Observations of the eclipse of the Sun on June 29, 1927

Observatory At Leiden,

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COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

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made at *the Observatory at Leiden.*

1. In planning the observations of the eclipse of the sun of June 29 last, which was partial at Leiden, ·926 of the diameter being covered in the maximum phase, the principal object was to test visual and photographic methods of determining the relative positions of the moon and the sun by observing the position angles of the cusps. Screens were attached by brass brackets to the eye ends of different telescopes, and on these pieces of polar-coordinate paper were fixed. On these papers are printed in brown ink concentric circles at distances of 1 mm., and radii at every two degrees, every tenth circle and every fifth radius being somewhat heavier. Circles were drawn in indian ink equal to, or a fraction of a millimeter larger than the solar image, and every tenth degree was marked outside these circles. It was expected that the position angles of the cusps could in this way be determined with a probable error not exceeding a degree. We may mention here at once the result of the discussion, viz: that the accidental error is very much smaller indeed, being less than a tenth of a degree on the average of all observers, but there may be systematic errors of several tenths of a degree, different for different observers. It was realised from the beginning that the centring of the solar image on the coordinate paper is very important. Since always at least 180° of the limb of the sun is visible, it was expected that this could be effected with sufficient accuracy. The rates of the driving clocks were so adjusted that the telescopes followed the sun, instead of the stars. At the beginning of the eclipse, however, when the zenith distance of the sun was over 80°, the change of refraction made frequent readjustment of the centring necessary. The direction of the parallel was determined by stopping the driving clock and letting a sunspot run over the screen, noting its position angle at the points where it crossed the successive concentric circles, or marking its successive positions on the screen in pencil. The directions thus determined were, of course, corrected for refraction.

The observations were repeatedly interrupted by clouds, especially the central phase which was the most important for our purpose, was entirely lost.

2. The tabular angles, with which the observations have been compared, were computed as follows.

The right ascensions and declinations of the sun and of the moon were taken from the Nautical Almanac, and corrected for parallax by the formulas given in NEWCOMB's *Spherical Astronomy*, using the equatorial horizontal parallax given in the Nautical Almanac. The following constant corrections were derived by extrapolation from the Greenwich results, and were applied

For the sun

$$\Delta\alpha = +0^s\cdot12 \qquad \Delta\delta = -0''\cdot3$$

For the moon

$$\Delta\alpha = +0^s\cdot45 \qquad \Delta\delta = -0''\cdot7.$$

From the differences $\Delta\alpha\cos\delta$ and $\Delta\delta$ (in the sense *Moon—Sun*) we compute the angle ψ and the distance s by

$$s\cos\psi = \Delta\delta$$
$$s\sin\psi = \Delta\alpha\cos\delta$$

Then calling the radii of the sun and the moon R and R' respectively and putting

$$\sigma = \frac{s}{R}, \qquad \alpha = \frac{R'^2 - R^2}{R^2},$$

we have

$$\cos\chi = \frac{1}{2}\sigma - \frac{1}{2}\frac{\alpha}{\sigma}.$$

Then 2χ is the angle at the centre of the sun between the two cusps. The position angles of the cusps from the centre of the sun are thus

$$\varphi_1 = \psi - \chi$$
$$\varphi_2 = \psi + \chi.$$

Now introducing the unknowns

$$x = d\,(\Delta\alpha \cos\delta)$$
$$y = d\,(\Delta\delta)$$
$$z = dR/R$$
$$u = d\,(R' - R)/R\,,$$

we have

$$\frac{\partial\psi}{\partial x} = \frac{57{\cdot}3}{s}\cos\psi \qquad \frac{\partial\psi}{\partial y} = -\frac{57{\cdot}3}{s}\sin\psi$$

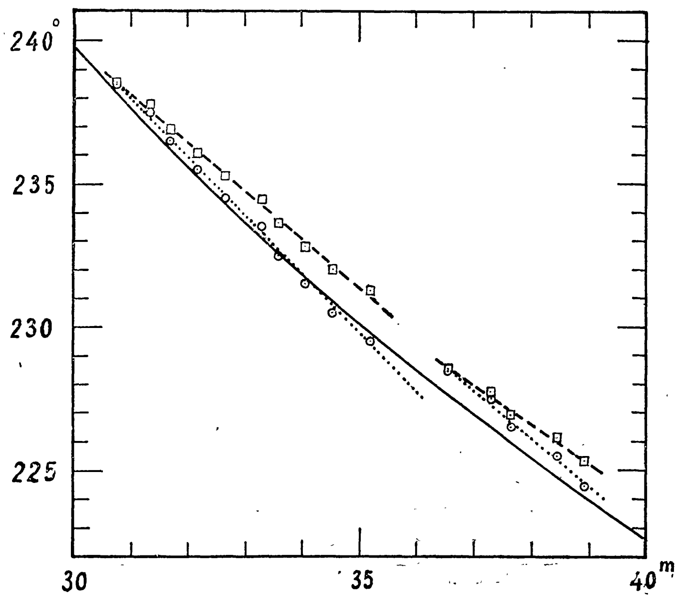
$$\frac{\partial\chi}{\partial x} = -\frac{57{\cdot}3}{2R}\cdot\frac{\sin\psi}{\sin\chi}\left(1+\frac{\alpha}{\sigma^2}\right) \qquad \frac{\partial\chi}{\partial y} = -\frac{57{\cdot}3}{2R}\cdot\frac{\cos\psi}{\sin\chi}\left(1+\frac{\alpha}{\sigma^2}\right)$$

