplex-type systems with an *arbitrary* hierarchical structure and an *arbitrary* number of bodies (Chapter 6), and apply these methods to the production of HJs through secular evolution in multiplanet systems (Chapter 7).

1.5.1 Chapter 2 – Relativistic dynamics around an SBH

In Chapter 2, we consider systems of the 'multiplex' type, and study the long-term evolution of orbits around an SBH in a NSC, with particular emphasis on Sgr A*, the SBH in the GC. About 20 B-type stars, known as the S-stars, have been observed to orbit around a single point in the GC. Two decades of observations have revealed their detailed orbital properties, and most importantly, the mass of the central object. Several studies have shown that this central object, beyond any reasonable doubt, should be an SBH with a mass of $\sim\ 4\times10^6\ \mathrm{M}_\odot$ (see Alexander 2005 and Genzel et al. 2010, and references therein).

As mentioned before, orbits around an SBH, when highly eccentric, can give rise to tidal disruption events, or high signal-to-noise GW signals in the case of the inspiral of a compact object onto the SBH. Here, we refer to both events simply as 'disruptions'. Objects on orbits that are initially highly eccentric can be brought close to the SBH and be disrupted already during the first orbit; the corresponding orbits are referred to the 'loss cone'. Because the orbital periods around the SBH are at most a few Myr (in the case of the GC), objects on initially loss cone orbits are rapidly depleted, and do not contribute to the long-term disruption rate (i.e. on Gyr time-scales).

However, because of gravitational interactions between objects on nearby orbits, orbits near the SBH are not static, and change due to various effects. First of all, random encounters, or 'fly-bys' of two (or more) objects change all orbital properties (the energy, angular momentum and orientation). When taking into account many of such encounters, the effect is to stochastically alter the orbital properties. Because these encounters are essentially random, this effect is referred to as 'non-resonant relaxation' (NRR). However, in the case of a NSC, objects move on (nearly) Keplerian orbits, and some encounters are, therefore, correlated. This gives rise to secular interactions, which are similar to LK cycles in the case of the hierarchical three-body problem. However, in this case, for a given 'subject' orbit, there are many 'inner' and 'outer' orbits, and the secular effects are much more complicated and irregular. Nevertheless, orbits can still be driven to high eccentricities, potentially leading to disruption events. This type of evolution is known as 'resonant relaxation' (RR).

Close to the SBH, where orbital periods are short and close encounters are very rapid (i.e. the velocity dispersion is large), RR is very effective at changing the orbital properties and dominates compared to NRR. Taking only into account the Newtonian effects of NRR and RR, objects on these close orbits would rapidly evolve to loss cone orbits. However, close to the SBH where orbital speeds v can reach a few per cent of the speed of light c, GR effects are important, and these affect the dynamics. To lowest order in v/c, the effect of GR is apsidal precession of the orbit; a familiar example is the relativistic precession of the orbit of Mercury around the Sun. In the case of a NSC, however, relativistic precession rates can be much higher compared to Mercury. Consequently, the 'efficiency' of the Newtonian torques associated with RR can be reduced substantially, thereby dampening, or completely mitigating the effects of RR.

This effect was first shown by Merritt et al. (2011), who carried out detailed N-body simulations of a small cluster of 50 stellar-mass black holes, each with a mass of $50\,\mathrm{M}_\odot$, orbiting an SBH with a mass of $10^6\,\mathrm{M}_\odot$. In their simulations, Merritt et al. (2011) noticed that as an orbit becomes more eccentric (due to RR), the relativistic precession rate increases, until at a certain semimajor axis and eccentricity, the orbit 'bounces' back to lower eccentricity. Because the relativistic precession is also known as Schwarzschild precession, Merritt et al. (2011) coined this phenomenon the 'Schwarzschild barrier' (SB). Equating the time-scales associated with Schwarzschild precession and RR, Merritt et al. (2011) obtained an expression for the semimajor axis and eccentricity associated with the SB, i.e. the 'location' of the SB.

Direct N-body simulations scale with N^2. Combined with the fact that the typical required time-step is very short (fractions of orbits which can be as short as a few yr, whereas the desired simulation time is on the order of a few Myr, or even Gyr to achieve a steady state), this means that further work was limited by the computational complexity of detailed simulations. In particular, realistic representations of the centres of NSCs require on the order of 10^3 stars, which is not feasible with currently available direct N-body codes.

