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Solving multiplication and division problems: latent variable modeling of students' solution strategies and performance

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Latent variable modeling of students' solution strategies and performance

Fagginger Auer, Marije F.
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Latent variable modeling of students' solution strategies and performance

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