$$\frac{\partial\chi}{\partial z} = 57{\cdot}3\,\frac{\sigma}{2\sin\chi} \qquad \frac{\partial\chi}{\partial u} = \frac{57{\cdot}3}{\sigma\sin\chi}\left(1+\frac{1}{2}\alpha\right)$$

In these formulas x, y and s are supposed to be expressed in seconds of arc, ψ and χ in degrees.

The method of observing has been different for the different observers.

3. *Observations at the 10-inch telescope.* Messrs. A. DE SITTER and W. DE SITTER took turns in observing the angles on the screen, and Mr. HINS read the chronometer. The diameter of the image of the sun on the screen was 216 mm. The observer followed the cusp, and at the moment when it crossed a division of the paper (even degrees) or the estimated middle point between two divisions (odd degrees), he gave a signal, consisting of a sharp knock with a pencil against the wooden observing chair. Mr. HINS then read the nearest half second from the chronometer and recorded the degree as called out by the observer. The two cusps were always observed alternately, so that the number of observations of each is the same, and the mean epoch is also the same. At the phases when the motion of the cusps was slow, observations were made not only of whole degrees, but also of estimated tenths. The driving clock of this telescope is very good, so that when once the solar image was centred on the screen, the centring remained perfect, except for the effect of refraction and equatorial adjustment. On the other hand the slow motion had a very large backlash, making it practically impossible to correct the centring continuously. The image was therefore carefully centred and then left to itself for a few minutes, generally five of six, after which time the centring was corrected again. From the observations during these intervals a linear formula was derived, by means of which the whole series was contracted to one normal place. For the angle to be compared with the ephemeris the value derived by this formula for the *beginning* of the series was chosen. The coefficients of the equations of condition were, however, taken for the middle of the series. In the cases where there is a noticeable curvature in the curve representing the observed angle as a function of the time, the differences $O-C$ were made for each observation, and from these a linear formula was derived. At the beginning of the eclipse, as has already been remarked, the centre of the sun is very soon carried away from the centre of the screen by the rapid change of the refraction. The accompanying figure gives the first two series of observations of the angle φ_1. The times given are Greenwich mean civil time. The zenith distance changed from 82°·1 to 80°·0 during the course of these two series. The circles give the observed angles. The squares represent the same angles corrected for the effect of the change of refraction. The full line gives the computed angles, the dotted



lines are the linear formulas derived from the observed angles, and the broken lines are what these would become if the correction for refraction were applied.

Table 1 gives the observed angles. The times are given in Greenwich mean civil time; t_1 is the beginning of the series, for which the angles φ_1 and φ_2 are interpolated from the observations; t_0 is the mean of

TABLE 1. OBSERVATIONS AT 10-INCH TELESCOPE.

t_1	t_0	Obsr.	φ_1	Nr.	φ_2	Nr.	$O-C$ φ_1	$O-C$ φ_2
h m	m		°		°		°	°
4 30·5	32·9	W	239·1	10	292·5	8	+ 0·36	+ 0·41
36·5	37·6	W	228·3	5	305·1	5	+ ·65	+ ·46
57·0	61·2	A	206·2	5	335·8	7	[+ 1·41	− 1·03]
5 31·0	33·2	A	342·0	10	140·4	9	+ ·57	+ ·50
37·0	39·4	A	354·8	10	140·7	9	+ ·41	+ ·31
43·5	46·5	W	5·4	12	137·2	11	+ ·58	+ ·15
52·0	54·6	A	17·5	10	130·5	10	+ ·59	+ ·04

The third series was made through clouds, and must be rejected.

the series, for which the coefficients are computed. The number of observations on which each normal place depends is also given. The last columms give the differences *O—C*. The observers A. DE SITTER and W. DE SITTER are referred to by A and W respectively. The chronometer was read by Dr. HINS.

The determinations of the parallel are given in Table 2. *K* is the correction to be applied to the observed angles. The second determination was rejected, and the mean — 1°·50 was adopted. It is already included in the angles as given above.

TABLE 2. PARALLEL, 10-INCH TELESCOPE.