In order to make progress, we developed a new special-purpose N-body code, TEST PARTICLE

INTEGRATOR (TPI). By making a number of simplifying assumptions about the motion of particles, this code allows efficient integration with a large number of particles around the SBH (order 10^3). It is based on a splitting of particles into field particles that move on (nearly) fixed Keplerian orbits around the SBH, and test particles that respond to the time-dependent potential of the field stars and the SBH. In this approach, interactions between field particles are ignored apart from effects associated with mass precession and relativistic precession. Furthermore, the mass of test particles is neglected, and, therefore, test particle-test particle interactions are ignored as well. Nevertheless, by comparing with slower, but more accurate N-body codes that do not make these assumptions, we show that TPI can be used to simulate the long-term angular momentum evolution of orbits around the SBH.

We apply TPI to the S-stars in the GC. We assume that the S-stars were formed through binary disruption. In this scenario, first proposed by Hills (Hills 1988), a stellar binary approaches the SBH from a wide but highly eccentric, or a hyperbolic orbit. Due to the tidal force of the SBH, the binary is unbound, and one of the stars is ejected and escapes from the SBH, whereas the other remains bound to the SBH in a tight and highly eccentric orbit. The escaped star is believed to give rise to hypervelocity stars (HVSs), a group of stars observed in the Milky Way with velocities on the order of 10^3 kms^{-1}. The majority of HVSs are consistent with originating from the GC, and their velocities (taking into account deceleration in the Galactic potential) are consistent with the escape speed of the SBH. Furthermore, their number is consistent with the number of S-stars (Perets et al. 2007).

The eccentricities of the orbits of the stars that remain bound to the SBH are expected to be very high, typically $e \sim 0.98$. However, this is inconsistent with the current observed eccentricity distribution, which shows that the typical eccentricity is high (~ 0.7), but not as high as ~ 0.98. To explain the statistically-lower observed eccentricities, we assume that the orbits of the S-stars, in particular their eccentricities, are perturbed by gravitational interactions with a background cluster of stellar-mass black holes. In TPI, the latter cluster is represented by 4800 field particles, ≈ 100 times larger compared to the simulations of Merritt et al. (2011), and predicted by previous studied of stellar populations in the GC (Hopman & Alexander 2006b).

In our simulations, the eccentricity distribution of the S-stars indeed evolves from a highly eccentric distribution corresponding to the initial assumed formation through binary disruption, to a less eccentric distribution which is statistically consistent with the observed eccentricity distribution of the S-stars. This gives indirect evidence for both the existence of the cluster of stellar-mass black holes, and the formation mechanism of the S-stars. In particular, when assuming an alternative background distribution of lower-mass main-sequence stars, the time-scale for the eccentricity distribution of the S-stars to relax to the observed distribution is much longer, and is inconsistent with the age of the S-stars.

As a next step, we determine specific coefficients from the simulations that describe diffusion in angular momentum. These diffusion coefficients are very useful in Fokker-Planck simulations, which can be used to describe the statistical dynamical evolution on very long time-scales (exceeding Gyr), i.e. time-scales that are currently inaccessible with direct N-body integrations. Furthermore, the coefficients yield analytical insight into the long-term orbital evolution. As a function of the angular momentum variable $\ell = \sqrt{1-e^2}$, we find that the SB can be associated with a rapid drop of the coefficients with decreasing ℓ. The value of ℓ for which this occurs, turns out to be approximately consistent with the previous expression for the SB derived by Merritt et al. (2011).

Finally, we give simple analytic expressions for the diffusion coefficients as a function of

ℓ, and we apply them to the Fokker-Planck equation to obtain the steady-state distribution.

1.5.2 Chapter 3 – Planetesimals in the GC

In the previous work, we considered the long-term dynamical evolution of orbits around an SBH, and applied our results to the S-stars. In this chapter, we focus on another type of bodies in the GC, i.e. planetesimals, and relate these to observations of flares observed from Sgr A* in the near infrared and X-ray (Baganoff et al. 2001, 2003; Genzel et al. 2003a; Dodds-Eden et al. 2011; Barrière et al. 2014). These flares occur approximately once per day, and are 3-100 times more luminous than the quiescent emission of the central radio source, Sgr A*.

One of the proposed explanations for the flares is that they result from the tidal disruption of planetesimals with radius $\gtrsim 10\,\mathrm{km}$ (Zubovas et al. 2012). The latter authors showed that if such a planetesimal passes within ~ 1 AU of the SBH, it is broken up into smaller fragments by tidal forces; the fragments subsequently vapourize because of friction with the ambient gas. When the vapourized material is mixed with the accretion flow onto Sgr A*, enough energy could be released to produce an observable flare.

