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## **Solving multiplication and division problems: latent variable modeling of students' solution strategies and performance**

Fagginger Auer, M.F.

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