Obsr.	*t*	*K*	Remarks.
	h m	°	
W. DE S.	4 25	— 1·50	
W. DE S.	40	— 1·29	through cloud
W. DE S.	6 0	— 1·52	
A. DE S.	»	— 1·51	
HINS	6 20	— 1·48	

4. *6-inch Equatorial.* The observations were made by Dr. BROUWER and Mr. SANDERS. The driving clock of this telescope is not very good, and continuous correction by the slow motions was necessary. The working of the slow motions was, fortunately, very smooth. The centring was thus continually corrected, and its effect may vary from one reading to the next, and is added to the accidental error. The diameter of the solar image was 180 mm. Mr. BROUWER adopted a kind of eye and ear method. He listened to the clock beating seconds, and at certain seconds made a reading of one of the angles φ_1 or φ_2, estimating tenths, and gave a signal; Mr. SANDERS then noted the second, and recorded the observed angles. The angles φ_1 and φ_2 were observed alternately. The readings thus made were combined to 26 means. The values of *O—C* were formed for these means, and these were then further combined to 10 normals. The observed angles are given in Table 3.

The determinations of the parallel are given in Table 4. The weights were assigned in accordance with the notes made by the observers at the time. The adopted mean —0°·75 is already included in the angles given in Table 3.

5. *The 4-inch guiding telescope of the Zeiss double camera* was used by Messrs. D. GAYKEMA and J. J. RAIMOND. The diameter of the image on the screen was 176 mm. Each of the observers confined his attention to one cusp, and by the signal key registered the time of its passage over the divisions on the screen (even degrees) and the estimated middle points (odd degrees) on a chronograph. Mr. GAYKEMA measured the angle φ_1 and Mr. RAIMOND φ_2. Both observers show a very marked personal error in placing the odd degree between the two even ones, both making the time of passage of the odd degree

TABLE 3. OBSERVATIONS AT 6-INCH TELESCOPE.

t_0	φ_1	Nr.	φ_2	Nr.	O—C φ_1	O—C φ_2
h m	°		°		°	°
4 40·6	222·1	5	311·2	4	—0·01	—0·86
43·5	218·0	6	314·5	4		
56·9	204·8	7	336·4	7	— ·01	— ·23
59·1	203·2	6	340·1	6		
5 1·3	201·9	5	343·4	4		
4·3	201·0	4	348·9	3	— ·39	+ ·01
6·3	200·3	4	353·6	6		
30·7	340·3	4	140·4	4		
32·1	344·1	5	140·8	4	— ·10	+ ·54
33·1	346·6	5	141·3	6		
34·7	349·9	4	141·0	4		
36·7	353·7	4	140·8	4	+ ·29	+ ·16
38·9	358·2	6	139·5	5		
40·1	0·2	6	139·2	6	+ ·40	+ ·25
41·3	2·1	6	138·8	6		
42·7	4·2	5	137·8	6		
44·0	6·1	5	136·8	5	+ ·51	+ ·29
45·3	7·8	6	136·2	5		
51·2	16·2	6	131·0	6		
52·9	18·6	5	129·5	5	+ ·55	+ ·32
54·8	21·4	5	127·4	5		
6 12·1	49·8	6	103·7	6		
14·1	54·7	6	99·4	6	+ 1·15	+ ·22
15·9	54·6	4	94·4	5		
16·9	63·2	3	90·9	3	+ 1.76	+ ·22
18·0	69·1	4	86·2	3		

TABLE 4. PARALLEL, 6-INCH TELESCOPE.

Obsr.	*t*	*K*	Weight
	h m	°	
BROUWER	5 40	—0·79	1
BROUWER	6 40	— ·77	1
BROUWER	»	— ·88	0
BROUWER	7 0	— ·69	1/2
SANDERS	»	— ·71	1/2
BROUWER	»	— ·75	1

TABLE 5. OBSERVATIONS AT 4-INCH FINDER.

t_0	φ_1	t_0	φ_2	t_0	O—C φ_1	O—C φ_2
h m	°	m	°	m	°	°
5 32·14	343·8					
32·97	345·8			33·1	—0·56	
33·93	347·8					
39·23	357·8					
40·47	359·8	39·70	139·5	40·9	—0·35	—0·09
41·67	1·8	42·05	137·8			
43·66	4·8					
45·66	7·8	45·02	135·8			
47·03	9·8	47·49	133·8	46·3	— ·11	— ·18
48·28	11·8					
49·80	13·8	49·80	131·8			
51·34	15·8	52·07	129·8	50·9	— ·13	— ·20
52·81	17·8					
54·16	19·8	54·14	127·8			
55·59	21·8	56·13	125·8	55·1	— ·10	— ·02
72·30	49·8			72·3	+ ·55	

later than the mean of the two even ones. For this reason, in forming means, double weight was given to the even degrees. Table 5 gives the mean of the registered times, reduced to Grw. mean civil time, of three successive angles, two even ones and the intermediate odd one.

The residuals $O-C$ were made for every individual observation, and combined to normal places as given in the table.