However, very little is known about the formation and evolution of planetesimals in galactic nuclei like the GC. One possibility is that they are formed in a large-scale spherical cloud orbiting the SBH. Another possibility is that they are born in debris discs around stars, and are stripped by the tidal force of the SBH or gravitational encounters with other stars. In this chapter, we investigate both scenarios by means of numerical integrations of the Fokker-Planck equation. We model the orbital energy evolution around the SBH, taking into account the effects of gravitational perturbations from various background perturbers.

We show that the predicted present-day disruption rates in the GC differ very little between the two scenarios and that this conclusion depends weakly on the details of the perturbers or other assumptions. In both scenarios, we find a disruption rate of $\sim 1\,\mathrm{d}^{-1}$ assuming that the number of planetesimals per (late-type) star is $N_{\mathrm{a}/\star} = 2 \times 10^7$. The number $N_{\mathrm{a}/\star} = 2 \times 10^7$ is consistent with debris discs observed around stars in the Solar neighbourhood. In the first scenario, in which the planetesimals are formed in a large cloud, this implies that the number of bodies formed is strongly correlated with the number of stars, and this requires finetuning of the quantity $N_{\mathrm{a}/\star}$. We favour the more natural explanation that planetesimals in galactic nuclei similar to the GC are formed in debris discs around stars, no differently than planetesimals around stars in the Solar neighbourhood.

1.5.3 Chapter 4 – Secular evolution of hierarchical quadruple systems

The first two chapters dealt with hierarchical systems of the multiplex type. In the remainder of this thesis, we shall focus on hierarchical systems of the simplex type, i.e. systems consisting of nested binary orbits. Each binary has two children, and the children can be either bodies or binaries themselves (see e.g. Fig 1.1). The simplest of these, the hierarchical triple, has been studied in detail before. However, this is not the case for hierarchical systems with more than three bodies.

Here, we begin with the simplest possible extension and consider four bodies in the '3+1' configuration, i.e. a hierarchical triple orbited by a fourth body. We assume that the system is sufficiently hierarchical, i.e. the ratios x_i of the binary separations are small. In that case, it is appropriate to expand the Hamiltonian in terms of the x_i. Subsequently, we orbit average the expanded Hamiltonian, and implement the equations of motion into a computer code, SECULARQUADRUPLE.

Subsequently, we study the secular evolution of highly hierarchical systems that are well described by the lowest order terms in the Hamiltonian, and characterise the evolution in terms of ratios of LK time-scales applied to different binary pairs.

1.5.4 Chapter 5 – Explaining the lack of circumbinary planets around short-period binaries

As mentioned before, planets have been observed around stellar binaries (i.e. circumbinary planets). The detection of transiting circumbinary planets is more tractable around short-period binaries, i.e. binaries with periods less than ~ 7 d. However, sofar, no such binaries have been found. As also mentioned before, short-period main sequence binaries have been suggested to form in triple systems, through a combination of LK cycles and tidal friction, i.e. LKCTF. Here, we apply the method developed in Chapter 4, and we show that coplanar circumbinary transiting planets are unlikely to exist around short-period binaries, due to secular evolution. This is constistent with the currently observed lack of transiting circumbinary planets around short-period binaries.

1.5.5 Chapter 6 – Secular evolution of hierarchical multiple systems

This chapter generalises the method and the algorithm developed in the earlier Chapter 4. Here, we derive the expanded Hamiltonian for systems consisting of nested binary (i.e. simplex) orbits, with an *arbitrary* number of bodies, and an *arbitrary* hierarchical configuration. This includes hierarchical triple and quadruple systems (in both the '2+2' and '3+1' configurations), but also much more complicated systems. For example, it applies to multistar systems with multiplanet subsystems around any of the stars. We develop an algorithm to implement the generised method, making long-term integrations of hierarchical systems with such complex hierarchies feasible, and present first applications. In particular, we apply our method to Mizar and Alcor and to 30 Ari.

1.5.6 Chapter 7 – Hot Jupiters in multiplanet systems

In the above, we mentioned a scenario for forming HJs in multiplanet systems through secular eccentricity excitation in multiplanet systems. Current studies of this variant of high-e migration have, until recently, been limited to a number of N-body integrations. Although accurate, such simulations are computationally costly, making large parameter space exploration unfeasible. Here, we apply the algorithm developed in Chapter 6 to study the formation of HJs in multiplanet systems with at least three planets. In particular, we investigate the dependence of the final orbital period distribution of the HJs, and compare this distribution to observations.