Hierarchical systems

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Hierarchical Systems

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Continuing in the family line of physicists

The endeavor to understand is the first and only basis of virtue.
—Baruch Spinoza

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