The determinations of the parallel are given in Table 6. The adopted mean $+0^\circ{\cdot}81$ is already included in the angles given in Table 5.

TABLE 6. PARALLEL, 4-INCH FINDER.

Obsr.	t	K
	h m	°
RAIMOND	4 20	+0·89
GAYKEMA	»	+ ·57
GAYKEMA	»	+ ·89
RAIMOND	7 10	+ ·74
RAIMOND	»	+ ·76
GAYKEMA	»	+ ·89
RAIMOND	»	+1·05

6. *Photographic observations.* Mr. VAN GENT took several exposures with the 13-inch photographic telescope, partly on slow plates, partly on photographic paper. The diameter of the solar image is 48·1 mm. A wire had been stretched in the telescope just before the plate, approximately in the direction of the parallel. The position angle of this wire was afterwards determined by trails of stars on two plates, one taken in the meridian and one at approximately the same hour angle as the sun during the eclipse. These gave $K = -0^\circ{\cdot}62$ and $-0^\circ{\cdot}60$ respectively. The mean $-0^\circ{\cdot}61$ was adopted. The plates and the papers were put in the Repsold machine, with the image of the stretched wire parallel to the axis of x, and the rectangular coordinates were measured of the two cusps and of the point where the shadow of the wire cuts the limb of the sun. From these three points the two angles ψ and χ were computed. There is, of course, the possibility of systematic error owing to the different nature of the measured points: the cusps on the one hand, and the indenture in the limb on the other hand. Such an error would probably affect the angle χ more than ψ. The observed angles, corrected for K, are given in Table 7. The plates are marked by P, the paper prints by p. Some of the latter had to be rejected, the measures being untrustworthy. The others were combined into means as given in the last column. The first mean, of a plate having the remark "cusp I badly measurable", and a paper, was treated as belonging to the papers.

TABLE 7. PHOTOGRAPHIC OBSERVATIONS.

t_0	Exp.	ψ	χ	$O-C$ ψ	$O-C$ χ
h m	s	°	°		
4 22·18	P 2	267·37:	47·27:	+0″·14	−0″·21
43·54	p 5	267·57	49·07		
59·98	P 2	272·35	69·37	+ ·15	− ·08
5 1·49	p 15	273·25	70·27	*Rejected*	
30·29	P 1	59·74	80·25	+ ·30	+ ·17
31·54	P 1	61·72	78·67	+ ·19	− ·06
33·53	P 1	64·30	76·78	+ ·13	+ ·16
35·03	P 1	65·78:	75·15:	+ ·01	− ·23
36·36	P 1	66·78:	74·05:		
36·27	p 20	69·37	70·63	*Rejected*	
40·26	p 20	71·05	70·43	*Rejected*	
45·92	P 1	71·78	63·35	− ·07	− ·06
47·00	P 1	72·32	62·02	+ ·10	− ·31
48·08	P 1	72·47	60·92	− ·07	− ·20
53·81	p 20	73·82	54·40	*Rejected*	
54·72	p 10	74·03	53·42		
55·56	p 10	74·25	52·35	+ ·02	+ ·07
56·65	p 10	74·33	51·05		
6 14·25	p 20	76·58	22·22		
15·33	p 30	76·40	18·95	+ ·12	− ·44
16·38	p 20	76·50	15·58		

7. *Discussions.* The first reductions were carried out by each observer of his own observations, the further reduction and discussion, beginning with the formation of the $(O-C)$'s already given, was undertaken by Dr. BROUWER, in consultation with the director of the observatory, who takes the responsibility for the final conclusions.

The division errors of the circles on the coordinate paper were determined, and were found to be as follows:

°	°	°	°	°	°
0	+0·10	120	−0·03	240	+0·01
30	− ·03	150	+ ·12	270	− ·02
60	− ·11	180	+ ·14	300	− ·07
90	+ ·02	210	+ ·09	330	+ ·11

It was not thought worth while to apply these corrections.

The effect of the irregularities in the moon's limb was computed from the data given by HAYN. The resulting corrections are smaller than a tenth of a degree, and might also safely have been ignored. They were, however, applied, and are already contained in the values of $O-C$ given above. These also contain the effect of refraction.

From the $O-C$ for φ_1 and φ_2 as given above, those for ψ and χ were derived, and equations of condition were formed for each mean.

The equations of condition are, for ψ:

$$a.\ 10^{-3}\,x + b.\ 10^{-3}\,y = O-C$$

and for χ:

$$a'.\ 10^{-3}\,x + b'.\ 10^{-3}\,y + c'z + d'u = O-C.$$

The equations for ψ contain only the unknowns

$x = d(\Delta\alpha \cos\delta)$ and $y = d(\Delta\delta)$. The coefficient of x is small at the beginning and end of the eclipse and large in the middle part, when the chord connecting the cusps swings round rapidly, ψ increasing by a degree in less than 10 seconds. This could, however, not be observed owing to clouds. The angle between the ecliptic, the moon's orbit and the equator being small at the date of the eclipse, the unknown x represents approximately the correction to the difference of longitude of the moon and the sun.

The equations for χ contain the four unknowns x, y, $z = dR/R$ and $u = d(R'—R)/R$. The coefficients of y are, however, so small that no solution of this quantity was attempted, but it was transferred to the right hand members. The angles χ are, of course, affected by imperfect centring of the solar image, the effect increasing with χ. It is smallest at the beginning and end of the eclipse, when the effect on ψ is at its maximum. Several solutions were made of the equations for χ. In solution I*a* all observations were used, in I*b* the photographic plates *P* and the visual observations at beginning and end (10-inch and 6-inch respectively) only. The results were:

Solution I*a*

$$x = -1''\!.5 \quad -\cdot 16\,y$$
$$z = -\cdot 0042 + \cdot 00000\,y$$
$$u = +\cdot 0012 + \cdot 00008\,y$$

Solution I*b*

$$x = -1''\!.8 \quad -\cdot 16\,y$$
$$z = -\cdot 0042 + \cdot 00000\,y$$
$$u = +\cdot 0010 + \cdot 00008\,y$$

The equations derived from the angles ψ require a large positive value of x. A solution based on all angles ψ gave

$$x = +3''\!.3 \qquad y = -0''\!.5,$$

and a solution based on those observed angles ψ, where $2\chi > 100°$, gave

$$x = +4''\!.0 \qquad y = +0''\!.2.$$

The weight of these results is very small, but they throw a doubt on the negative value of x derived from the χ's (the weight of which is also small). A solution was therefore made from the χ's of the unknowns z and u only, giving

Solution II

$$z = -\cdot 0036 + \cdot 00006\,y + \cdot 00038\,x$$
$$u = +\cdot 0013 + \cdot 00009\,y + \cdot 00007\,x$$

Finally a solution was made assuming no correction to the diameter of the moon, i. e. taking $u = -z$, and introducing a constant empircial correction c to all angles χ. The result, derived from the photographic plates and the observations at the 10-inch and 6-inch telescopes at the beginning and end of the eclipse, was:

Solution III

$$x = -0''\!.0$$
$$z = -u = -0\cdot 0033$$
$$c = +0°\!\cdot 29$$

Another set of solutions was made, based on seven normal places, formed as follows, the numbers given being those of the first column of Table 8.

Normal places.	t_0	Constituent observations 10-in.	6-in.	4-in.	*P*	*p*
	h m					
I	4 39	1, 2	1			1
II	5 1		2, 3		1	
III	32	4	4		2, 3, 4	
IV	38	5	5, 6	1	5	
V	46	6	7	2	6, 7	
VI	53	7	8	3, 4	8	2
VII	6 16		9, 10			3

The solutions for the ψ's quoted above were derived from these normal places. From the equations for the χ's we found

Solution IV.

$$x = -0''\!.4 \quad -\cdot 16\,y$$
$$z = -\cdot 0032 + \cdot 00001\,y$$
$$u = +\cdot 0007 + \cdot 00007\,y$$

The weight of x is again very small. Taking $y = x = 0$, we find

Solution V.

$$z = -\cdot 0033$$
$$u = +\cdot 0010$$

Finally a solution was made from the photographic observations alone, as well from the ψ's alone, the χ's alone and the ψ's and χ's combined. This last solution gave

Solution VI.

$$x = +0''\!.9 \qquad z = -\cdot 0028$$
$$y = -0\cdot 3 \qquad u = +\cdot 0004$$

All these solutions agree in giving a value of z of about $-0\cdot 0030$ and a value of u in the neighbourhood of $+0\cdot 0010$, the values of x and y being very uncertain, but probably small. Table 8 gives the coefficients of the equations of condition, the original right hand members $O—C$, and the residuals for χ left by solution II, which is practically identical to V. Those from the other solutions do not differ appreciably from these. The residuals for ψ are, of course, equal to the original $O—C$. The substitution of the values of x and y found from the various solutions would not alter these residuals to any essential amount. Table 9 gives the same for the normal places I to VII.

It will be seen that the residuals ψ are strongly systematic, especially so for the 10-inch and the 6-inch. In the latter case there is a strongly marked increase with the time. We find the following means

	Mean ψ	Mean χ O—C	Mean χ Res. II
	°	°	°
10-in.	+ 0·42	— 0·11	— 0·04
6-in.	+ 0·0130 t'	— ·20	— ·03
4-in.	— 0·15	+ ·02	+ ·07
P	+ ·09	— ·03	— ·06
p	+ ·09	— ·22	+ ·04
$(P+p)$	+ ·09	— ·08	— ·03

In the case of the 6-inch t' is the time counted in minutes from $5^h 15^m 0$.

In the case of the 10-inch there is no difference between the two observers. It is very difficult to explain these evident systematic errors. The fact that they are larger in ψ than in χ shows that they are nearly the same for the two angles φ_1 and ψ_2, though different for different instruments. An influence of defective centring could hardly be systematic. Moreover it would affect the χ's more than the ψ's, since only five out of the 31 means have $2\chi < 90°$. A possible explanation would be a difference in the equatorial adjustment of the telescope when driven by the clock and when unclamped, in consequence of which the position of the parallel, determined with the telescope unclamped, would not be the correct one to use in the reduction of the observations. It is, however, hardly credible that this effect could be so large, and still less so that it could vary so strongly with the time as does the value of ψ for the 6-inch telescope. This variation, at all events, is probably of the nature of a personal error, depending perhaps on the position angles and the direction of motion of the cusps.

If the systematic residuals are due to personal or instrumental errors, it is desirable to eliminate them. The following corrections were therefore applied to all observations:

10-inch:	$\delta\psi = -0°40$	$\delta\chi = +0°.10$
6-inch:	$\delta\psi = -0·0130\,t'$	$\delta\chi = +0·20$
4-inch:	$\delta\psi = +0·15$	$\delta\chi = 0$
P and p	$\delta\psi = -0·10$	$\delta\chi = +0.10$

The following solutions were then made from the normal places I—VII.

From all normal places we found:

From ψ's alone *Solution* VII*a*.	From χ's alone *Solution* VII*b*.
$x = +1''·2$	$x = +0''·7 \quad -·15\,y$
$y = +0·4.$	$z = -\ ·0027 + ·00001\,y$
	$u = +\ ·0020 + ·00007\,y$

From the ψ's and χ's combined we find:

$$x = +0''·7 \qquad z = -''·0027$$
$$y = +0·2 \qquad u = +·0021.$$

The last normal place is isolated, and is responsible for the greater part of the right hand members of the normal equations for x, z and u. Solutions were therefore made from the normal places I to VI only. These gave:

From ψ's alone *Solution* VIII*a*.	From χ's alone *Solution* VIII*b*.
$x = +0''·5$	$x = +0''·9 \quad +·03\,y$
$y = +0·4$	$z = -\ ·0030 - ·00007\,y$
	$u = +\ ·0021 + ·00012\,y.$

From χ's and ψ's combined:

Solution VIII*c*.

$$x = +1''·1 \qquad z = -·0031$$
$$y = +0·4 \qquad u = +·0022.$$

The result is the same as from all normal places. The values of z found from these solutions is practically the same as from the uncorrected observations, and also the value of u does not differ much. It comes nearer to the value $u = -z$, adopted in solution III. The solution VIII*c* was substitued, and the residuals were formed. The means were, for the separate instruments:

	Mean ψ	Mean χ
	°	°
10-inch:	0·00	— 0·05
6-inch:	— ·01	+ ·06
4-inch:	— ·01	+ ·01
$P+p$	+ ·01	— ·06

It is noticeable that, if these mean values of χ be added to those found before, we have

10-inch:	Mean $\chi = -0°15$
6-inch:	— ·14
$P+p$	— ·16,

or practically the same for all instruments, with the exception of the 4-inch finder. We apply to the residuals of the χ's of solution VIII*c* the corrections

10-inch:	$\delta\chi = +0°05$
6-inch:	— ·06
P and p	+ ·06,

and we correct the right hand members of the normal equations accordingly. We then find the following corrections to the solution VIII*c*, from the ψ's and χ's combined:

Normals I—VII *Solution* IX	Normals I—VI *Solution* X
$\delta x = -0.''15$	$\delta x = +0''.1$
$\delta y = -0·05$	$\delta y = +0·0$
$\delta z = +·00015$	$\delta z = -·0004$
$\delta u = +·0000$	$\delta u = +·0001.$

TABLE 8. EQUATIONS OF CONDITION AND RESIDUALS.

N°.	t_0	a	b	a'	b'	c'	d'	$O-C$ ψ	$O-C$ χ	Res. II χ	Final residuals ψ	Final residuals χ	φ_1	φ_2
								10-inch telescope						
	h m							°	°	°	°	°	°	°
1	4 32·9	−2	+36	+59	+4	+94	+65	+0·38	+0·02	+0·28	−0·03	+0·26	−0·29	+0.23
2	37·6	−2	+40	+47	+3	+67	+59	+·56	−·10	+·06	+·15	+·08	+·07	+·23
3	5 1·2	+6	+90	+33	0	+21	+90	[+·19	−1·22]	[−1·26]	[−·24	−1·23]	[+·99	−1.47]
4	33·2	+58	−114	−28	−15	+14	+124	+·54	−·04	−·15	+·13	−·10	+·23	+·03
5	39·4	+32	−81	−30	−12	+21	+88	+·36	−·05	−·08	−·04	·00	−·04	−·04
6	46·5	+20	−62	−33	−11	+32	+69	+·36	−·22	−·20	−·04	−·10	+·06	−·14
7	54·6	+14	−49	−36	−10	+43	+60	+·32	−·28	−·19	−·08	−·10	+·02	−·18
								6-inch telescope						
1	4 42·0	−2	+45	+42	+2	+54	+58	−·44	−·43	−·31	−·02	−·29	+·27	−·29
2	58·0	+2	+78	+34	0	+24	+80	−·12	−·11	−·09	+·07	−·09	+·16	−·02
3	5 4·0	+12	+108	+32	−3	+18	+104	−·19	+·20	+·10	−·09	+·14	−·23	+·05
4	32·0	+65	−121	−28	−15	+14	+135	+·22	+·32	+·24	−·02	+·23	−·25	+·21
5	36·8	+42	−94	−30	−12	+19	+104	+·22	−·06	−·15	−·06	−·06	·00	−·12
6	40·7	+29	−77	−31	−12	+23	+83	+·33	−·08	−·09	·00	−·03	+·03	−·03
7	44·0	+24	−68	−32	−11	+28	+75	+·40	−·11	−·07	+·02	−·02	+·04	·00
8	53·0	+15	−51	−36	−11	+41	+60	+·44	−·11	−·02	−·05	+·05	−·10	·00
9	6 14·0	+8	−32	−81	−20	+146	+85	+·66	−·47	−·07	−·11	−·01	−·10	−·12
10	17·4	+7	−30	−158	−38	+302	+157	+·99	−·77	+·14	+·18	+·07	+·11	+·25
								4-inch finder						
	5 33·1	+57	−113	−28	−14	+15	+125						(−·23)	
1	40·9	+29	−76	−31	−11	+24	+82	−·23	+·12	+·13	−·08	+·04	−·12	−·04
2	46·3	+21	−62	−33	−11	+31	+69	−·14	−·03	+·04	+·01	−·06	+·07	−·05
3	50·9	+17	−54	−35	−11	+38	+62	−·17	−·04	+·06	−·02	−·03	+·01	−·05
4	55·1	+14	−48	−37	−11	+45	+59	−·06	+·04	+·13	+·09	+·08	+·01	+·17
	6 12·3	+8	−33	−67	−11	+115	+72						(+·46)	
								photographic plates						
1	5 0·0	+4	+84	+34	−2	+23	+86	+·15	−·08	−·09	+·02	−·06		
2	30·3	+80	−130	−27	−16	+12	+148	+·30	+·17	+·14	+·18	+·07		
3	31·5	+69	−124	−28	−15	+13	+138	+·19	−·06	−·03	+·07	−·14		
4	33·5	+54	−111	−29	−14	+15	+123	+·13	+·16	+·08	+·02	+·12		
5	35·7	+44	−98	−30	−13	+18	+109	+·01	+·23	+·14	−.10	+·23		
6	45·9	+21	−63	−33	−11	+31	+70	−·07	−·16	−·09	+·03	−·03		
7	47·0	+20	−60	−33	−11	+32	+68	+·10	−·31	−·22	·00	−·17		
8	48·1	+19	−58	−33	−11	+34	+66	−·07	−·20	−·11	+·03	−·05		
								photographic papers						
1	4 42·8	−2	+46	+40	+2	+52	+57	+·14	−·21	−·08	+·03	−·05		
2	5 55·6	+13	−47	−38	−11	+49	+58	+·02	+·07	+·18	−·08	+·20		
3	6 15·4	+8	−31	−93	−24	+172	+98	+·12	−·44	+·05	+·02	+·10		

TABLE 9. NORMAL PLACES.

Normal	a	b	a'	b'	c'	d'	$O-C$ ψ	$O-C$ χ	Res. V χ	Final Residuals ψ	Final Residuals χ
							°	°	°	°	°
I	−2	+42	+47	+3	+67	+60	+0·16	−0·14	−0·02	+0·03	0·00
II	+6	+92	+33	−2	+22	+90	−·05	+·05	−·01	·00	·00
III	+65	−120	−28	−15	+14	+134	+·28	+·08	+·02	+·05	−·03
IV	+35	−85	−30	−12	+21	+93	+·13	+·03	+·01	−·05	+·04
V	+21	−63	−33	−11	+30	+70	+·13	−·17	−·13	·00	−·08
VI	+15	−51	−36	−11	+42	+61	+·08	−·08	−·01	−·02	+·02
VII	+8	−31	−111	−27	+207	+113	+·60	−·56	+·01	+·03	−·06

It is not worth while to apply these corrections. As the final result we adopt

$$x = +0''\cdot8 \quad \pm \ 10'' \rho_0$$
$$y = +0\cdot3 \quad \pm \ 6 \ \rho_0$$
$$z = - \ \cdot0030 \pm \cdot008 \ \rho_0$$
$$u = + \ \cdot0022 \pm \cdot007 \ \rho_0.$$

For the probable errors I have adopted about the mean between those corresponding to the weights as found in the solutions VIII*c* and IX*c*, ρ_0 being the probable error of one normal place, expressed in degrees.

We find the following values for the average residuals of ψ and χ in the various solutions. The averages have also been taken for the photograhic observations alone, and also for φ_1 and φ_2 for the visual observations alone. It should be remembered that these should be $\sqrt{2}$ as large as those of ψ and χ, since $\psi = \frac{1}{2}(\varphi_2 + \varphi_1)$, $\chi = \frac{1}{2}(\varphi_2 - \varphi_1)$.

TABLE 10. AVERAGE RESIDUALS.

Sol.	Residuals ψ	Residuals χ	Visual Obsns φ_1	Visual Obsns φ_2	Photogr. Obsns ψ	Photogr. Obsns χ
O—C	± 0°27	± 0°19	± 0°45	± 0°27	± 0°12	± 0°19
II		± ·12				
V		± ·12	± ·36	± ·38		
VI	± ·27	± ·12			± ·10	± ·14
(*O—C*)*	± ·06	± ·16	± ·18	± ·18	± ·06	± ·17
Final	± ·06	± ·10	± ·11	± ·11	± .05	± ·11

*) *O—C* with the systematic corrections given on *p.* 76 only.

The averages for the separate instruments are, for the final solution:

	ψ	χ	φ_1	φ_2	n
10-inch:	± 0°08	± 0°12	± 0°13	± 0°17	9
6-inch:	± ·07	± ·11	± ·15	± ·12	13
4-inch:	± ·05	+ ·05	± ·05	± ·09	7,4

n is the average number of pointings combined to one mean.

These average residuals give rise to several remarks. Considering first the visual observations, we see that there is no difference in accuracy between the observed angles φ_1 and φ_2. The solutions without systematic corrections (V is practically the same as II) have the effect of equalising the averages for φ_1 and φ_2, without decreasing them. The residuals of these solutions are still, like the original *O—C*, due for the greater part to the systematic errors. The solutions in which x and y were taken equal to zero only affect the *difference* $2\chi = (\varphi_2 - \varphi_1)$. The application of the systematic corrections reduces the average residuals to one half, and the final solution brings a considerable further reduction. It is important to note that the result of the solutions with, and those without the systematic corrections is essentially the same. This increases our confidence in the reality of the values found for the unknowns, as wel as of the systematic corrections. Assuming this, then the fact that the residuals for ψ are so much smaller than those for χ shows that most of the accidental errors have opposite signs in φ_1 and φ_2. Since in the great majority of the observations the difference $\varphi_2 - \varphi_1$ was large (exceeding 120° in two thirds of the cases), this proves that a great part of the accidental errors is due to defective centring.

In the case of the photographic observations the influence of the systematic errors is smaller than for the visual observations. Here also the residuals for ψ are better than for χ, which must be ascribed to the circumstance that the angle ψ is derived from the measures of two similar objects, viz: the two cusps, while χ is found from the combination of these with the pointing on the intersection of the limb by the shadow of the thread, which is a very dissimilar object.

From the final residuals we would, taking the mean of ψ and χ, or that of φ_1 and φ_2 divided by $\sqrt{2}$, find for the probable error of one *O—C* ± 0°·07. We can take for ρ_0 one half of this, or ± 0°·035. But it appears safer to increase this to ± 0°·05.

8. *Conclusion.* It has already been pointed out that the values found for the unknowns are very little affected by the systematic corrections adopted. Introducing the value of ρ_0 found above, we have thus

$$x = +0''.8 \pm 0''.5$$
$$y = +0.3 \pm 0.3$$
$$z = -\ .0030 \pm .0004$$
$$u = +\ .0022 \pm .00035$$

The weights of x and y are disappointingly small, owing to the loss of the middle of the eclipse through clouds.

Applying these corrections to the adopted corrections to the right ascension and declination of the moon and sun, and the adopted semidiameters, (which are, for the sun: $R_0 = 959''.63$, and for the moon: sin $R'_0 = 0.272274$ sin $\pi_{☾}$, or, using BROWN's value of $\pi_{☾}$, $R'_0 = 931''.87$), we find:

Corrections to the differences of right ascension and declination of the moon and the sun as given in the *Nautical Almanac*:

$$\Delta\alpha_{☾} - \Delta\alpha_{\odot} = +0^s.39 \pm 0^s.04$$
$$\Delta\delta_{☾} - \Delta\delta_{\odot} = -0''.1 \pm 0''.3.$$

Semidiameters of sun and moon, in the projected image of the eclipsed sun on a screen, or a photographic plate:

$$R_{\odot} = 956''.75 \pm 0''.40$$
$$R_{☾} = 931.10 \pm 0.60.$$